Resistively Loaded Dipole Characteristics

Peter HAJACH [1], *René HARŤANSKÝ* [2]

[1] Dept. of Radio and Electronics, FEI STU, Ilkovičova 3, 812 19 Bratislava, Slovakia
[2] Dept. of Mechatronics, TnU Alexandra Dubčeka, Študentská 1, 911 50 Trenčín, Slovakia

hajach@kre.elf.stuba.sk, rene@yhman.tnuni.sk

Abstract. *Resistively loaded dipole is very often used to measuring the electromagnetic field in EMC testing. The dipole is a traveling wave antenna, where its resonance is eliminated. The paper includes the computation of the current distribution and radiation pattern of this dipole. The characteristics were obtained by moment method, Hallen method and Wu-King method. The unique modification of presented methods has been made. The comparison of used methods has been presented in paper conclusions.*

Keywords

Antenna, electromagnetic field, electromagnetic compatibility, analytical and numerical methods of antenna analysis.

1. Introduction

In electromagnetic testing applications, we have to measure the electric field in a wide range of intensities and frequencies. Therefore, very precise measuring equipment has to be built.

Resistive dipole is almost always used as the electromagnetic probe. The usual Hallen's method is unable to reach the correct results when computing the characteristics of the dipole. Therefore, three other methods of resistive dipole design are compared in this paper: the modified Hallen's method, the moment method and the Wu-King method. The current distribution is computed using all the methods. The results of methods are mutually compared. The advantages and disadvantages of these methods are discussed in the paper.

2. Problem Formulation

The structure of resistively loaded dipole consists of two triangles created from a resistive material. The length of the dipole is $2l$ and width at the center equals to $2a$. This geometry causes that only traveling wave rises up along the dipole. There is no reflected wave traveling in the opposite direction.

The axial component $A_z(z)$ of the vector potential on the surface of a cylindrical antenna of the internal impedance per unit length $Z_i(z)$ carries a total axial current $I_z(z)$.The antenna is excited at $z=0$ by the electromotoric force U_0. The vector potential satisfies one-dimensional wave equation in the form:

$$\left(\frac{\partial^2}{\partial z^2}+k^2\right)A_z(z)=\frac{jk^2}{\omega}\left[Z_i(z)I_z(z)-U_0\delta(z)\right], \qquad (1)$$

where

$$A_z(z)=\frac{\mu_0}{4\pi}\int_{-l}^{l}\dot{I}_z(z')\frac{e^{-jk\sqrt{(z-z')^2+a^2}}}{\sqrt{(z-z')^2+a^2}}dz' \ . \qquad (2)$$

There are several methods of solving eqn. 2. The first one is also known as Wu-King's method [1].

In order to obtain a triangular current distribution along the dipole in wide frequency range, the dipole has to be made from a resistive material such way, so that its internal impedance per unit length should follow the formula:

$$Z_i(z)=\frac{\Psi\sqrt{\mu_0/\varepsilon_0}}{2\pi(l-|z|)}=\frac{60\Psi}{l-|z|}, \qquad (3)$$

while l is the length of one dipole arm and z is a point at the dipole arm. The axis of the dipole is oriented along the z axis. The constant ψ is given by:

$$\begin{aligned}\Psi \ &\doteq \ \frac{j}{kl}\left(1-e^{-j2kl}\right)+\\ &2\left[\sinh^{-1}\left(\frac{l}{d}\right)-Ca(2ka,\,2kl)-j\,Sa(2ka,\,2kl)\right]\end{aligned}, \qquad (4)$$

where $Ca(x,y)$ and $Sa(x,y)$ are general sine a cosine integrals [2]. Using the above-mentioned equations, the current distributions along the dipole arms can be described as:

$$I_z(z)=\frac{U_0}{60\Psi\left[1-j/(kl)\right]}\left(l-|z|\right)e^{-jk|z|} \ . \qquad (5)$$

Since the solution of exp($-jk$ $|z|$) no longer satisfies the original wave equation, the antenna does not support a standing wave. The antenna is of much wider frequency characteristics than an antenna with a lumped resistor located at a quarter wavelength from ends.

