ADAPTIVE ORDER-STATISTIC LMS FILTERS

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Abstract

The LMS-based adaptive order-statistic filters are presented in this paper. The adaptive Ll-filters as extension of the adaptive L-filter for two-dimensional filtering of noisy greyscale images is studied too. Their adaptation properties are studied by three types of noise, the additive white Gaussian noise, the impulsive noise or both, respectively. Moreover, the impulsive noise has the fixed noise value (Salt & Pepper noise). The problem of pixel value multiplicity and determination its position in the ordered input vector for adaptive Ll-filter is shown in this article. The two types of images with different of image complexity are used to demonstration of the power of time-spatial ordering.

Keywords

Order-statistics, adaptive LMS filtering, image filtering, L-filter, Ll-filter

1. Introduction

The adaptive signal processing has been significantly evolved almost in the past three decades. The designed filters have been applied in a several of problems such as the channel equalisation, echo cancellation in the telephone channels etc.

The widely family of the adaptive linear filters with finite impulse response (FIR) has been improved by ordered information of the observed signal and this class is called the adaptive LMS L-filters [1-3]. Furthermore, they employ the least mean square (LMS) adaptation algorithm to updating the weight coefficients and along with ones minimise the mean square error (MSE). On the other hand, the minor order statistic filters so-called adaptive LMS Ll-filters incorporate the marginal and the time-spatial order information [4-6]. For different types of noise and image complexity was determined the optimal adaptive LMS L or Ll-filter. A specific affinity to the linear order LMS filters represents a class of non-linear microstatistic or adaptive LMS Volterra filters [7-10].

The outline of this paper is as follows. Section 2 is devoted to the simple description of the adaptive unconstrained LMS L-filter and section 3 to the adaptive unconstrained LMS Ll-filter, respectively. Moreover, section 3 contains the two solutions of the determination of a pixel position for pixels with multiplicity value in magnitude. In the section 4 are involved our experiments and achieved results. The last section contains the conclusion, evaluation of results and future tasks that can have improve the filtration results.

2. The adaptive LMS L-filter

The observed vector $\mathbf{x}(n)$ can be expressed as the sum of two components, the original image $\mathbf{d}(n)$ and the noise $\boldsymbol{\eta}(n)$, respectively. The $N=(2\xi+1)^2$ defines the filter dimension and the noisy two-dimensional observation vector ordered in the lexicographic order is given by

$$\begin{aligned}\mathbf{x}(n) = \big(&x(k-\xi,l-\xi), x(k-\xi,l-\xi+1),\cdots \\ &\cdots, x(k-\xi,l+\xi),\cdots,x(k+\xi,l+\xi)\big)^T\end{aligned} \tag{1}$$

where n is the running index, which is used instead of the original pixel coordinates k and l.

Let $\mathbf{x}_r(n)$ is the ordered noisy input vector for *n-th* observation (e.g. in the ascending order) by next law

$$x_1(n) \le x_2(n) \le \cdots \le x_N(n) \tag{2}$$

and the final ordered input vector is given by

$$\mathbf{x}_r(n) = \big(x_1(n), x_2(n), \cdots, x_N(n)\big)^T . \tag{3}$$

For simplicity, the LMS unconstrained solution for the weight coefficients updating is used. Thus, the updating formula is written as follows

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu\varepsilon(n)\mathbf{x}_r(n) \ , \tag{4}$$

where $\varepsilon(n)$ is the estimation error at *n-th* pixel, i.e. $\varepsilon(n)=d(n)-y(n)$. The equation (4) determines the adaptive unconstrained LMS L-filter [3]. The output of this L-filter is defined for each processed pixel as follows

$$y(n) = \mathbf{w}^T \mathbf{x}_r(n) \ , \tag{5}$$

where $\mathbf{w}^T$ is the adapted coefficient vector and $\mathbf{x}_r(n)$ is the ordered input vector for *n-th* observation.

