Synthesis of Optimized Piecewise-Linear Systems Using Similarity Transformation
Part III: Higher-Order Systems

Jiří POSPÍŠIL, Zdeněk KOLKA, Stanislav HANUS
Jaromír BRZOBOHATÝ [1]
Dept. of Radio Electronics, [1] Dept. of Microelectronics
Brno University of Technology, Czech Republic

Abstract

State models of dynamical systems used as prototypes in their practical realization are optimized from the viewpoint of minimum eigenvalue sensitivities with respect to the change of the individual parameters. In the paper the previously published optimization design procedure for the second-order linear and piecewise-linear (PWL) autonomous dynamical systems [2] is extended also for higher-order systems. Results obtained give the possibility to realize the third-order basic chaotic or the fourth-order hyper-chaotic oscillators.

Keywords

Dynamical systems, piecewise-linear systems, sensitivity optimization, chaos, hyper-chaos

1. Introduction

State models of dynamical systems can be used as prototypes in practical realizations of electronic chaotic oscillators. Recently published design procedure [1], [2] represents the results optimized from the viewpoint of the minimum relative eigenvalue sensitivities. The method used is based on the general topological conjugacy [3], [5], where the original and the resultant systems are described by the general state matrix form

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{b}\,h\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}\right) \tag{1}$$

in which the elementary PWL feedback function is

$$h\left(\mathbf{w}^{\mathrm{T}}\mathbf{x}\right) = \tfrac{1}{2}\left(\left|\mathbf{w}^{\mathrm{T}}\mathbf{x}+1\right| - \left|\mathbf{w}^{\mathrm{T}}\mathbf{x}-1\right|\right) \tag{2}$$

Both the systems are qualitatively equivalent and their mutual relation is given by the similarity transformation [1]. It provides possibility to derive the general form of the corresponding optimization condition for minimum sum of the relative eigenvalue sensitivity squares with respect to the change of the individual model parameters. Its detailed applications in the second-order linear and PWL systems, where both the real and complex conjugate eigenvalues are considered, are introduced in paper [2].

The present paper shows the extension of these results also for the higher-order systems, where the optimization procedure can be applied to the state models based on the block-decomposed state matrix [4]. In the next parts the optimized state models of the third- and fourth-order systems are shown which are suitable for modeling of their chaotic and hyper-chaotic behavior, respectively.

2. Third-order system

Utilizing results for the second-order systems, the third-order model with upper block-triangular state matrix containing complex decomposed second-order sub-matrix [4] can be designed. Suppose one pair of the complex conjugate eigenvalues and one real eigenvalue in both outer and inner regions of the elementary PWL function (2), i.e.

$$\nu_{1,2} = \nu' \pm j\nu'',\ \nu_3\text{: real};\quad \mu_{1,2} = \mu' \pm j\mu'',\ \mu_3\text{: real}.$$

Then the state matrix and the vectors in (1) have the form

$$\mathbf{A} = \left[\begin{array}{cc|c} \nu' & -\nu'' & -b_1 \\ \nu'' & \nu' & -b_2 \\ \hline 0 & 0 & \nu_3 \end{array}\right],\quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix},\quad \mathbf{w} = \begin{bmatrix} 1 \\ w_2 \\ 1 \end{bmatrix}, \tag{3a}$$

where

$$b_1 = \mu' - \nu',\qquad b_3 = (\mu_3 - \nu_3), \tag{3b}$$

while the parameters b_2 and w_2 are given by formulas [2]

$$b_2 = \frac{(\mu'-\nu')^2}{\nu''-\mu''K},\qquad w_2 = \frac{\nu''-\mu''K}{\mu'-\nu'} \tag{4}$$

Then the state matrix associated with the inner region has the lower block-triangular form

$$\mathbf{A}_0 = \left[\begin{array}{cc|c} \mu' & -\mu''K & 0 \\ \mu''K^{-1} & \mu' & 0 \\ \hline b_3 & b_3 w_2 & \mu_3 \end{array}\right] \tag{5}$$

