Estimation of Spread Spectrum Signal Parameters Utilizing Wavelet Transform Analysis

Nikolaos LEONIS, George KATSOULIS, Angelos AMDITIS, Nikolaos UZUNOGLU

Microwave and Fiber Optics Laboratory, National Technical University of Athens, Athens 15773, Greece

nuzu@cc.ece.ntua.gr

Abstract. *This paper investigates the application of wavelet transform (WT) to the extraction of particular features of direct sequence spread spectrum signals. The WT is exploited to the point that not only its capabilities but also its limitations are exposed to achieve the specific task of identifying spread spectrum signal parameters. The capabilities focus on the detection of the chipping sequence, while the limitations refer primarily to the advent of direct sequence CDMA signals. In the latter case no progress has yet been made to distinguish the different chipping sequences by the WT, whose resultant effect on the carrier appears as a single-phase change.*

Keywords

Spread spectrum, wavelet signal analysis.

1. Introduction

Current developments not only in military but also in commercial communications are characterized by a widespread application of spread spectrum modulation. This technology promises considerable benefits in both commercial and military domains. However, its application in military communications is more crucial, since it guarantees a means of camouflage to the friendly signals that renders them invisible to the non – cooperative receiver [1], [2], [3]. Even in case that a hostile observer is able to intercept such a signal, its further demodulation and exploitation are extremely difficult. Direct sequence spread spectrum technology, usually combined with pulse compression methods, is also applied to radar signals, to give them Low Probability of Detection (LPD) and Low Probability of Intercept (LPI) characteristics [4], [5].

In all the aforementioned cases, the key parameter of this extra-modulation applied to the signal is the pseudo noise (pn) – sequence, also called chipping sequence. This sequence is responsible for the spreading and the camouflaging of the signal. Only the receiver knowing this sequence can get synchronized with the transmitted signal and further demodulate it. In most advanced spread spectrum communication systems, the pn-sequence synchronization between transmitter and receiver is the first and most important synchronization that has to be achieved [1], [6].

The vulnerability of a telecommunication system compared to radar, is that since in telecommunications the signal emitted from user A needs to be analyzed by another user, say B, the pn-sequence has to be generated by a periodic mechanism, so that it can be independently reproduced from both users. Unless this condition is satisfied, the establishment of a direct sequence spread spectrum telecommunication network is impossible.

From the above discussion it is obvious that the detection of the pn-sequence is crucial, for both the cooperative and uncooperative receiver. The pn-sequence is multiplied by the signal, providing an extra modulation to it. In case that the type of spread spectrum modulation is of PSK type, either BPSK or QPSK, the multiplication of the information signal with the PSK spreading waveform, will result in abrupt phase changes to the signal waveform that constitute critical points to transmitted signal's waveform.

One of the most important applications of the wavelet transform, is the detection of discontinuities to either 1-D or 2-D signals. The discontinuities in a 2-D signal, e.g. an image, are the edges of the objects in the image, while the discontinuities in a 1-D signal are the abrupt changes either in phase or in frequency. In order to detect similar changes in a 1-D signal, the classical Fourier transform method is of no use. These changes resulting from the spread spectrum modulation techniques, cause their respective signals to be non – stationary. Unless the flexibility of time – frequency analysis techniques is applied to the exploitation of spread spectrum signals, no further action can be taken to extract useful information from them [7], [8], [9]. For the detection of changes in frequency either the Short Time Fourier Transform (STFT) or Wavelet Packet methods can be used. For this purpose the application of wavelet transform is of limited benefit, due to the requirement of uniform frequency resolution across a wide bandwidth [10].

For the detection of changes in phase, the wavelet transform provides a mean of time-frequency analysis that detects the successive phase changes appearing at a direct sequence spread spectrum signal through time. Initially, the chip duration is found as the minimum time difference between two successive phase changes. Then, since each phase change corresponds to a change of sign of the pn-sequence, starting by a chip value or either +1 or –1, at each interval of one chip duration the chip value is let to be unchanged when no phase change is encountered, while the chip value is changed in the opposite case. Thus either the modulating chipping sequence or its inverse is detected. This method is particularly interesting from calculations' reduction point of view. It permits the chipping sequence detection by the interceptor without prior carrier demodulation. This fact can also be used from the communicator as a simple method to achieve pn – sequence synchronization at the receiver, a fact that will not be dealt in this paper.