3. The adaptive LMS Ll-filter

Recently, as extension of the adaptive LMS L-filters were designed the adaptive LMS Ll-filters [4-6]. The main problem of adaptive L-filters is that they do not operate with time or spatial information. Furthermore, the Ll-filter class incorporates the time-spatial information along with order information in magnitude. The input vector for *n-th* observation is given by

$$\mathbf{z}(n) = \left(\zeta_1^T(\mathbf{x}(n)), \zeta_2^T(\mathbf{x}(n)), \cdots, \zeta_N^T(\mathbf{x}(n))\right)^T \tag{6}$$

The $\mathbf{z}(n)$ is (N^2 x 1) vector, where each (N x 1) vector $\zeta_i^{\mathrm{T}}(\mathbf{x})$ is defined by

$$\zeta_i^T(\mathbf{x}) = \left(x_{(i)1}, x_{(i)2}, \cdots, x_{(i)N},\right)^T \tag{7}$$

and its partial values are computed by next rule

$$x_{(i)j} = \begin{cases} x_{(i)}, & \text{if} \quad x_{(i)} = x_{(j)} \\ 0, & \text{otherwise} \end{cases}. \tag{8}$$

The vector $\mathbf{z}(n)$ includes both time and ordering information and the coefficient vector is given by

$$\begin{aligned} \mathbf{c} = \Big(c_{(1)1}, \cdots, c_{(1)N} \Big| c_{(2)1}, \cdots, c_{(2)N} \Big| \cdots \\ \cdots \Big| c_{(N)1}, \cdots, c_{(N)N} \Big)^T . \end{aligned} \tag{9}$$

The LMS updating algorithm for adaptive unconstrained LMS Ll-filter has almost identical form as the adaptive unconstrained LMS L-filter and this equitation is written as follows

$$\mathbf{c}(n+1) = \mathbf{c}(n) + 2\mu\varepsilon(n)\mathbf{z}(n). \tag{10}$$

The output of the adaptive Ll-filter is defined as follows

$$y(n) = \mathbf{c}^T \mathbf{z}(n), \tag{11}$$

where $\mathbf{c}^{\mathrm{T}}$ is the adapted coefficient vector and $\mathbf{z}(n)$ is the ordered input vector for *n-th* observation in time and value.

3.1 Determination of a pixel position

The determination of truth position for image pixels with same value in magnitude is solved by two methods.

A-solution

The first method so-called A is for a simple description explained by two vectors. One of them, the observed vector contains the image pixels with associated time-spatial indices *i*. Likewise, the ordered vector contains the image pixels ordered in magnitude, where their order is defined by a order index *j*. The determination rule (8) is extended by rule (12). To the first pixel of multiplicity value with time-spatial index *i* is assigned the order index *j* (*j-th* pixel in the ordered vector with equal magnitude as). Moreover, to the next pixel of the same multiplicity value is assigned the order index *j*+1.

$$\begin{aligned} &\text{if } \; x_{(i)} = x_{r(j)} \\ &\quad \text{then } \; z_{(i,j)} = x_{(i)} \\ &\qquad\quad x_{r(j)} = 0 \\ &\quad \text{break} \end{aligned} \tag{12}$$

This principle is shown in the Fig.1a, where from five pixels are only three displayed with equal magnitude. The arrows locate their index position in the ordered vector.

B-solution

The second method so-called B is based on the A-method along with detection of image pixels equal in magnitude and at the same time in time-spatial and ordering indices of the both processed vectors. The determination of pixel position in the ordered input vector $\mathbf{z}(n)$ is for these image pixels determined by

$$\begin{aligned} &\text{if } \; x_{(i)} = x_{r(j)} \; \text{ and } \; i = j \\ &\quad \text{then } \; z_{(i,j)} = x_{(i)} \end{aligned} . \tag{13}$$

The graphical principle of the B-solution is shown in the Fig.1b, where it is seen that for image pixel with equal time-spatial and ordering indices is reserved the position on the diagonal with *i,j* - indices. The positions of non-determined image pixels by (13) are obtained by rule (12). Moreover, the ordered input vector is finally achieved by row-row or column-column ordering of this matrix.