The optimizing coefficient K is expressed as the real root of the quadratic equation [2]

$$K^2 - 2K(M+1) + 1 = 0,$$
$$K = 1 + M \pm \sqrt{M(M+2)},$$

where the auxiliary parameter M is given in the form

$$M = \frac{(\mu' - \nu')^2 + (\mu'' - \nu'')^2}{2\mu''\nu''} > 0 \ , \quad (\mu'', \nu'' \neq 0).$$

Such a way, the model having very low eigenvalue sensitivities in both the outer and inner regions of the PWL feedback function is obtained. The complete state equations of the optimized third-order PWL autonomous system can be rewritten into the following final form

$$\dot{x}_1 = \nu' \left[x_1 + x_3 - h(x_1 + w_2 x_2 + x_3)\right] - \nu'' x_2 + \mu' \left[h(x_1 + w_2 x_2 + x_3) - x_3\right] \quad (6)$$

$$\dot{x}_2 = \nu'' x_1 + \nu' x_2 + b_2 \left[h(x_1 + w_2 x_2 + x_3) - x_3\right] \quad (7)$$

$$\dot{x}_3 = \nu_3 \left[x_3 - h(x_1 + w_2 x_2 + x_3)\right] + \mu_3 \, h(x_1 + w_2 x_2 + x_3) \quad (8)$$

where the basic individual parameters are separated. However, the parameters b_2 and w_2 are given by more complex formulas (4) but the final required effect, i.e. the minimum eigenvalue sensitivities, has been achieved by this arrangement. The corresponding integrator-based block diagram, which is suitable as the prototype [6] for practical realization of optimized universal chaotic oscillator, is on Fig. 1.

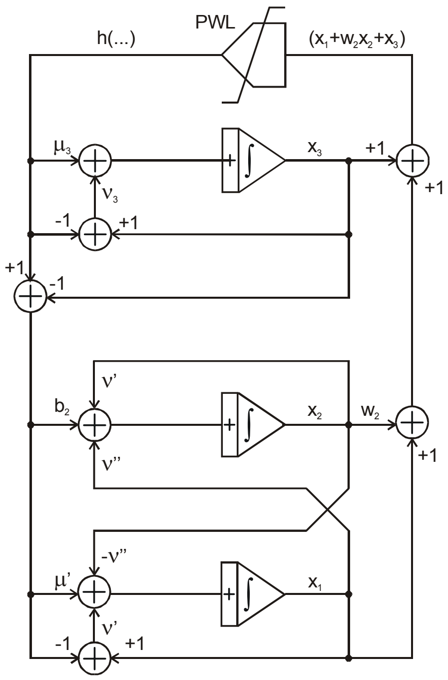

Fig. 1 Integrator-based circuit structures of the 3rd order state models with block-triangular state matrices and complex decomposed 2nd order sub-matrix

3. Fourth-order system

Using the same design principles the optimized state model of the fourth-order system can also be designed having again upper block-triangular state matrix. In the 4th -order case it contains two complex decomposed 2nd -order sub-matrices having two pairs of the complex conjugate eigenvalues in both outer and inner regions, i.e.

$$\nu_{1,2} = \nu_1' \pm j\nu_1'' , \quad \nu_{3,4} = \nu_3' \pm j\nu_3'' ,$$

$$\mu_{1,2} = \mu_1' \pm j\mu_1'' , \quad \mu_{3,4} = \mu_3' \pm j\mu_3'' .$$

Then the state matrix and vectors have the following decomposed form

$$\mathbf{A} = \left[\begin{array}{cc|cc} \nu_1' & -\nu_1'' & -b_1 & -b_1 w_4 \\ \nu_1'' & \nu_1' & -b_2 & -b_2 w_4 \\ \hline 0 & 0 & \nu_3' & -\nu_3'' \\ 0 & 0 & \nu_3'' & \nu_3' \end{array}\right], \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 1 \\ w_2 \\ 1 \\ w_4 \end{bmatrix}, \quad (9)$$

where $b_1 = \mu_1' - \nu_1'$ and $b_3 = \mu_3' - \nu_3'$, while the other parameters are given by the modified formulas (4), i.e.