In the case of the radar pulse, both the frequency modulation of the linear FM chirp and the BPSK direct sequence spreading modulations are detected across the time frames and scales of the wavelet transform. The image provided that displays the continuous wavelet transform of the radar pulse is very revealing, working like "x-raying" the pulse and exposing its modulation characteristics.

2. Continuous Wavelet Transform (CWT), Discrete Wavelet Transform (DWT): Ability to Detect Signal Discontinuities

The most famous among the wavelet transforms are the CWT and the DWT. In fact it is the same transform, with the DWT issued by discretization of the parameters of the CWT. The CWT permits simultaneous time-frequency analysis with the signal being analyzed to basic functions called wavelet functions. All wavelet basis functions are generated by dilations and translations of a single function $\psi(t)$, called 'mother wavelet'. The functions generated by this method have the form:

$$\psi_{a,b}(t) = \psi\left(\frac{t-b}{a}\right) , \tag{1}$$

where a represents the scale or dilation parameter and b represents time-shift or translation parameter. To compare this transform with the well-known Fourier transform, the basis functions in the wavelet transform case are not sinusoidal sequences of infinite length. On the contrary they have compact support, e.g. they have finite time duration. This duration is related to the time resolution of the transform. The Fourier transform of a wavelet atom, reveals the frequency resolution of the wavelet. Varying a, results to another basis function at a different scale. If a is increased, the time resolution is decreased, the center frequency of the wavelet is also decreased, while the frequency resolution provided by this wavelet is increased. This is due to the inverse relation between the duration of a signal and the bandwidth of its frequency content. Varying b, the entire wavelet at the scale a is positioned at the next time-interval, and the resemblance of the signal to be analyzed with the wavelet at this interval is obtained. So both parameters a and b permit a time-frequency localization of the signal with the time-frequency tiling shown in Fig. 1.

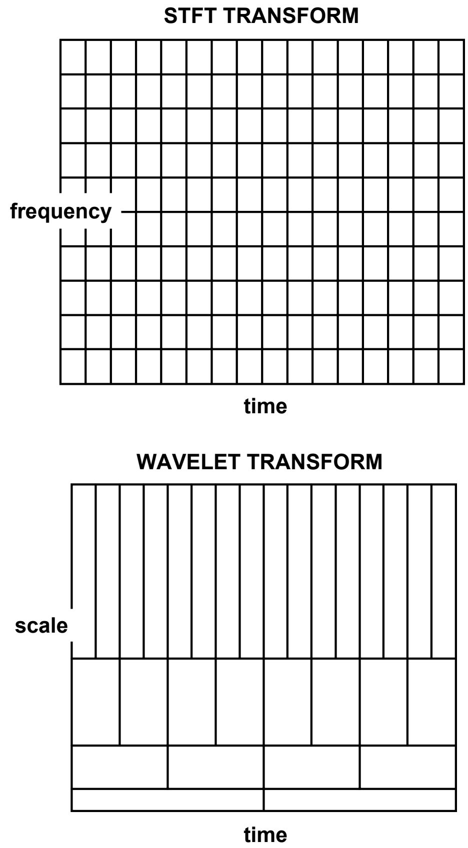

Fig. 1. Comparison of time-frequency tiling between the STFT and the Wavelet Transform.