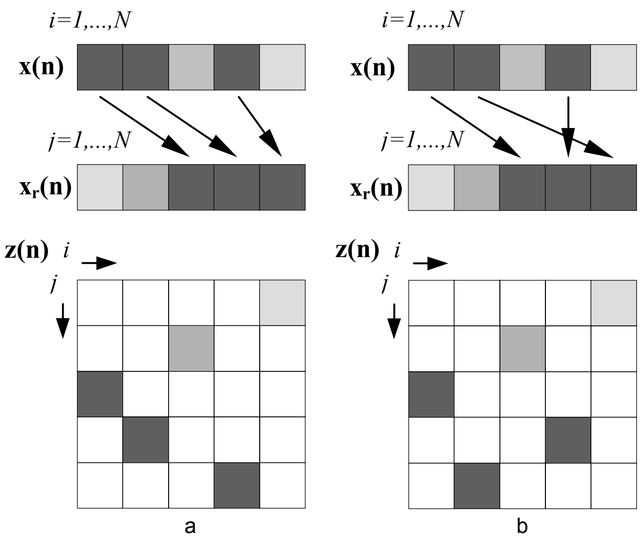

Fig. 1 Determination of a pixel position. (a) A-Solution. (b) B-Solution.

4. Experimental results

The experimental part is divided to the three sets of experiments. All sets of experiments were carried out on two types of image complexity, namely, the second frame of the Trevor sequence, so-called 2nd Trevor and image of the Bridge. Both images are shown in the Fig.2. The 2nd Trevor contains only a few image details; however on the other hand, one contains many homogeneous areas. The image of the Bridge in comparison with 2nd Trevor has opposite structure.

a b

Fig. 2 Original noise-free images. (a) 2nd Trevor. (b) Bridge.

To the determination of convergence properties of studied adaptive order-statistic LMS filters, the noise-free images were degraded by three types of noise.

First type of noise was the additive Gaussian white noise with standard deviation σ=20 (marked as G20). Moreover, as a second noise model was used the impulsive noise with the fixed noise value and with equal probability of black and white impulses p=20% (so-called Salt & Pepper or Black & White, marked as BW20). Finally, last used noise type was the mixed noise that consisted of both G20 and BW20 noises (marked as G20BW20). The evaluating criterions MAE and MSE were used for all sets of experiments [2-3].

First set of experiments was carried out on G20 noise and the results are presented in the Table 1. As it is seen, the adaptive Ll-filters offer better results as another used filters.

Method	2nd Trevor		Bridge	
	MAE	MSE	MAE	MSE
Noised	15.395	379.598	15.308	376.366
Median	7.402	91.261	11.269	227.178
3x3 LMS	6.849	75.296	9.897	161.940
3x3 L	6.936	80.439	11.557	233.080
3x3 $Ll_{(A)}$	6.761	74.521	**9.742**	**159.411**
3x3 $Ll_{(B)}$	**6.759**	**74.500**	9.745	159.451

Tab. 1 The filtration results of noisy 2nd Trevor and Bridge images corrupted by the Gaussian noise with standard deviation σ=20 (G20)

Between A and B solution is not more different in MAE or MSE criterions for the 2nd Trevor or Bridge images. The best results are marked in bold and outputs of these filters are shown in Fig.3. Moreover, the adaptive Ll-filters offer better results than adaptive L-filter. From point of view of image complexity, the B-solution of adaptive Ll-filter is preferable than A-solution and reversal. This figure contains the noisy images too.

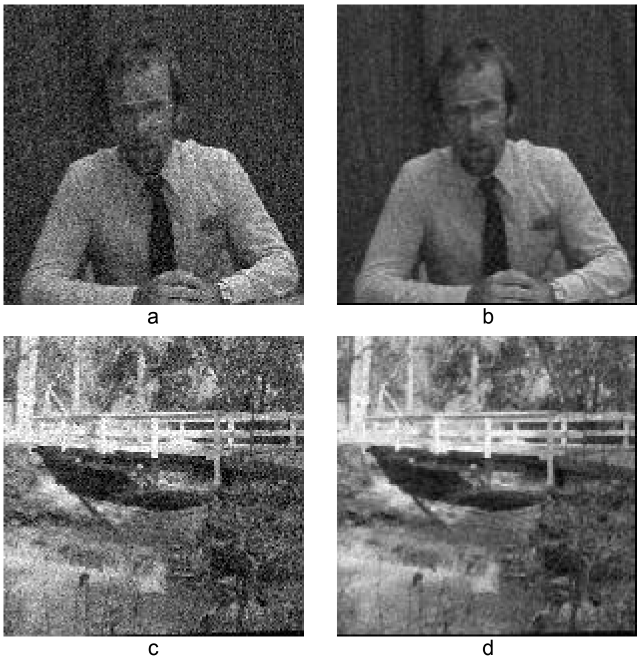

Fig. 3 The suppression of the Gaussian noise type. (a) The noisy 2nd Trevor image. (b) The Filtered 2nd Trevor image by the adaptive 3x3 LMS $Ll_{(B)}$-filter. (c) The noisy Bridge image. (d) The filtered Bridge image by the adaptive 3x3 LMS $Ll_{(A)}$-filter.

