Throughput Analysis of an Adaptation Rule in the HARQ Environment

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Abstract. *In this paper we analyze the adaptation rule, which estimates the channel state and switches between hybrid ARQ (automatic-repeat-request) and pure ARQ. Convolutional code was chosen as FEC (forward-error-correction) in hybrid ARQ part and go-back-N ARQ scheme is used in both cases. The adaptation rule is based on counting ACKs and NAKs and its throughput analysis is made.*

Keywords

Automatic-repeat-request (ARQ), forward-error-correction (FEC), hybrid ARQ, channel model.

1. Introduction

Basic error control techniques are ARQ and FEC. Both have some advantages and drawbacks. Hybrid ARQ (HARQ) methods combine these two in order to increase the performance. In this paper we consider HARQ type-I, which is the combination of ARQ and convolutional code with Viterbi decoding.

Convolutional encoding with Viterbi decoding is a FEC technique that is particularly suited to a channel in which the transmitted signal is corrupted mainly by additive white gaussian noise (AWGN) and has been widely used in space communication. By deleting some channel symbols at the output of the decoder a punctured code can be created. Punctured convolutional codes are useful to change the code rate and its performance.

ARQ scheme used is Go-Back-N (GBN) due to its good throughput and low complexity since it does not require buffering at the receiver side. In HARQ, throughput is moved thanks to FEC into the area of lower Eb/N0, but due to code redundancy the throughput in less noisy channel is lower than throughput of pure ARQ. To make it more powerful we have to consider the adaptive scheme that would switch among convolutional, variety of punctured convolutional codes and classic ARQ according to the state of communication channel. In this paper we want to analyze the adaptation rule, which estimates the channel state and switches between the methods. The analysis is made for marginal case, switching between convolutional code and pure ARQ. The adaptation rule is based on counting ACKs and NAKs.

2. Analyzed Scheme

The forward channel has two states, low error state (L state) and high error state (H state), as shown in Fig. 1. The channel state model in Fig. 1 is used to describe the way of switching between methods.

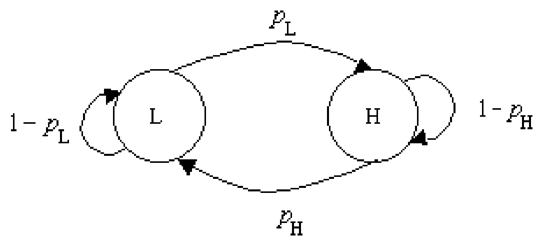

Fig. 1. Channel state model

Corresponding to the two channel states, there are two operation modes in the proposed scheme. In channel state L, the transmitter follows the classic GBN ARQ and its throughput can be expressed by [5]

$$\eta_L = \frac{1 - P_e}{1 + N \cdot P_e}. \qquad (1)$$

P_e is block error probability and can be expressed

$$P_e = 1 - (1 - P_b)^n \qquad (2)$$

where P_b is a bit error probability in BSC (Binary Symmetric Channel) and n is block length.

In the channel state H, the transmitter works in HARQ transmission mode, which operates like classic GBN ARQ scheme except for encoding data with convolutional code. Its throughput is [5]

$$\eta_H = \frac{1 - P_{He}}{1 + N \cdot P_{He}}. \qquad (3)$$

In this paper we used constraint length K = 3 convolutional code with generator polynomials (7, 5) and code rate 1/2.

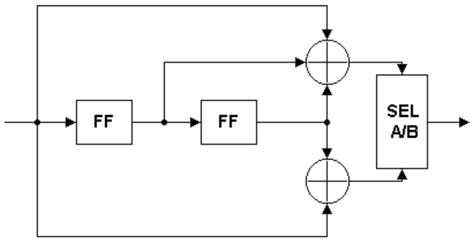

Fig. 2. Convolutional encoder

The analysis of convolutional codes is generally difficult to perform, these heuristically constructed codes can be analyzed through their transfer functions, which can be obtained from the state diagram of the convolutional code. Transfer function for used code is

$$T(D,N) = \frac{ND^5}{1-2ND}. \quad (4)$$

For hard-decision Viterbi decoding the bit error probability can be written [4]

$$P_{Hb} < \left.\frac{dT(D,N)}{dN}\right|_{D=\sqrt{4P_b(1-P_b)},N=1}. \quad (5)$$

If the transmitter is in ARQ operation mode (channel state L) successive NAKs are counted. If the counted amount of NAKs is greater than α, the transmitter would consider that the channel is too noisy and transits it from L state to H state. The transition probability is

$$p_L = P_e^{\alpha}. \quad (6)$$

On the other hand, in HARQ operation mode, successive ACKs are counted and when the amount is greater than β, the transmitter would consider that the channel is transiting from H state to L state. The transition probability is

$$p_H = (1-P_{He})^{\beta}. \quad (7)$$

The channel state model does not define a channel environment. The channel is considered to be disturbed by random noise, which results in independent errors. It is also assumed that the feedback channel is noiseless. This channel estimator is quite simple, based on counting the ACKs and NAKs and might not be enough flexible.