$$b_2 = \frac{(\mu_1' - \nu_1')^2}{\nu_1'' - \mu_1'' K_1}, \qquad w_2 = \frac{\nu_1'' - \mu_1'' K_1}{\mu_1' - \nu_1'} , \quad (10)$$

$$b_4 = \frac{(\mu_3' - \nu_3')^2}{\nu_3'' - \mu_3'' K_3}, \qquad w_4 = \frac{\nu_3'' - \mu_3'' K_3}{\mu_3' - \nu_3'} . \quad (11)$$

Here, $K_i^2 - 2K_i (M_i + 1) = 0$, i.e. $K_i = 1 + M_i \pm [M_i (M_i + 2)]^{1/2}$ and

$$M_i = \frac{(\mu_i' - \nu_i')^2 + (\mu_i'' - \nu_i'')^2}{2\mu_i''\nu_i''} > 0, \ \mu_i'', \nu_i'' \neq 0 \ ; \quad i = 1,3 .$$

The state matrix associated with the inner region has again the lower block-triangular form, i.e.

$$\mathbf{A}_0 = \left[\begin{array}{cc|cc} \mu_1' & -\mu_1'' K_1 & 0 & 0 \\ \mu_1'' K_1^{-1} & \mu_1' & 0 & 0 \\ \hline b_3 & b_3 w_2 & \mu_3' & -\mu_3'' K_3 \\ b_4 & b_4 w_2 & \mu_3'' K_3^{-1} & \mu_3' \end{array}\right], \quad (12)$$

so that also this model has very low eigenvalue sensitivities in both the outer and inner regions of the PWL feedback function. The complete state equations of the optimized fourth-order PWL autonomous system can be rewritten into the following final form

$$\dot{x}_1 = \nu_1' \left[x_1 + x_3 + w_4 x_4 - h(x_1 + w_2 x_2 + x_3 + w_4 x_4)\right] - \nu_1'' x_2 + \mu_1' \left[h(x_1 + w_2 x_2 + x_3 + w_4 x_4) - (x_3 - w_4 x_4)\right]$$

$$\dot{x}_2 = \nu_1'' x_1 + \nu_1' x_2 + b_2 \left[h(x_1 + w_2 x_2 + x_3 + w_4 x_4) - (x_3 + w_4 x_4)\right] \quad (14)$$

$$\dot{x}_3 = \nu_3' \left[x_3 - h(x_1 + w_2 x_2 + x_3 + w_4 x_4)\right] - \nu_3'' x_4 + \mu_3' \, h(x_1 + w_2 x_2 + x_3 + w_4 x_4) \quad (15)$$

$$\dot{x}_4 = \nu_3'' x_3 + \nu_3' x_4 + b_4 \, h(x_1 + w_2 x_2 + x_3 + w_4 x_4) \quad (16)$$

The corresponding integrator-based block diagram, which is suitable as the prototype for the practical realization of the optimized universal hyper-chaotic oscillator, is shown in Fig. 2.

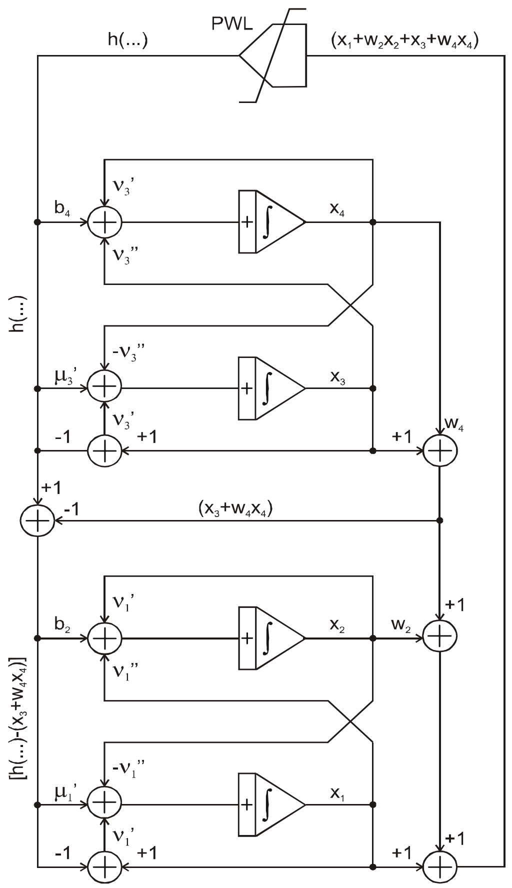

Fig. 2 Integrator-based circuit structures of the 4th order state models with block-triangular state matrices and complex decomposed 2nd order sub-matrices.