The CWT of a signal $f(t)$ with respect to the wavelet $\psi(t)$ is defined as:

$$\begin{aligned} W(a,b) &= \int_{-\infty}^{+\infty} f(t)\cdot\tfrac{1}{\sqrt{|a|}}\cdot\psi\left(\frac{t-b}{a}\right)\cdot dt = \\ &= \int_{-\infty}^{+\infty} f(t)\cdot\tfrac{1}{\sqrt{|a|}}\cdot\psi_{a,b}(t)\cdot dt \ , \end{aligned} \tag{2}$$

where

$$\psi_{a,b}(t) = \tfrac{1}{\sqrt{|a|}}\cdot\psi\left(\frac{t-b}{a}\right) . \tag{3}$$

From the definition of the wavelet transform, it is obvious that it can be interpreted as the cross-correlation between the signal $f(t)$ and each scaled by a version of $\psi(t)$, at a time lag equal to the time shift b since:

$$W(a,b) = < f(t), \psi_{a,0}(t-b) > = R_{f,\psi_{a,0}}(b) \ , \tag{4}$$

where < , > denotes the inner product of the functions. From this viewpoint, the wavelet decomposition calculates at each scale a the resemblance between the signal and the scaled by 'a' wavelet [11]. By shifting the wavelet function by b, the resemblance to the next time interval is calculated. By successively changing a and b we obtain the whole time-frequency analysis of the signal, at different scales and time intervals. The display of the wavelet time-frequency analysis is called "scalogram", in contrast with the corresponding time-frequency analysis provided by the STFT, that is called "spectrogram". Given the relation connecting the cross-correlation between two signals to their convolution,

$$R_{x,y}(l) = x(l) * y(-l) \,, \tag{5}$$

the wavelet transform can be expressed as:

$$W_{a,b} = f(b) * \psi_{a,0}(-b) \,. \tag{6}$$

Thus, for any given a, the wavelet transform $W(a, b)$ of the signal $f(t)$ can be considered the output of a filter with impulse response the time-reversed wavelet function at scale a $\psi_{a,0}(-b)$. Since from the well-known Fourier transform property [12]

$$FT\left\{\psi\left(\frac{t}{a}\right)\right\} = |a| \cdot \Psi(a \cdot \omega) \,, \tag{7}$$

and time reversal does not change the magnitude of the Fourier transform of a real-valued wavelet $\psi(t)$, when moving from a lower scale a to a higher one (when we increase a), the center frequency and the bandwidth of the filter is decreased (the frequency resolution is increased), and the wavelet duration is increased (the time resolution is increased). So the filter bank created is not uniform. On the contrary, what is constant in the wavelet transform time – frequency analysis is not the bandwidth of the filters but the ratio of the center frequency to the filter bandwidth (Constant Q-factor filter bank).

The transition from the CWT to the DWT is performed by letting [8]

$$a = 2^k \text{ and } b = 2^k \cdot l \,, \tag{8}$$

where k and l are integers. In that case the wavelet functions become:

$$\psi_{j,l}(t) = 2^{-\frac{j}{2}} \cdot \psi\left(2^{-j} \cdot t - l\right) \,. \tag{9}$$

Here another function is used, scaling function [7] where

$$\phi_{j,l}(t) = 2^{-\frac{j}{2}} \cdot \phi\left(2^{-j} \cdot t - l\right) \,, \tag{10}$$

This new function is added to obtain a complete signal representation that permits the complete signal analysis by creating a perfect reconstruction wavelet filter bank. In this construction the wavelet functions correspond to the high-pass filters while the scaling functions correspond to the low-pass filters. The perfect reconstruction filter bank permits, by appropriately combining the coefficients obtained, to recover the original signal. The signal representation obtained by using the DWT is:

$$f(t) = \sum_k c_{j_o,k} \cdot \phi_{j_0,k}(t) + \sum_k \sum_{j=j_0}^{\infty} d_{j,k} \cdot \psi_{j,k}(t) \,, \tag{11}$$

where $c_{j,k}(t)$ are the scaling function or approximation coefficients and $d_{j,k}$ are the wavelet or detail coefficients. To perform wavelet analysis to the signal we start from the samples themselves of the signal, which constitute the finest analysis level. By increasing j, the analysis becomes coarser. As the level of analysis j is increased, the approximation coefficients provide coarser and coarser approximation of the signal. The detail coefficients provide the detail lost in each approximation, while moving from the finer scale j-1 to the coarser scale j. The detail coefficients, related to high-pass filters, are ideal in identifying high-frequency abrupt changes or discontinuities localized in time.