The second way of computing the current distribution on a resistively loaded antenna is based on numerical solving the one-dimensional wave equation given in (1), using the re-formatted moment method [3]. The integral equation can be transformed to a set of algebraic equations, which can be solved in a very simple way. After several rearrangements, eqn. (1) will be as follows:

$$\sum_{n=1}^{N}[I_n]\cdot[Z_{mn}]=[U_m]\,, \tag{6}$$

where I_n is the column matrix of unknown currents [A], Z_{mn} denotes the impedance matrix [Ω] describing the influence of n-th current on the point of interest m. Next, the column matrix U_m of the supply voltages [V] equals to one for the position $N/2$ of the segment and is zero elsewhere.

The matrix element Z_{mn} of unknown impedance can be solved as follows:

$$Z_{mn}=\frac{1}{j\omega\varepsilon_0}\int_{z_{n-1/2}}^{z_{n+1/2}}\Xi\, dz'+Z_i(z_m)(z_{m+1/2}-z_{m-1/2})\,, \tag{7}$$

where

$$\Xi=k^2\frac{e^{-jk|\vec{R}|}}{4\pi|\vec{R}|}+\frac{\partial^2}{\partial z^2}\frac{e^{-jk|\vec{R}|}}{4\pi|\vec{R}|}\,. \tag{8}$$

The current distribution along the dipole can be reached by solving the equation (6) and using equations (7) and (8).

Hallen's method is the last method suitable for solving the integral equations. Hence, even the current distribution can be computed by Hallen's method. However, the imperfect conductivity of antenna conductor has to be taken into account. In this case, $I_z(z)$ is of the following form:

$$\begin{aligned}I(z')=&\frac{jU_o}{\Omega\mu c\left[F_0(l)+\frac{1}{\Omega}F_1(l)\right]}\cdot\\&\left\{\left[f_0(z)+\frac{1}{\Omega}f_1(z)\right]\cdot\left[G_0(l)+\frac{1}{\Omega}G_1(l)\right]-\right.\\&\left.-\left[g_0(z)+\frac{1}{\Omega}g_1(z)\right]\left[F_0(l)+\frac{1}{\Omega}F_1(l)\right]\right\}\end{aligned}\,, \tag{9}$$

where $\Omega = 2\ln(2l/a)$ is a thickness coefficient. The other variables can be evaluated using the following formulas:

$$\begin{aligned}F_0(z)&=\cos(kz) & G_0(z)&=\sin(k|z|)\\f_0(z)&=F_0(z)-F_0(l) & g_0(z)&=G_0(z)-G_0(l)\\f_1(z)&=F_1(z)-F_1(l) & g_1(z)&=G_1(z)-G_1(l)\end{aligned} \tag{10}$$

Hallen's coefficients can be evaluated using the equations for F_0, F_1, G_0, G_1. The complete description can be found in [5]. The parameter $Z_i(z)$ is included and equations are presented below:

$$\begin{aligned}F_1(z)&=\\&-[\cos(kz)-\cos(kl)]\ln\left[1-(z/l)^2\right]+\\&+\tfrac{1}{2}\cos\,[kz\,\{\,C[2k(l+z)]+C[2k(l-z)]+\\&\quad+j\,Si[2k(l+z)]+j\,Si[2k(l-z)]\,\}]-\\&-\tfrac{1}{2}\sin\,[kz\,\{\,Si[2k(l+z)]-Si[2k(l-z)]-\\&\quad-j\,C[2k(l+z)]+j\,C[2k(l-z)]\,\}]-\\&-\cos\,[\,kl\,\{\,C[k(l+z)]+C[k(l-z)]+\\&\quad+j\,Si[k(l+z)]+j\,Si[k(l-z)]\,\}]+\\&+j\frac{4\pi Z_i(z)l}{\mu c}\left[\frac{z}{2l}\sin(kz)-\frac{\cos(kl)}{kl}(1-\cos kl)\right]\end{aligned}\,. \tag{11}$$