Second set of experiments was dedicated to filtration of images corrupted by BW20 noise. The filter results are presented in Table 2, and the best filter outputs in bold are shown in Fig.4.

Method	2nd Trevor		Bridge	
	MAE	MSE	MAE	MSE
Noised	22.839	3562.5	23.221	3517.2
Median	**3.637**	**55.255**	**8.896**	235.949
3x3 LMS	16.848	498.059	20.151	699.176
3x3 L	4.082	56.367	13.075	320.911
3x3 $Ll_{(A)}$	4.595	75.314	9.303	214.162
3x3 $Ll_{(B)}$	4.513	73.178	9.173	**207.592**

Tab. 2 The filtration results of noisy 2nd Trevor and Bridge images corrupted by the Salt & Pepper with probability p=20% (BW20)

The best result was obtained by 3x3 median filter, but for images with high image complexity (Bridge), the adaptive $Ll_{(B)}$-filter offers better improvement in MSE parameter than median filter. Furthermore, the adaptive L-filter in compare with modifications of adaptive Ll-filters is better in both MAE and MSE parameters only for images with low number of image details (2nd Trevor).

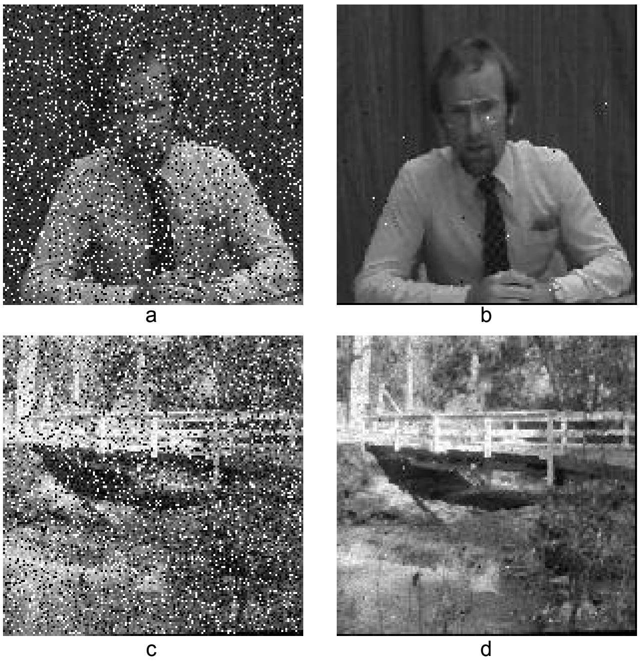

Fig. 4 The suppression of the Salt & Pepper noise type. (a) The noisy 2nd Trevor image. (b) The filtered 2nd Trevor image by the 3x3 median filter. (c) The noisy Bridge image. (d) The filtered Bridge image by the adaptive 3x3 LMS $Ll_{(B)}$-filter.

Finally, the next noise type G20BW20 was used in third set of experiments. The filter results are introduced in Tab. 3. It can be observed from this table that the best result has been achieved by adaptive L-filter for 2nd Trevor image, but for filtration of image of Bridge is advantage to use some modification of adaptive Ll-filters. The best filter outputs (in bold) are shown in Fig.5.

Method	2nd Trevor		Bridge	
	MAE	MSE	MAE	MSE
Noised	34.017	3551.8	34.345	3499.9
Median	9.194	157.218	13.327	343.178
3x3 LMS	17.063	501.731	21.280	762.010
3x3 L	**8.937**	**149.241**	15.062	406.731
3x3 $Ll_{(A)}$	9.455	173.259	13.108	327.888
3x3 $Ll_{(B)}$	9.448	173.187	**13.068**	**326.699**

Tab. 3 The filtration results of noisy 2nd Trevor and Bridge images corrupted by the mixed Gaussian and Salt & Pepper noise (G20BW20)

As reference filter from the linear filters was used the adaptive unconstrained LMS filter, and from non-linear filters the median filter. For all types of filters was applied the square window of 3x3 dimension. The adaptation parameter μ was chosen $\mu = 1x10^{-7}$. All test images have had size 256 by 256 pixels and the experimental results was obtained after $(256\text{-}2)^2$ iterations, what is a number of sliding filter windows over noisy image.