A wavelet transform having n vanishing moments, can be interpreted as a multiscale differential operator of order n [7]: This is because if the wavelet $\psi(t)$ has n vanishing moments, there exists a fast decaying function $\theta(t)$ such that:

$$\psi(t) = (-1)^n \cdot \frac{d^n \theta(t)}{dt^n} \,. \tag{12}$$

Substituting this equation to the dilated and translated version of wavelet $\psi_{a,b}(t)$ we get:

$$\psi_a(-b) = a^n \cdot \frac{d^n \overline{\theta}_a(t)}{dt^n} \,,$$

where

$$\overline{\theta}_a(t) = a^{-\frac{1}{2}} \cdot \theta\left(-\frac{t}{a}\right) \,. \tag{13}$$

Substituting this equation to the convolutional interpretation of the wavelet transform given in (6) we get:

$$W(a,b) = a^n \cdot f * \frac{d^n \overline{\theta}_a}{dt^n}(b) = a^n \cdot \frac{d^n}{db^n}\left(f * \overline{\theta}_a\right)(b) \,. \tag{14}$$

It is for this reason that the wavelet transform can locate singularities in a signal. The singularities are detected as high amplitude detail coefficients at fine scales. In the case of a direct sequence spread spectrum signal, the singularities carry the essential information of the change of sign of the chipping sequence. The ability of the CWT to detect phase changes in a BPSK signal is also explained in [13], as a means to identify the modulation of an intercepted digital signal. In the below sections we expand this ability of the CWT to identify and to detect characteristics of a PSK digital signal, in which is applied extra spreading modulation.

3. Application of DWT to a BPSK Signal with a BPSK Spreading Modulation

The application of the DWT to a BPSK signal with BPSK spreading modulation is shown in Fig. 2.

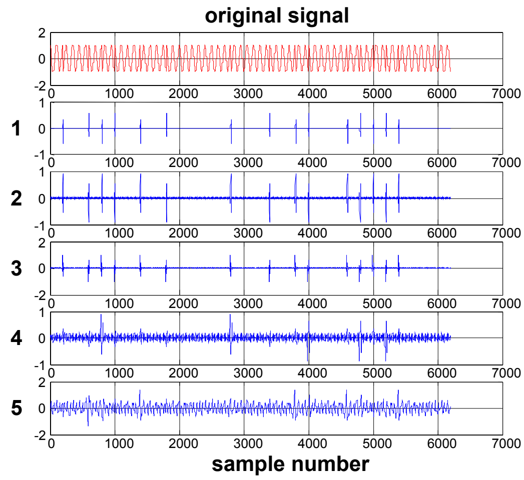

Fig. 2. Detail coefficients for the DWT of a BPSK direct sequence spread spectrum signal analyzed using the Daubechies 2 (*db2*) wavelet.

The information signal to be analyzed is a constant envelope data-modulated carrier with amplitude A_c, frequency f_c and phase θ equal to either zero or π. This signal is described by the equation [12]:

$$s_d(t) = \sqrt{2} \cdot A_c \cdot \cos(2\pi f_c t + \theta) \ , \tag{15}$$

and can also be written as

$$s_d(t) = \sqrt{2} \cdot A_c \cdot d(t) \cdot \cos(2\pi f_c t) \ , \tag{16}$$

where $d(t)$ represents the binary data and is modeled as a polar random binary wave:

$$d(t) = \pm 1 \quad \text{for} \quad (n-1) \cdot T_b \leq t \leq n \cdot T_b \ . \tag{17}$$