The function G_1 is of the form:

$$\begin{aligned}G_1(z)&=\\&-[\sin(k|z|)-\sin(kl)]\ln\left[1-(z/l)^2\right]-\\&-\tfrac{1}{2}\cos\,[kz\,\{\,Si[2k(l+z)]+Si[2k(l-z)]-\\&\quad-2\,Si(2k|z|)-j\,C[2k(l+z)]-\\&\quad-j\,C[2k(l-z)]+2j\,C(2kz)\}]-\\&-\tfrac{1}{2}\sin\,[kz\,\{\,C[2k(l+z)]-C[2k(l-z)]+\\&\quad+j\,Si[2k(l+z)]-\\&\quad-j\,Si[2k(l-z)]-2j\,Si(2kz)\}]-\\&-\tfrac{1}{2}\sin\left\{k|z|\left[4\ln\left(\frac{|z|}{l+|z|}\right)-2C(2kz)\right]\right\}-\\&-\sin[\,kl\,\{\,C[k(l+z)]+C[k(l-z)]+\\&\quad+j\,Si[k(l+z)]+j\,Si[k(l-z)]\}]+\\&j\frac{4\pi Z_i(z)l}{\mu c}\left[\frac{\sin(k|z|)}{2kl}-\frac{|z|\cos(kz)}{2l}-\right.\\&\quad\left.-\frac{\sin(kl)}{kl}[1-\cos(kz)]\right]\end{aligned}\,. \tag{12}$$

Here, k is wave number [m^{-1}], l denotes half length of the dipole [m], γ is Euler constant and $C(x)$ is a function depending on the generalized cosine integral by the form $C(x) = \gamma + \ln(x) - C_i(x)$. Next, $C_i(x)$ and $S_i(x)$ are the generalized cosine and sine integrals, respectively:

$$Ci(x)=-\int_x^{\infty}\frac{\cos(u)}{u}du\,,$$

$$Si(x)=\int_0^{x}\frac{\sin(u)}{u}du\,.$$

Not only the current distribution computation (inner antenna task) is to be presented in the paper. The paper is focused in radiation pattern estimation, which requires the knowledge of currents.

The current distribution of the resistively loaded dipole, obtained by three different ways, has been already given. The one obtained by Wu and King leads to radiation pattern characteristics for E - plane:

$$F(\theta)=\{-jkl\sin^2\theta+(1+\cos^2\theta)- [2j\cos\theta\sin(kl\cos\theta)+(1+\cos^2\theta)\cdot \cos(kl\cos\theta)]e^{-jkl}\}/kl\sin^3\theta \quad (13)$$

Radiation pattern characteristic for H - plane is not presented in the paper, since it is circular [1].

According to the current distribution computed by moment method and Hallen method, we can determine the field intensity in the dipole neighborhood exactly from Maxwell equations and we can derive the formula for radiation pattern:

$$F(\theta)=E(\Theta)/E_{\max}(\Theta) \quad (14)$$

since

$$E=\frac{j\omega\mu_0}{4\pi r}\sin\theta.e^{-jkr}\int_{-l}^{+l}I(z')e^{jkz'\cos\theta}dz' \quad (15)$$

Obviously, both the current distribution and the radiation pattern have been obtained by different ways. In order to compare the advantages and disadvantage of the presented methods, the solved example is presented next.

