CHUA'S CIRCUIT IN SPREAD SPECTRUM COMMUNICATION SYSTEMS

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Abstract

Communication system via chaotic modulations is demonstrated. It contains the well-known chaotic circuits as its basic elements – Chua's circuits. The proposed system has some standard properties of spread spectrum communication. The following advantage is found in simulations: a) Transmitted signals have broad spectra. b) Secure communications are possible in the sense that the better parameter matching is required in order to recover the signal. c) The circuit structure of the communication system is most simple and communication systems are easily built at a small outlay. Finally computer simulations are given to examine the validity of this system.

Keywords

Chua's circuit, chaos, communication systems, synchronization

1. Introduction

Conventional narrowband communication systems have serious disadvantages. Namely, narrowband signals are sensitive to selective fading caused by multipath propagation and the high-transmitted power spectral density causes high levels of interference with other users. Disadvantages of narrowband systems mentioned above could be avoided by applying spread spectrum (SS) techniques, in which the spectrum of the information signal is spread over a wide bandwidth for transmission.

An alternative approach to making a transmission wideband is to represent the transmitted signals by inherently nonperiodic chaotic signals. Because chaos provides a pseudo-noise characteristic to its output patterns and can be reproduced with special synchronization, it has applicability as a spreading signal in spread spectrum communications. Chaos can be used in multiple ways in both analog and digital communications.

Over past ten years, four new chaos-based spreading techniques have been developed: chaotic masking, chaotic modulation, chaos shift keying and predictive Poincaré control modulation. First two techniques spread analog information data by chaotic signals, and the remaining spread binary information data. In chaotic masking, the analog information signal $s(t)$ is spread by adding it to the output $x(t)$ of chaotic system. Resulting signal $s(t)+x(t)$ is modulated and transmitted [1]. In chaotic modulation, the analog information signal $s(t)$ is injected into a chaotic circuit. This injection modifies the dynamics of the chaotic circuit, and so the information signal $s(t)$ is modulated [2]. Due to space limitations, we have not concern with spread binary information data in this article.

Chaos masking and chaos modulation can be used in the analog communications realm to provide signal spreading and low probability of detection. Chaotic signals are characterized by a wideband power spectrum, while in the time domain they appear "random", as shown in Fig. 7.

Dynamical systems can produce a number of different steady state behavior including DC, periodic (see Fig. 5a), and chaotic (see Fig. 5b) solutions [3]-[10]. Chaotic systems are characterized by "sensitive dependence on initial conditions"; a small perturbation eventually causes a large change in the state of the system. Equivalently, chaotic signals decorrelate rapidly with themselves.
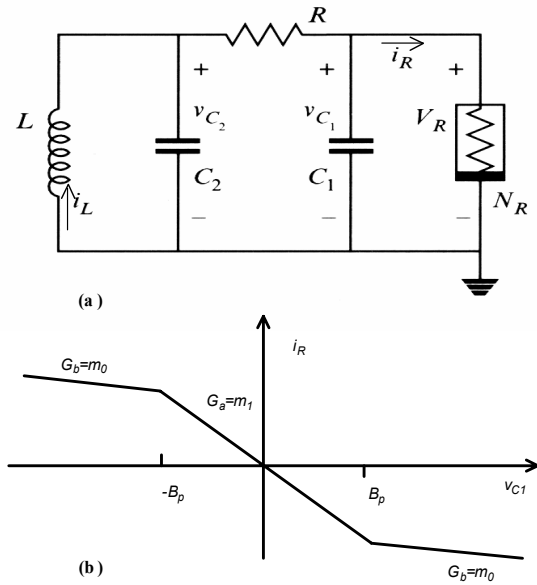
From the telecommunication point of view the important property of chaotic signals is that they are nonperiodic wideband signals that can be generated by very simple circuitry. This means that continuous-time chaotic systems can be used to generate wideband carriers for chaos-based analog and digital communication systems. The main idea in article is to use the chaotic modulation to transmit the informational signals and to use chaotic synchronization mechanism to recover the signals.

In section 2 the Chua's circuit is described and its behavior analyzed. In section 3, this circuit used for the secure communication is shown. In section 4, in order to examine the validity of this scheme, some computer simulation results and performance analysis is given. Finally, effects of parameter mismatch and channel noise are discussed.