4. Conclusion

General optimization condition for the second-order PWL dynamical systems is utilized for design of their state models with low eigenvalue sensitivities and also extended for the optimized synthesis of higher-order systems. The results achieved are important for the practical realization of autonomous chaotic dynamical systems (oscillators) in the form of simple electronic circuits having separately adjustable parameters. It has been also proved by the first laboratory experiments.

Acknowledgement

This research is particularly supported by the Grant Agency of the Czech Republic, Grant projects No. 102/99/0203 and No. 102/01/0229. It represents the part of the Research Program of the Czech Ministry of Education – CEZ:. J22/98: 262200011.

References

[1] KOLKA, Z. Synthesis of Optimized Piecewise-Linear Systems Using Similarity transformation – Part I: Basic principles. Radioengineering. 2001, vol. 10, no. 3, p. 5-7.

[2] POSPÍŠIL, J., KOLKA, Z., HORSKÁ, J: Synthesis of Optimized Piecewise-Linear Systems Using Similarity transformation – Part II: Second-Order Systems. Radioengineering. 2001, vol. 10, no. 3, p. 8-10.

[3] CHUA, L. O., WU, C. W. On Linear Topological Conjugacy of Lure's Systems. IEEE Trans. CAS 43, 1996, p. 158-161.

[4] POSPÍŠIL, J., BRZOBOHATÝ, J., KOLKA, Z., HORSKÁ, J. Decomposed Canonical State Models of the Third-Order Piecewise-Linear Dynamical Systems", Proc. ECCTD'99, Stresa, 1999, p. 181-184.

[5] POSPÍŠIL, J., BRZOBOHATÝ, J., KOLKA, Z., HORSKÁ, J. Simplest ODE Equivalents of Chua's Equations. International Journal of Bifurcation and Chaos. 2000, vol. 10, no. 1, p. 1-23.

[6] HANUS, S. Realization of Third-Order Chaotic Systems Using Their Elementary Canonical State Models. In Proc. Rádioelektronika'97, Bratislava, 1997, p. 44-45.

About authors...

Jiří POSPÍŠIL was born in Brno, Czechoslovakia, in 1939. M.Sc. and Ph.D. (equiv. degrees): 1963 and 1973, respectively; DSc. (equiv. degree.): 1988, all in el. eng. TU Brno, Czechoslovakia. 1964: Assist. Prof., Military Acad. of Brno, Dept of El. Eng.; 1970-1972: Visit. Prof., Military Tech. College, Cairo, Egypt; since 1974: TU Brno, Dept of Radio Electronics; 1980: Assoc. Prof.; 1989: Prof.; Research and pedagogical interest: Circuits and Systems Theory, *PWL* Dynamical Networks, Dynamical Systems Modeling. IEEE: M.-1992, S.M.- 1995.

Zdeněk KOLKA was born in Brno, Czechoslovakia, in 1969. He received the M.S. (92) and Ph.D. (97) degrees in electrical engineering, both from the Brno University of Technology. At present he is an Assistant Professor at the Dept. of Radio Electronics. He is interested in PWL modeling, circuit simulation, and nonlinear dynamical systems.

Jaromír BRZOBOHATÝ was born in Brno, Czechoslovakia, in 1935. M.Sc. and Ph.D. (equiv. degrees): 1960 and 1980, respectively, both in el. eng., TU Brno, Czechoslovakia. 1960-1963: researcher in Metra Blansko; 1963: As. Prof.; 1987: Prof.; 1985-1989: Dean of Faculty of El. Eng.; 1980-1993: Head of Dept of Microelectronics. Research and pedagogic interest: Circuits Theory, Microelectronics, *PWL* Dynamic Networks. IEEE: M.-1988, S.M.-1990.

Stanislav HANUS was born in Brno, Czech Republic, in 1950. He received Ing. (MSc.) and CSc. (PhD.) degree from the Brno University of Technology. He is associate professor at the Dept. of Radio Electronics. His research is concentrated on Wireless and Mobile Communications.