3. Computation of Current Distribution

Exploiting the described methods, the current distribution for the dipole $2l$ = 73.5 mm and $2a$ =1 mm was computed. Using (1), the equivalent impedance constant can be evaluated Ψ = 6.68025 – j 2.42923 Ω. (valid for frequency from $kl = \pi/2$). Using the constant Ψ, the internal impedance per length unit can be solved:

$$\mathrm{Re}\{Z_i(z)\}=\frac{60\,\mathrm{Re}\{\Psi\}}{l-|z|}=\frac{60.6{,}68025}{36{,}75-|z|}\Omega.mm^{-1} \quad (16)$$

Using the above equation, the current distribution can be evaluated by Wu-King method, moment method and Hallen method. The complex amplitude of current (amplitude and phase) is shown in Figures 1 and 2.

When comparing results of computations to measurements, the amplitude of current distribution is very similar for all the three presented methods. To compare these to measurement (realized in [2]), the more accurate results are obtained by moment method, where the current in the point of supply equals to 2.3 mA, and at the end of arms is 0 mA. This fulfills requirements set by equation (5). The other two methods contain a lot of simplification and this causes that their results are not consequent. This can be proofed by Fig. 2, where the dependence of current distribution phase on the dipole arm position for frequency $kl = \pi/2$, what actually is f = 2.03945 GHz as can be seen. The current distribution phase varies from 40° at the center of dipole to -80° at the end of arms.

To obtain the previous (presented) results we had to consider (especially when using Hallen method) the shape of the resistively loaded dipole. Tightening the dipole influenced the thickness coefficient Ω as:

$$\Omega'(z)=\ln\frac{2l}{(l-z)a/l} \quad (16)$$

Of course, we also have to consider the fact that the resistively loaded dipole cross-section is not circular – the thickness coefficient correction constant has to be used. After that Ω(z) is equal to:

$$\Omega(z)=\Omega'(z)\cdot 0.75 \quad (17)$$

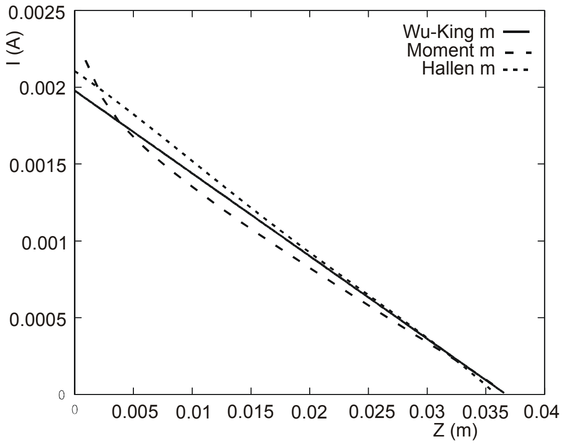

Fig. 1. The current amplitude distribution along the dipole arm for frequency $kl = \pi/2$.

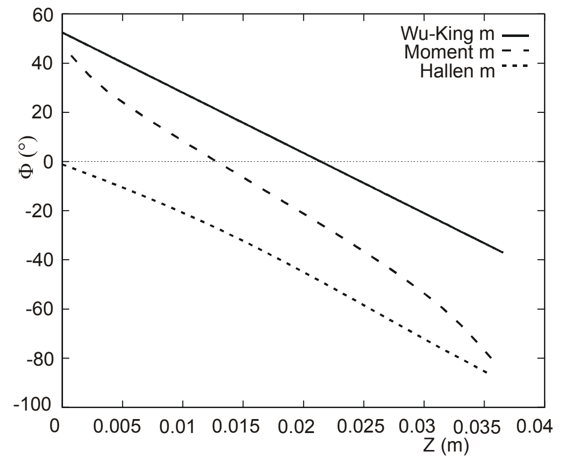

Fig. 2. The current phase distribution along the dipole arm for frequency $kl = \pi/2$.

4. Computation of the Radiation Pattern

The resistively loaded dipole computation will continue in this chapter. Its sizes are: $2l$=73.5 mm and $2a$=1 mm, frequency is $kh = \pi/2$. The computation is based on previous chapter results and on equations (13) and (15). The computation has been made by software package *Mathematica* running on operating system *Linux*. The results can be seen in Fig. 3. Dashed line symbolizes the computed characteristic after Wu - King and the solid one is the characteristic by Hallen and moment method. It is obvious that after Wu -King the emitting beam by cut frequency is narrower as after other methods. According to the previous chapter results and to the measurement made in [2], we can say that the results of moment and Hallen methods are closer to reality and more accurate, compared to Wu - King.