2. Chua's Circuit

The main reason for using chaotic signals for secure communications is because they are asymptotically stable. Chua's circuit (see Fig. 1) is the simple and robust circuit, which exhibits the complex dynamics of bifurcation and chaos.

The circuit consists of linear elements (a resistor, an inductor, two capacitors) and a nonlinear element (called

Fig. 1 (a) Chua's circuit (b) V-A characteristic of the nonlinear element (Chua's Diode).

Chua's Diode). The Chua's diode has been fabricated as a microelectronic IC chip [11]. The state equations of Chua's circuit are given by:
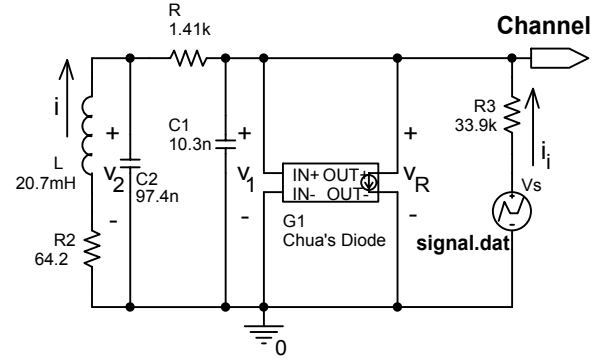
$$C_1 \frac{dv_{C1}}{dt} = \frac{1}{R}(v_{C2} - v_{C1}) - i_R(v_{C1})$$

$$C_2 \frac{dv_{C2}}{dt} = \frac{1}{R}(v_{C1} - v_{C2}) + i_L \qquad (1)$$

$$L \frac{di_L}{dt} = -v_{C2}$$

where v_{C1}, v_{C2} and i_L are voltage across the capacitor C_1, the voltage across the capacitor C_2, and the current through the inductor L, respectively and $i_R(v_{C1})$ is the current versus voltage characteristic of the nonlinear element shown in Fig. 1b. Parameters used are $C_1 = 10.3$ nF, $C_2 = 97.4$ nF, $L = 20.7$ mH, $R = 1.41$ kΩ, $B_p = 1.85$ V, $G_a = -0.87$ mS, $G_b = -0.52$ mS, and $r = 64.2$ Ω. Then, (1) has a double scroll attractor.

3. Communication System

The system used for the secure communication is shown in Fig. 2 (transmitter) and in Fig. 3 (receiver). A Chua's circuit (see Fig. 1) is used for each the transmitter and the receiver. A signal $i_i(t)$ is given as input to Chua's circuit in the transmitter. The voltage $v_1(t)$ is the output of the transmitter, which is used by the receiver to get $i_d(t)$.

The current signal $i_i(t)$ contains the information signal $v_s(t)$ that we want to transmit. In the receiver $i_d(t)$ is obtained, which varies in proportion to the information signal $v_s(t)$. That is, the informational signal can be recovered using the current detectors in Fig. 4.

Fig. 2 Transmitting communication system, which utilizes Chua's circuit.

The nonlinear resistors G_1 (Fig. 2) and G_{c1} (Fig. 3) have the following three-segment piecewise-linear v-i characteristics, respectively [12]

$$i_R = f(v_R) = G_b v_R + \tfrac{1}{2}(G_a - G_b) \cdot$$
$$\cdot \{|v_R + B_P| - \|v_R - B_P\|\} \qquad (2)$$

$$i'_R = f'(v'_R) = G'_b v'_R + \tfrac{1}{2}(G'_a - G'_b) \cdot$$
$$\cdot \{|v'_R + B'_P| - \|v'_R - B'_P\|\} \qquad (3)$$

The state equations for transmitter are as follows:

$$C_1 \frac{dv_1}{dt} = \frac{1}{R}(v_2 - v_1) - f(v_1) + \frac{v_S - v_1}{R_3} \qquad (4)$$