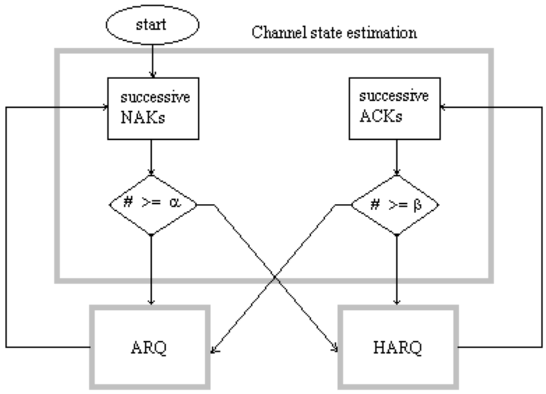

Fig. 3. Proposed scheme

3. Throughput Analysis

The throughput of proposed scheme can be expressed as an average of the throughput values of the two operation modes [2]

$$\eta = T_L \cdot \eta_L + T_H \cdot \eta_H \quad (8)$$

where T_L (resp. T_H) is the final probability that the channel is in L state (H state) for semi-markov automat. These probabilities can be expressed

$$T_L = \frac{\alpha \cdot P_L}{\alpha \cdot P_L + \beta \cdot P_H}, \quad (9)$$

$$T_H = \frac{\beta \cdot P_H}{\alpha \cdot P_L + \beta \cdot P_H}. \quad (10)$$

Using transition matrix we can write equations with two unknowns P_L and P_H

$$[P_L\, P_H] = [P_L\, P_H] \cdot \begin{bmatrix} 1-p_L & p_L \\ p_H & 1-p_H \end{bmatrix}, \quad (11)$$

$$P_L + P_H = 1. \quad (12)$$

Solving of the equations gives

$$P_L = \frac{p_H}{p_L + p_H}, \quad (13)$$

$$P_H = \frac{p_L}{p_L + p_H}. \quad (14)$$

The main problem is to define the values of system parameters α and β. These values can be found by simulations and would differ according to desired performance.

The throughput versus bit error probability performance of the proposed scheme with different values of α and β are shown in Fig. 5 and Fig. 6. The ARQ scheme is GBN with delay N = 10, block length n = 512 bits and HARQ uses the above-mentioned convolutional code with Viterbi hard-decision decoding (throughput analysis is made in BSC so no soft-decision was considered).

In Fig. 5 the effect of parameter α can be seen. It is obvious that the optimal value of α in the proposed scheme is 2. If it is greater than 2 its performance is quite low in the area of low bit error probability.

In Fig. 6, α is equal to 2 and the effect of β can be seen. The greater β the better performance in the area of $P_b > 10^{-4}$. On the other hand, when increasing the value of β the throughput for $10^{-5} < P_b < 10^{-4}$ is getting poorer. From Fig. 6 the best value of β seems to be 500, but optimal value of β cannot be found exactly, since it depends on the channel environment the scheme would be used in.

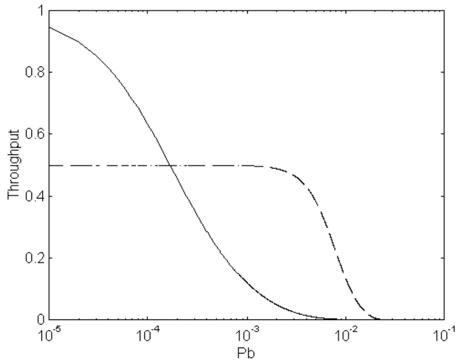

Fig. 4. Throughput versus bit error probability: solid is ARQ GBN scheme with N = 10 and block length 512 bits; dashed is HARQ using convolutional code k/n = ½, K = 3 with Viterbi hard-decision decoding.

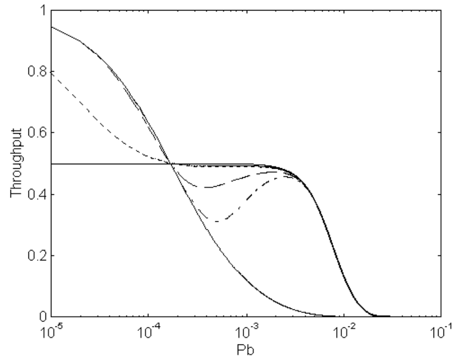

Fig. 5. Throughput versus bit error probability: proposed scheme N = 10, block length n = 512 bits; dash-dot: α = 1, β = 100; dash: α = 2, β = 100; short dash: α = 3, β = 100.

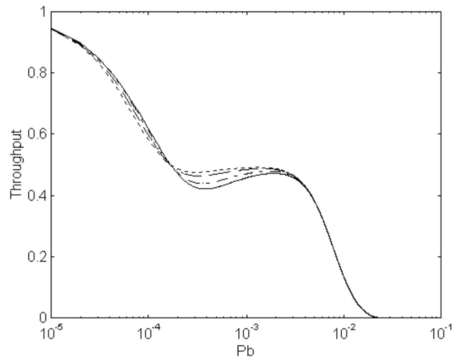

Fig. 6. Throughput versus bit error probability: proposed scheme N = 10, block length n = 512 bits; solid: α = 2, β = 100; dash-dot: α = 2, β = 150; dash: α = 2, β = 300; short dash: α = 2, β = 500.

4. Conclusion

The adaptation rule in the proposed scheme estimates the channel environment by counting ACKs and NAKs and therefore might not be flexible enough. The throughput analysis has shown that this rule could be used for switching between operation modes, however. In the paper we wanted to analyze the behavior of the scheme and therefore we used simple convolutional code with hard-decision decoding. In a real system probably soft-decision decoding as well as much more robust code would be used, which would improve the throughput. Next attention could be focused on general adaptation rule that could be used for switching among more operation modes and could be based on proposed scheme.

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