This signal is remodulated using a spreading waveform

$$c(t) = \pm 1 \quad \text{for} \quad (n-1) \cdot T_c \leq t \leq n \cdot T_c \tag{18}$$

to give:

$$s_t(t) = \sqrt{2} \cdot A_c \cdot c(t) \cdot d(t) \cdot \cos(2\pi f_c t) \ . \tag{19}$$

where $T_c << T_b$. The phase changes that this signal contains are equal to $\pm\pi$. As it will be explained later, this amount of phase change characterizing the BPSK modulation is the most favorable for the wavelet transform to analyze, compared with the other modulation methods, where phase changes of less extent are contained in their respective signals. To this analysis the *db2* wavelet was used. It is obvious that the *db2* wavelet transform efficiently detects the occurrences of the phase changes to the signal. The consistency between the peaks at scales 1, 2 and 3 and the phase changes is evident. For scales greater than 3, the wavelet detail coefficients become "noisy" and not useful for the purpose of the analysis considered.

The *db2* wavelet is a wavelet having 4 vanishing moments. By increasing the number of wavelet vanishing moments and successively using to the analysis *db3*, *db4* and so on, we still got the same good results, with the difference that detection can be performed, each time at greater levels of analysis. For *db6*, the detail coefficients not only at level 1 but also at levels 2, 3, 4, and 5 have peaks corresponding to phase changes as shown in Fig. 3.

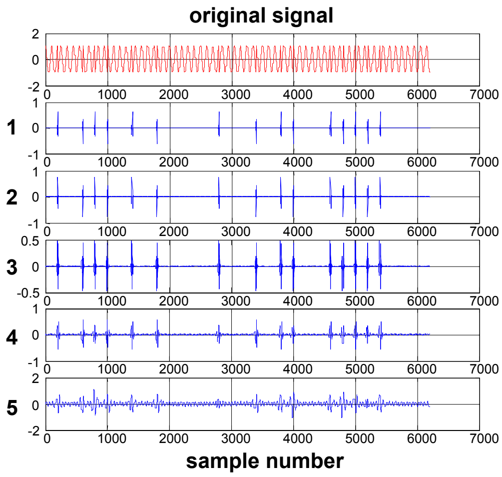

Fig. 3. Detail coefficients for the DWT of a BPSK direct sequence spread spectrum signal analyzed using the Daubechies 6 (*db6*) wavelet. The phase changes are traced from the detail coefficients at all levels shown (1 to 5).

This phenomenon can be further explained by realizing that a sudden change or discontinuity is high-frequency information, with the rupture containing the high-frequency part. For a singularity to be detected, the wavelet used must be sufficiently regular, which implies a longer filter impulse response. For the 'Daubechies' wavelet, an increase to the wavelet order means increased number of vanishing moments, regularity, and filter length [7], [8].

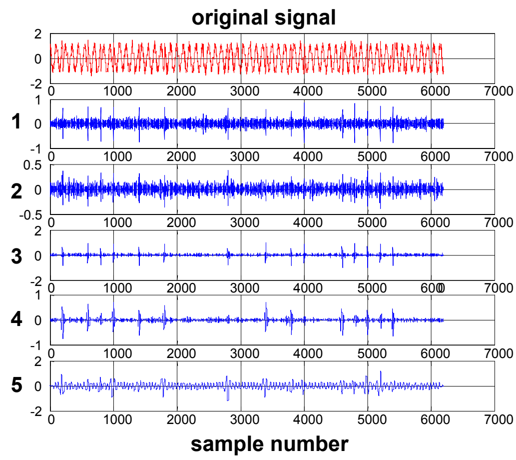

Fig. 4. Noisy version of the ds-bpsk signal with noise variance $\sigma^2 = 0.2$ analyzed using the Daubechies 4 (*db4*) wavelet and the DWT. The detail coefficients at more than one level need to be consiered in order to efficiently track all the phase changes in the signal.