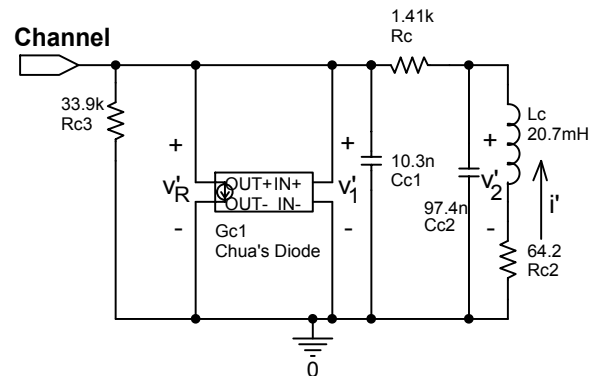$$C_2 \frac{dv_2}{dt} = \frac{1}{R}(v_1 - v_2) + i \qquad (5)$$

$$L \frac{di}{dt} = -v_2 - R_2 i \qquad (6)$$

The circuit equations for receiving system are given by:

$$C_{c1} \frac{dv'_1}{dt} = \frac{1}{R_c}(v'_2 - v'_1) - f'(v'_1) - \frac{v'_1}{R_{c3}} \qquad (7)$$

$$C_{c2} \frac{dv'_2}{dt} = \frac{1}{R_c}(v'_1 - v'_2) + i' \qquad (8)$$

$$L_c \frac{di'}{dt} = -v'_2 - R_{c2} i' \qquad (9)$$

Fig. 3 Receiving communication system which utilizes Chua's circuit.

For proper operation of the system, both the transmitter and the receiver should be exactly matched i.e. $R_c = R$, $C_{c1} = C_1$, $C_{c2} = C_2$, $L_c = L$ and $G_{c1} = G_1$. Due to the voltage buffer, we have $v_1' = v_1$ and we can subtract (8) from (5) and subtract (9) from (6) to obtain:

$$C_2 \frac{d(v_2 - v_2')}{dt} = -\frac{1}{R}(v_2 - v_2') + (i - i') \qquad (10)$$

$$L \frac{d(i - i')}{dt} = -(v_2 - v_2') \qquad (11)$$

which are the state equations of a parallel *RLC* circuit with linear positive *R*, *L* and *C*. This implies that the origin is globally asymptotically stable (all eigenvalues have negative real parts) and $(v_2 - v_2') \rightarrow 0$ as $t \rightarrow \infty$.
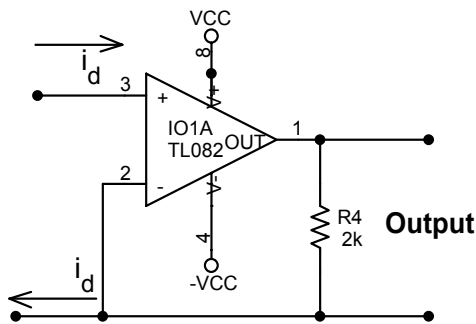

Fig. 4 Current detector.

Subtracting (7) from (4), we get

$$i_d(t) - i_i(t) = (v_2 - v_2')/R \qquad (12)$$

i.e., $i_d(t) \rightarrow i_i(t)$ as $t \rightarrow \infty$. That is, (v_2, i)-subsystem and the (v_2', i')-subsystem synchronizes and $v_{R4}(t) \rightarrow v_S(t)$ as $t \rightarrow \infty$.

In summary, the operation of this system is as follows: the voltage across the capacitor C_1 is affected by $i_i(t)$. This transmitted voltage $v_1(t)$ then affects the voltage across C_{c1} in the receiver. Since both the transmitter and the receiver are matched then current flowing into the second Chua's circuit must match the current injected into the first Chua's circuit. Since $i_i(t)$ determines the voltage $v_1(t)$ which in turn determines the current through C_{c1} and G_{c1}, then the current in the receiver must be equal to $i_i(t)$.

4. Simulation Results

We simulate the communication system via Chua's circuit in Fig. 2 and Fig. 3 by PSpice software. The system is tested using various kinds of signals (audio signal *gong*, sinusoidal signal, and triangular signal).

From these simulation results, we conclude the followings:

- The communication systems are easily built (see Fig.2 and Fig.3).

- The waveforms of the transmitted chaotic signal $v_1(t)$ can mask the input signal $v_s(t)$, if its amplitude is small (see Fig. 5). Otherwise an oscillatory phenomenon appears. From the practical viewpoint, however, oscillatory phenomena are undesirable. In such case analysis is valuable in that, it enables to verify amplitude of input signal for which oscillatory phenomenon is absent.

