# Application of Shape-Independent Orthogonal Transforms for Image Interpolation

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## Abstract

*In the contribution we develop a new method for object-oriented interpolation of images. It is an important tool in image processing, since using interpolation we can considerably decrease amount of data, necessary for image reconstruction. Application of this interpolation enables to down-sample the object separately. The selected object can be processed at the different sampling level. This approach allows object oriented zoom, e.g. Moreover, object - oriented approach is a novel idea that helps to understand the content of image. Method is created from cosine approximation implemented to coding with shape independent basis functions.*

## Keywords

Interpolation, orthogonal transforms, basis functions, threshold segmentation

## 1. Introduction

In spite of simplicity of a hardware realization and no requirements of prior information for block - based algorithms, there is an increasing pressure for abandoning, or at least adapting, a block based methods in image processing because of their disadvantages - blocking artefacts. However, very efficient hardware equipment for image processing enables utilization of more sophisticated algorithms, e.g. ones that incorporate an object-oriented approach.

Interpolation is a very efficient way to decrease amount of necessary data. In our approach we reduce the amount of data sequence $B_1 N_1 \times B_2 N_2$ to size $N_1 \times N_2$ and use separable 2D interpolation, as proposed in [1], [4].

We combine advantages of these two approaches – object - oriented description and interpolation. Image is segmented into regions by suitable homogeneity criterion. Regions are supposed to be quasi-stationary and to have uniform texture. Resulting description is composed of segment internal gray-level signal and image partition. This is reduced (for later interpolation), coded and transmitted. Coding of images in this way seems to us more natural and amiable to human psycho-visual perception.

In this paper we propose an appropriate interpolation scheme for arbitrarily shaped regions with internal texture structure. Texture structure of segment $A$ in rectangle $L$ is interpolated using shape - independent sine transform functions, defined on rectangle $L$.

The rectangle $L$ represents the rectangle circumscribing the image segment $A$. Thus texture representation is independent on the exact contour information.

Let us have a segment $A$ of 2D discrete image $L$. We transmit all $n_1$ $B_1$ pixels from $n_2$ $B_2$ rows, which is called separable interpolation - we can use the same algorithm for columns and for rows. The algorithm is as follows:

In 1D interpolation with orthogonal transform holds for sequence $x(m)$ created from $x(n)$, where we suppose the sequences elements are numbers representing the image gray level values [1] following formula:

From original sequence of length $N$

$$x(n) = \{x(0), x(1), \ldots, x(N-1)\} ,$$

we create the sequence $x(m)$ of length $M = B \times N$, where $B$ is the order of interpolation, or increasing factor of the sequence. Created sequence is

$$x(m) = \{x(0), {}^{1}0, {}^{2}0, \ldots {}^{B-1}0, x(1), {}^{1}0, \ldots, {}^{B-1}0, x(2), \ldots\} ,$$

or

$$x(m) = \begin{matrix} x(n) & \text{for } m = Bn \\ 0 & \text{other} \end{matrix} \tag{1}$$

Resulting interpolated sequence is

$$X(k) = \frac{1}{BN} \sum_{m=0}^{BN-1} x(m) u(m,k), \quad k = 0,1,\ldots BN-1 \tag{2}$$

where $u(m, k)$ is a separable orthogonal basis function set fulfilling conditions [1]. E.g., forward discrete sine transform is

$$X(k) = \frac{1}{BN} \sum_{m=0}^{BN-1} x(m) \sin\left[ (m+1)(k+1) \frac{\pi}{BN+1} \right] \tag{3}$$

## 2. Selection of suitable basis function

As a next step, we have to find the set of the best-suited (most convenient) basis functions. We will denote this set as follows:

$$u_{k_1,k_2}\left(B_1n_1, B_2n_2\right), \quad k_1 = 0,1,\ldots N_1 - 1, \quad k_2 = 0,1,\ldots N_2 - 1,$$
$$n_1 = 0,1,\ldots N_1 - 1, \quad n_2 = 0,1,\ldots N_2 - 1.$$

After each step of interpolation, we will compute the residuum

$$r^{(v)}\left(B_1n_1, B_2n_2\right) = f\left(B_1n_1, B_2n_2\right) - g^{(v)}\left(B_1n_1, B_2n_2\right), \quad \left(B_1n_1, B_2n_2\right) \in A \tag{4}$$
$$f\left(B_1n_1, B_2n_2\right) = x\left(B_1n_1, B_2n_2\right)w\left(B_1n_1, B_2n_2\right), \quad x \in L \Rightarrow f \in A$$

where $x$ denotes the down sampled image interpolated with zero - values as described in (1),

$$w\left(B_1n_1, B_2n_2\right) = \begin{matrix} 1 & \text{for} & \left(B_1 n_1, B_2 n_2\right) \in A \\ 0 & \text{for} & \left(B_1 n_1, B_2 n_2\right) \in L \setminus A \end{matrix} \tag{5}$$