The next step is to use the same signal as above, with zero-mean AWGN added. To investigate the performance of the DWT at this case, the noise variance was gradually increased, until a very 'dirty' signal with noise variance $\sigma^2 = 0.2$ was finally obtained. In that case the wavelet coefficients at levels 1 and 2 are also very noisy, and by themselves they are not able to demonstrate consistency with the phase changes appearing at the BPSK signal. However, the phase changes can still be traced if not only the peaks of the detail coefficients at levels 1 and 2, but also those of the detail coefficients at levels 3, 4 and 5 are appropriately combined [14]. The noise variance $\sigma^2 = 0.2$ was experimentally found to be the limiting value that still permitted the detection of the phase changes. The noisy signal with this value of variance analyzed using the *db4* wavelet is shown in Fig. 4.

Next the pulse shaping function used was changed from rectangular to raised cosine. This is usually the case for either BPSK or QPSK communication signals, where the raised cosine pulse shaping functions are used to reduce inter-symbol interference. This case is shown in Fig. 5 where the "haar" wavelet is used for the analysis.

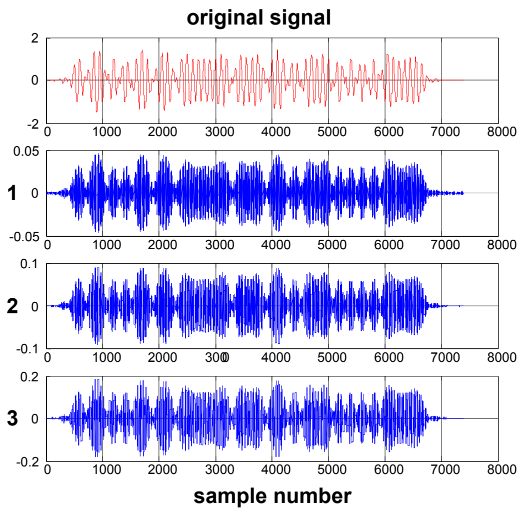

Fig. 5. Detail coefficients for the DWT of a BPSK direct sequence spread spectrum signal with raised cosine pulse shaping function, analyzed using the Daubechies 2 (*db2*) wavelet.

The application of this pulse shaping function drastically smoothens the abrupt phase changes noticed before. In fact, the prior abrupt phase changes are now converted to a diminution of amplitude of the signal around the change of sign area, that appears to the signal as an effect of the sign changing of the pn-sequence. In that case, trying to locate the phase changes by using peaky output of detail wavelet coefficients is useless. Fortunately, another method can be used. Since the wavelet transform works as a correlation between the signal analyzed and the wave - shaped wavelet, the areas where the signal amplitude is reduced due to the change of sign of the pn-sequence, also have lower amplitude wavelet coefficients. This effect is clearly shown in Fig 4., where the envelope of the wavelet coefficients at levels 1, 2 and 3 have local minima in the areas of the signal that coincide with the change of sign of the raised cosine shaped pn-sequence.

The next step is to derive the chipping sequence from the phase changes' detection obtained. No matter what spreading code is used in a direct sequence spread spectrum communication system, this code needs to be periodic. The necessary periodicity of the spreading code is a vulnerability that can be exploited. The important point is that any periodic sequence can be generated by a linear feedback shift register. For this type of register, the recursive relation

$$b_i = \sum_{m=1}^{r} g_m \cdot b_{i-m} \ , \tag{20}$$

where g_0, g_1, ..., g_r represents the generative polynomial and r the number of registers, relates every chip generated with the last r chips. Thus for an r-stage linear feedback shift register, the solution of $2r$ equations will provide the feedback connections of the generator [1].

4. Application of CWT to a MSK Signal with BPSK Spread Spectrum Modulation

In Fig. 6 the application of the *db4* continuous wavelet transform for the scales a = 1 to a = 100 is shown.

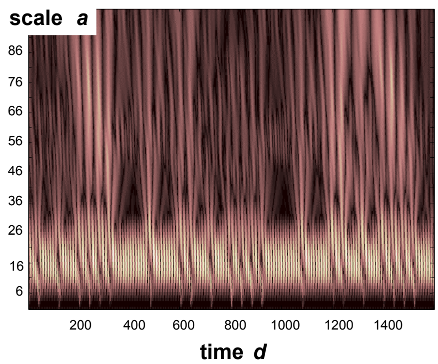

Fig. 6. Application of the Daubechies 4 (*db4*) CWT to a direct sequence MSK signal for a=1 to a=100. The levels 1 to 5 reveal the periodicity of the pn-sequence. The different coloring in levels 30 to 100 shows the frequency modulation existing in the signal.