- The transmitted signals have broad spectra, and can mask the spectra of the input signals (see Fig. 6).

- The informational signals are recovered with sufficient quality from the transmitted signals (see recovered audio signal in Fig. 7 and the top trace in Fig. 8).

5. Parameter Mismatch and Channel Noise

This system has the high sensitivity of the parameter mismatch. When the parameters of the transmitting and receiving systems are not identical, the retrieved signal will be corrupted by chaotic noise. We show in Fig. 8 how the recovered signal degrades for a 4.25% mismatch between the parameters R and R_c. This is the reason why the system is secure, in the sense that the parameters have to be matched very closely in order for the retrieved signal. Finally, we show in Fig. 9 how the recovered signal degrades when white noise whit different SNR is added in the channel.
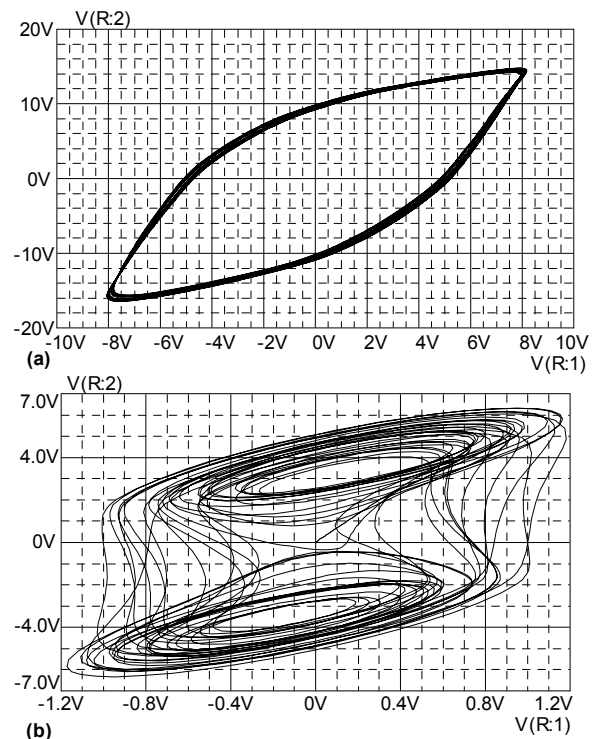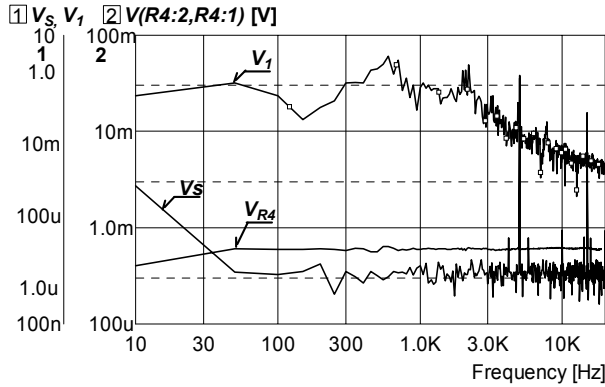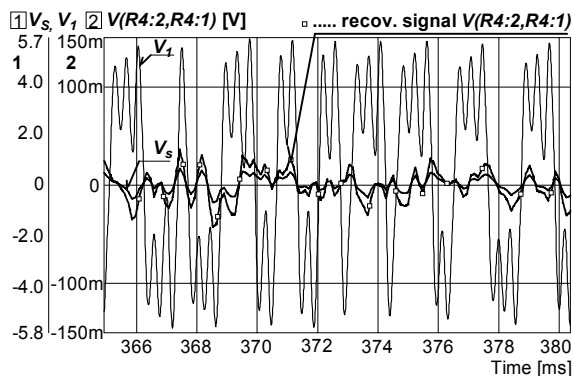