Since the residuum is orthogonal to the selected basis function, it follows from

$$r^{(v+1)}\left(B_1n_1, B_2n_2\right) = r^{(v)}\left(B_1n_1, B_2n_2\right) - \Delta c \; u_{k_1,k_2}\left(B_1n_1, B_2n_2\right), \quad \left(B_1n_1, B_2n_2\right) \in A$$

that in each iteration the residual error energy cannot increase

$$\sum_{(B_1n_1, B_2n_2) \in A} \left[r^{(v+1)}\left(B_1n_1, B_2n_2\right)\right]^2 = \sum_{(B_1n_1, B_2n_2) \in A} \left[r^{(v)}\left(B_1n_1, B_2n_2\right)\right]^2 - \left[\Delta c\right]^2 \sum_{(B_1n_1, B_2n_2) \in A} u^2{}_{k_1,k_2}\left(B_1n_1, B_2n_2\right) \tag{6}$$

The convergence of the iteration is thus guaranteed if the additional basis function is selected such that the remaining error energy is definitely reduced in each step.

Considering (4), this claim is identical to the selection of that basis function which maximizes

$$\Delta E_A^{(v)} = \left[\Delta c\right]^2 \sum_{(B_1n_1, B_2n_2) \in A} u^2{}_{k_1,k_2}\left(B_1n_1, B_2n_2\right) \tag{7}$$

$$\Delta c = \frac{\sum\limits_{(B_1n_1, B_2n_2) \in A} r^{(v)}\left(B_1n_1, B_2n_2\right) u_{k_1,k_2}\left(B_1n_1, B_2n_2\right)}{\sum\limits_{(B_1n_1, B_2n_2) \in A} \left[u^2{}_{k_1,k_2}\left(B_1n_1, B_2n_2\right)\right]}$$

$$\Delta E_A^{(v)} = \frac{\left[\sum\limits_{(B_1n_1, B_2n_2) \in A} r^{(v)}\left(B_1n_1, B_2n_2\right) u_{k_1,k_2}\left(B_1n_1, B_2n_2\right)\right]^2}{\sum\limits_{(B_1n_1, B_2n_2) \in A} \left[u^2{}_{k_1,k_2}\left(B_1n_1, B_2n_2\right)\right]}$$

Thus, the basis function has to be selected for residuum approximation maximizing $\Delta E_A{}^{(v)}$ in each iteration. The selected coefficient $c_{k1,k2}$ of the interpolation function is now updated adding the just determined value $\Delta c$ [2].

In this case it is reasonable to start with

$$g^{(0)}\left(B_1n_1, B_2n_2\right) = \frac{\sum\limits_{(B_1n_1, B_2n_2) \in A} f\left(B_1n_1, B_2n_2\right)}{\sum\limits_{(B_1n_1, B_2n_2)} w\left(B_1n_1, B_2n_2\right)} \tag{8}$$