The information signal to be analyzed is a Minimum-Shift Keyed signal. The form of MSK signal used here is the "Type I" MSK that is defined by the equation [13]:

$$s_d(t) = \sqrt{2} \cdot A_c \cdot \left[d_I(t) \cdot \cos\left(\frac{\pi \cdot t}{2 \cdot T_b}\right) \cdot \cos(2\pi f_c t) - d_Q(t) \cdot \sin\left(\frac{\pi \cdot t}{2 \cdot T_b}\right) \cdot \sin(2\pi f_c t) \right] , \tag{21}$$

where $d_I(t)$, $d_Q(t)$ are modeled as polar random binary waves

$$d_I(t), d_Q(t) = \pm 1 \ \text{ for } \ (n-1) \cdot T_b \le t \le n \cdot T_b \ , \tag{22}$$

and represent the in-phase and quadrature data streams respectively, A_c is the carrier amplitude and f_c is the carrier frequency. MSK signal can be viewed either as frequency or as phase modulation [15]. In the latter case, the phase changes it contains are only equal to $\pm\pi/2$. This information signal is remodulated as before being multiplied by the polar random binary wave $c(t)$ that represents the pn-sequence. So the direct sequence MSK signal to be analyzed has the form:

$$s_d(t) = \sqrt{2} \cdot A_c \cdot c(t) \cdot \left[d_I(t) \cdot \cos\left(\frac{\pi \cdot t}{2 \cdot T_b}\right) \cdot \cos(2\pi f_c t) - d_Q(t) \cdot \sin\left(\frac{\pi \cdot t}{2 \cdot T_b}\right) \cdot \sin(2\pi f_c t) \right]. \quad (23)$$

Since the spreading waveform is still of BPSK type, the changes of phase of interest are still of $\pm\pi$ type. The signal used was intensively constructed to contain several repetitions of the chipping sequence. As before, the discontinuities to the signal waveform caused by the change of sign of the chipping sequence, give high detail coefficients in the analysis levels 1 to 5. The color display of the signal's CWT at scales 1 to 3 reveals the periodicity of the chipping sequence. The CWT calculates a "resemblance index" between the signal and the wavelet [10]. If the signal is similar to itself at different scales, the wavelet coefficients are also similar at different scales. However, the self-similarity does not concern the scales from 38 to 100. On the contrary, what is observed at these scales is that there are areas with high amplitude coefficients, while some other areas have low amplitude coefficients. This is due to the different frequencies used in the MSK modulation, which provide an uneven scalogram at these scales relative to time.

In the case that the same signal as above is used with zero-mean AWGN added, the same results as in the previous paragraph are obtained. The more dirty the signal becomes, the more it is necessary to appropriately combine by thresholding detail coefficients at more than one level, in order to trace signal's phase changes. This process can be efficiently applied up to a noise variance of $\sigma^2 = 0.02$. It is worth to notice in this case, a considerable reduction to the maximum value of the noise variance allowed, for the wavelet transform to be able to properly work. The pn-sequence is still of BPSK type. What changes is that in the last example the carrier and the pn-sequence were perfectly synchronized. So the phase changes appeared as transitions from the most possible positive point of the waveform (+1) to the most possible negative point (-1) of the waveform and vice-versa. In the MSK case, the signal undergoes frequency/phase modulation and this synchronization is no longer a fact. The transitions are no longer from maximum to maximum but also between intermediate points of the waveform. The efficiency of the wavelet transform can be increased if we increase the sampling frequency used. In that case more points are devoted to the phase change area and the detail coefficients obtained are more peaky.