Fig. 5 Different steady state behavior of the simulated communication system. (a) Undesirable periodic solution (amplitude of input signal $v_s(t)$ is more than 4,3V). (b) Chaotic solution (amplitude of input signal $v_s(t)$ is less than 4,3V).
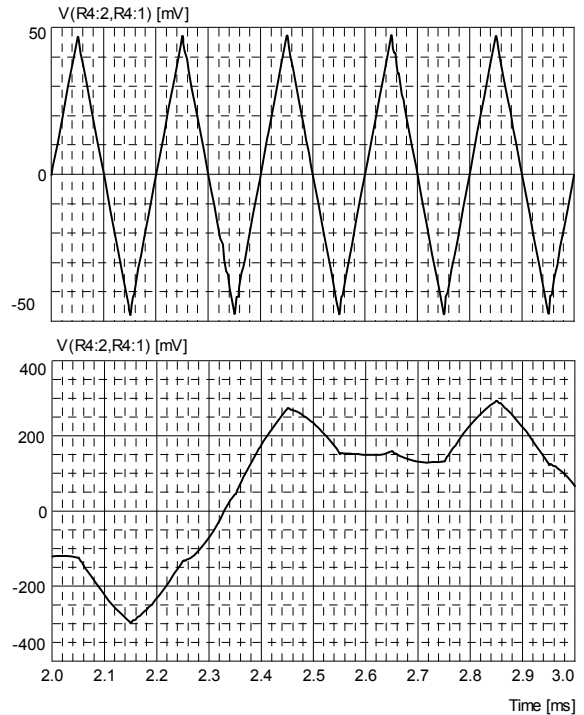
Fig. 6 Spectra of the transmitted chaotic signals v_1, the input triangular signal v_S with frequency 5 kHz, amplitude 0.8 V and recovered signal v_{R4}.
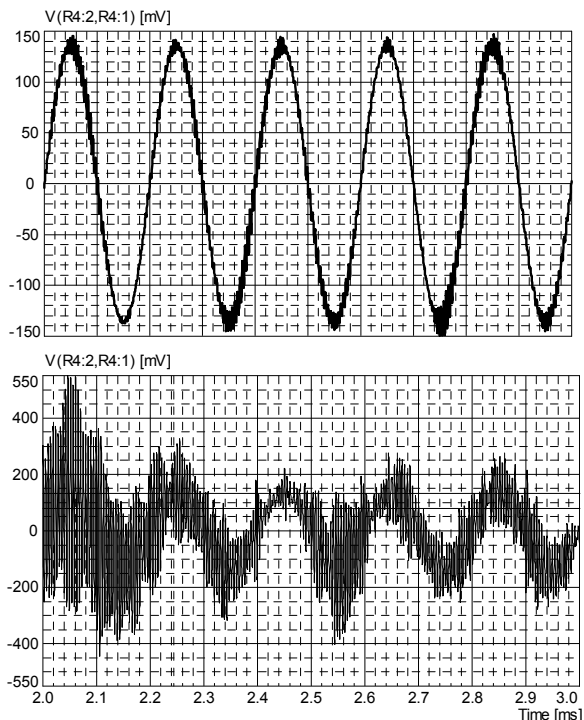
6. Conclusion

Research into the use of chaos for spread spectrum communication is increasingly common [13] and [14]. For analog communications, chaos masking and chaos modulation are used to spread the voice signals in order to prevent eavesdropping and otherwise increase security. We demonstrated the feasibility of using Chua's circuits to implement a secure communication system. We showed how the amplitude of input signal can bias the dynamic behavior of chaotic circuit. If the amplitude is large oscillatory phenomenon appears. From the practical viewpoint, however, oscillatory phenomena in our communication system are undesirable. In such a case analysis was implemented, which verify amplitude of input signal for which the oscillatory phenomenon is absent. We have shown that this system has advantages of transmitting a spread-spectrum signal, and having enhanced sensitivity to parameter variations. Finally we have shown that the informational signals are recovered with high quality, as the retrieved signals are corrupted by the noise.

Fig. 7 Time waveform of the transmitted chaotic signal v_1, input audio ("gong") signal v_S and recovered (□ …..) signal v_{R4}.

Fig. 8 The top trace is the recovered signal when both the transmitter and the receiver are matched. The bottom trace is the recovered signal with 4.25% resistor mismatch.

Fig. 9 The top trace shows the recovered signal with 63 dB of SNR. The bottom trace shows the recovered signal with 50 dB of SNR.

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