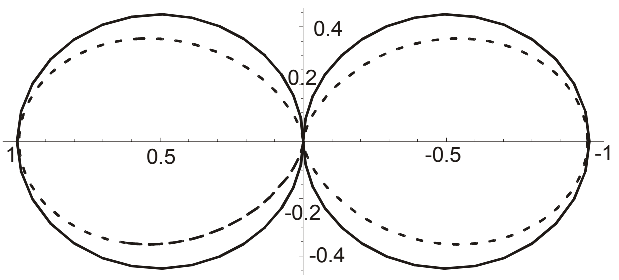

Fig. 3. Radiation pattern of half wave resistive structure

5. Conclusion

From the previous computations, especially from the presented figures, it goes that the most perspective method to solve the resistive antenna problems is the modified moment method.

Hallen's equation is suitable especially to solve the antennas from perfect conducting materials. To solve the resistive antennas, the equation has to be modified. The product is unique and not very often presented result. This solution can also be used to design the other types of antennas, which are made of real materials (the material conductivity is finite).

In order to compare the advantages and disadvantages of methods, the brief table is presented. The basic quantitative and qualitative parameters used by each method are included. Time, degree and precision of each method are compared. The best one is marked by “1”, the second one by “2” and the worst one by “3” (see Tab. 1).

	Wu - King method	Moment method	Hallen method
Time of calcul.	1	3	2
Degree of calcul.	1	3	2
Precision of calcul.	3	1	2

Tab. 1. Comparison of advantages of using methods.

References

[1] WU, T. T., KING, R. W. P. The cylindrical antenna with nonreflecting resistive loading. *IEEE Transactions on Antennas and Propagation*. 1965, vol. 13, no. 3, p. 369-373.

[2] KANDA, M. Time-domain sensors and radiators. In *Time Domain Measurements in Electromagnetics*. E. K. Miller, ed. New York: Van Nostrand, 1986.

[3] HARTANSKY, R., KOVAC, K., HALLON, J. Effective modification of moment method for antenna simulation. In *Proceedings of 9th Conference on Microwave Techniques*. Pardubice (Czechia), 1997, p. .262-265

[4] MAGA, D., WAGNER, J. Numerické riešenie elektromagnetických polí v elektrických strojoch (Numerical solution of electromagnetic fields in electrical engines). *Odborný časopis pre elektrotechniku a energetiku (Special journal on electrical engineering and power engineering)*. 1999, roč. 5, special issue ELOSYS 99, pp. 77 – 79

[5] KING, R. W. P. *Theory of linear antennas*. Cambridge MA: Harvard Univ. Press, 1956.

[6] MILJAVEC, D., JEREB, P. Synchronous reluctance motor and induction motor using the same stator frame. In *Proceeding of ICEM 96*. Vigo (Spain), 1999

About Authors...

Peter HAJACH (doc., Ing., Ph.D.) was born in Bratislava, Slovak Republic, in June 1946. He received - Ing. degree in electrical engineering from the Slovak Technical University of Bratislava in 1969, and the CSc. (Ph.D.) degree in radio-electronics from the STU Bratislava, in 1984. He is currently the associate professor at the Radio-electronic department of the STU Bratislava. His research interests include microwave antennas.

René HARŤANSKÝ (Ing., PhD.) was born in Banská Bystrica on May 13th, 1969. He received the M.Sc. degree with honors in 1992 and then PhD. degree in electrical engineering from the Faculty of Electrical Engineering of the Slovak Technical University (FEE STU). He is currently the assistant professor at the Dept. of Mechatronics TnU of Alexander Dubcek. His research interests are the computer modeling EMC phenomena, antennas and propagation.