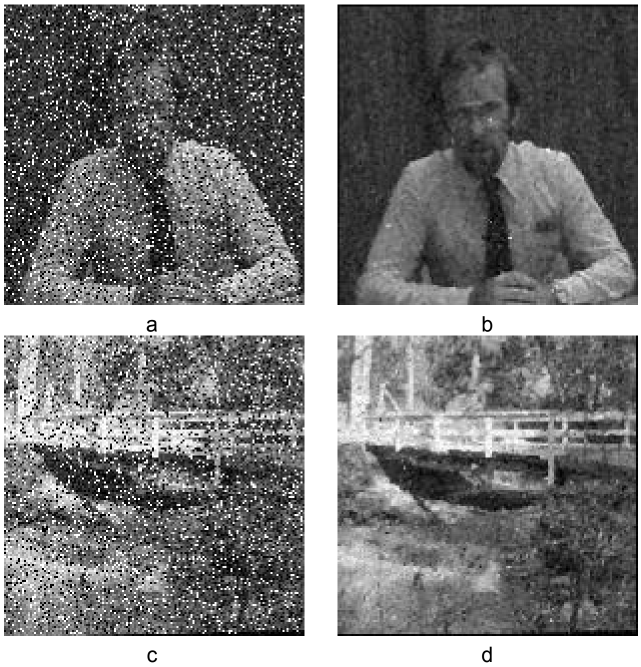

Fig. 5 The suppression of the mixed Gaussian and Salt & Pepper noise types. (a) The noisy 2ndTrevor image. (b) The filtered 2nd Trevor image by the adaptive 3x3 LMS L-filter. (c) The noisy Bridge image. (d) The filtered Bridge image by adaptive 3x3 LMS $Ll_{(B)}$-filter.

5. Conclusion

In this paper, the two adaptive order-statistic LMS filters were described [1,4-5]. Thus, the adaptive LMS L-filter and the adaptive LMS Ll-filter as the time-spatial extension of adaptive LMS L-filter, resp. Moreover, for the adaptive LMS Ll-filter, two methods of the determination of a pixel position in the ordered input vector for pixels with multiplicity in magnitude are studied. The adaptation properties of used adaptive filters were studied by three types of noise and two types of images with different image complexity. By experiments, it has been shown that the adaptive unconstrained LMS Ll-filter is better than adaptive unconstrained LMS L-filter or adaptive unconstrained LMS filter for additive Gaussian noise type. It has been found that by processing of more image complexity of images, the adaptive Ll-filter offer better filter results than other used filters. The more efficiency of adaptive $Ll_{(B)}$-filter considering to adaptive $Ll_{(A)}$-filter is observed by filtering of the impulsive noise with fixed noise value. Thus, for the modifications of adaptive LMS Ll-filters that employ the time-spatial information along with ordering information will be this influence more expressive for larger dimension of filter masks. Finally, the adaptive Ll-filters proved than its noise-suppression characteristics are able to reduce the mixed noise too. Moreover, their future modifications in the adaptation process can achieve better filter results than simple LMS algorithm.

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Róbert HUDEC was born in Revúca, Slovakia 1974. He graduated from the Technical University in Košice in 1998, and then he started Ph.D. study at Department of Electronics and Multimedia Communications, Faculty of Electrical Engineering and Informatics, Technical University in Košice. His work includes adaptive LMS filters and filtration of images corrupted by mixed noise for static images and satellite communication systems.

Stanislav MARCHEVSKÝ received the M. S. degree in electrical engineering from the Faculty of Electrical Engineering, Czech Technical University in Prague, in 1976 and Ph.D. degree in radioelectronics from Technical University in Košice in 1985. From 1987 he is the associate professor at the FEI TU in Košice. His research interest includes image processing, neural network and satellite communication systems.