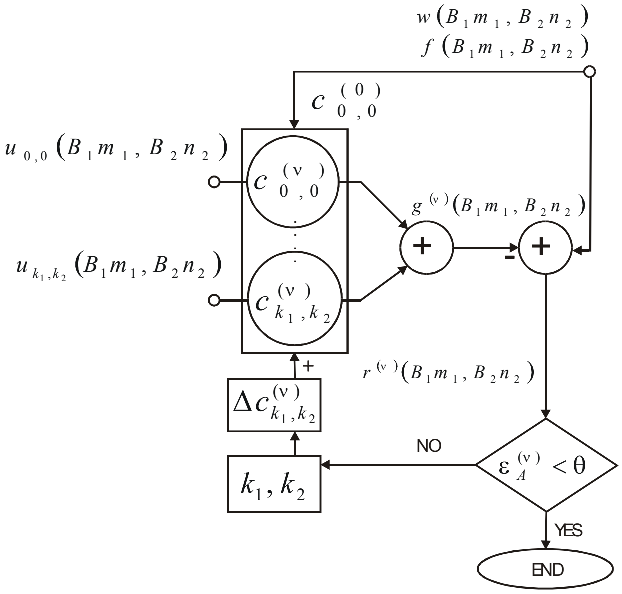



**Fig. 1** Diagram of interpolation procedure, $\varepsilon_A$ is MSE of residuum

## 3. Segmentation

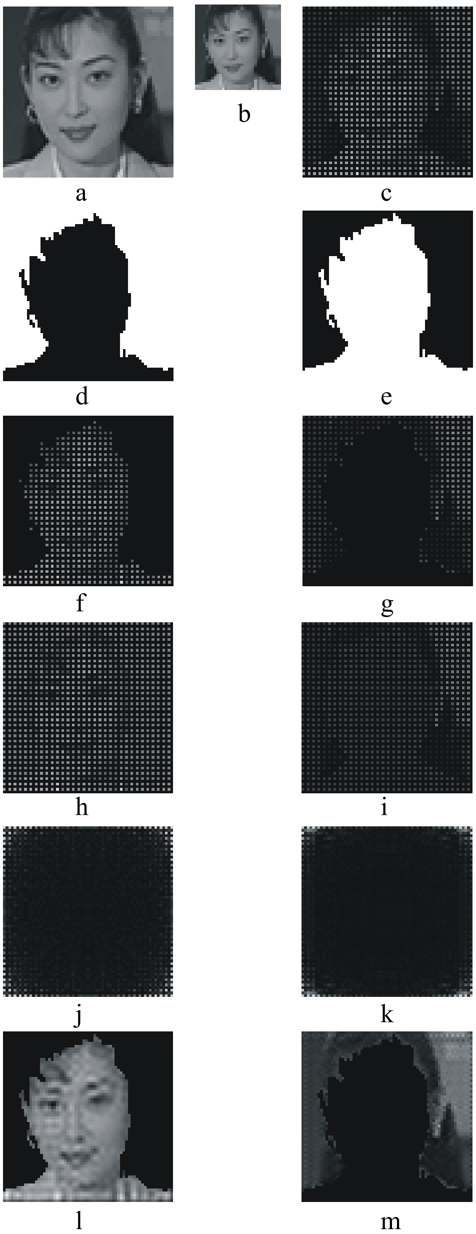
Much work has been reported in literature for extraction of object by segmentation. Using fuzzy set theoretic approach, some work on segmentation has been reported like fuzzy thresholding, fuzzy clustering and fuzzy edge detection.

To partition an image into the meaningful regions, we use different measures of fuzziness, such as index of fuzziness, index of non-fuzziness or entropy. The Shannon's entropy involves the measure of fuzziness of an image so that the threshold can be determined by minimizing the entropy measure. It is very different from the classical entropy measure, which measures probabilistic information.

To find the threshold for separating the object from the background we use the membership function of a pixel defined as an absolute difference between a gray level and the average gray level of its belonging region [3].

From the histogram solved average gray levels $\mu_1$ and $\mu_2$ are considered as target values of the background and the object for the given threshold value $t$.

The relationship between a pixel and its belonging region possess the property that the smaller the absolute difference between a grey level of a pixel and its correspon-



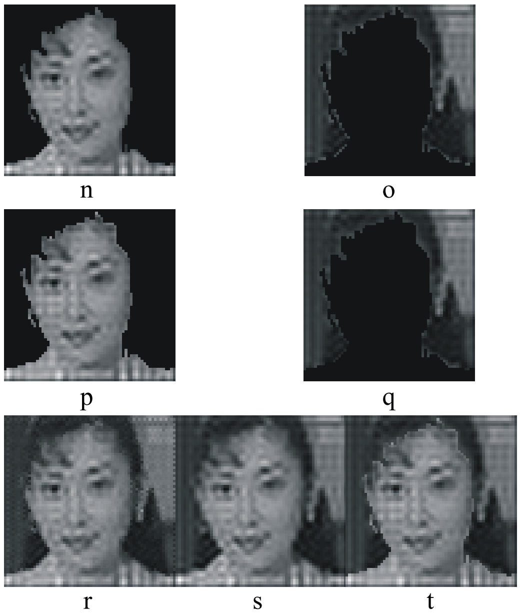




**Fig. 2** The results of 2nd order interpolation of 32×32 image to 64×64 using DST 1: a) original 64×64, b) sub-sampled 32×32 c) zero-inserted image b, d) object mask, e) background mask, f), g) zero-inserted masked, h), i) zero-inserted and masked with DC coefficients, j), k) spectra of h, I, l), m) objects interpolated – new approach, n), o) conventional interpolation [1], p), q) object oriented conventional interpolation, r) result of new approach, MSE 177, s) result of conventional interpolation [1], MSE 236, t) result of . object oriented conventional interpolation, MSE 238

ding target value is, the larger the membership value the pixel has:

$$\mu_x\left(x_{n_1,n_2}\right)=\begin{cases}\dfrac{1}{1+\left|x_{n_1,n_2}-\mu_1\right|/C} & \text{if } x_{n_1,n_2}\le t\\[2ex] \dfrac{1}{1+\left|x_{n_1,n_2}-\mu_2\right|/C} & \text{if } x_{n_1,n_2}> t\end{cases} \qquad (9)$$

where $C$ is a constant value such that $1/2 \le \mu(x_{n1,nz}) \le 1$. To increase the sensitivity of the criteria we modified $C$ for ackground ($C_1$) and for object ($C_2$) with respect to the bandwidths of gray levels belonging to object and background areas. As criteria function there was used the entropy measure based on Shannon's function.

# 4. Conclusion

Fig. 2 shows the results of 2nd order interpolation of the image of the size 32×32 to the size 64×64 using DST 1.

We can see from [1], that standard interpolation described by eqns. (1) – (3) is in most cases better than interpolation be mean, median, Legendre's polynomials and splines. For that reason we present only a comparison with standard interpolation by discrete orthogonal transforms.

The approach presented is novel and a deep analysis of interpolation results using objects of different character should be the next step of our work.

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