5. Application of the DWT to a QPSK-Spread Direct Sequence QPSK Signal

Next the application of the DWT to the detection of the phase changes of a QPSK spread direct sequence QPSK signal is demonstrated. Fig. 7 shows the detail coefficients of the discrete wavelet transform obtained by using the *db4* wavelet to a noiseless signal version.

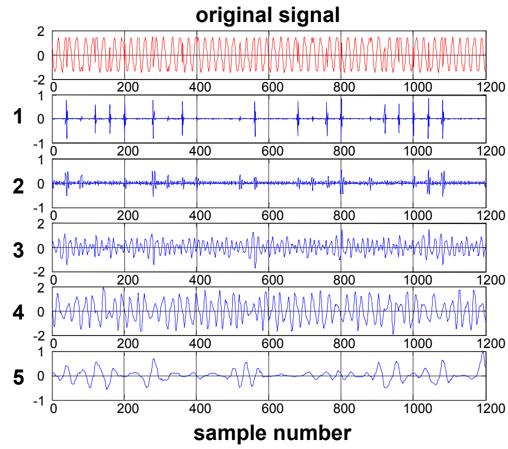

Fig. 7. Application of the Daubechies 4 (*db4*) CWT to a QPSK spread direct sequence QPSK signal. The less amount of phase change compared with the BPSK spreading case results in less amplitude detail coefficients. Even for a noise-free signal the detail coefficients at more than one level need to be considered in order to efficiently trace all the phase changes in the signal.

The signal to be analyzed was constructed to have the form [2]:

$$s_t(t) = \sqrt{2} \cdot A_c \cdot \left[c_I(t) \cdot d_I(t) \cdot \cos(2\pi f_c t) - c_Q(t) \cdot d_Q(t) \cdot \sin(2\pi f_c t) \right]. \quad (24)$$

The meaning of the above equation is that not only the data sequence is separated to in-phase and quadrature information symbols, but also that two different pn-sequences are used: One to remodulate the in-phase information symbols and one to remodulate the quadrature information symbols. So the QPSK spreading modulation appears to the signal as an extra phase modulation to the carrier, equal to 0, $\pi/2$, π or $3\pi/2$. Thus from chip to chip transmitted, phase changes of either $\pm\pi$ or $\pm\pi/2$ are encountered. This is the reason why from this very first application of the DWT to the noiseless signal, it becomes apparent that the detection of the phase changes is more difficult, compared with all the cases treated before. The reduction to the amount of phase change results to a corresponding reduction to the amplitude of the detail coefficients. The need to consider not only the first level of details, but to apply a decision rule based on more levels of detail coefficients in order to efficiently detect the phase changes is obvious. The next step is to proceed by working as in the previous cases and adding zero-mean AWGN of gradually increased noise variance to the signal. The DWT was experimentally found to properly work up to a noise variance of $\sigma^2 = 0.02$.

Another drawback in this case is that the phase changes encountered are the simultaneous effect of the changes of signs of two chipping sequences, that of the in-phase and that of the quadrature component. So the chipping sequences cannot be detected as in the case of the BPSK spreading modulation. The two components in quadrature need to be separated from each other and processed independently. The chip duration can still be found as the minimum time difference observed between two successive phase changes, as well as the identification of the exact modulation used [13]. The situation just described is worsened in the case that the intercepted signal is not a single direct sequence spread spectrum channel but a CDMA signal. The co – existence of many signals code – multiplexed in a common bandwidth by individual chipping sequences, renders this method useless in terms of extracting these chipping sequences. The phase changes encountered are the combined result of the multiplication of all the chipping sequences with the carrier so they become indistinguishable from each other if seen from the wavelet transform perspective. So from the communicator's side, one should aim to multiplex his signal into a CDMA scheme to render it not exploitable for an enemy interceptor. On the other hand from the interceptor's side, one should seek to intercept a single uplink channel, hopefully not CDMA, in order to extract the chipping sequence of the particular user of this channel.

6. Application of the CWT to a BPSK Spread Radar Pulse with Linear FM Chirp Compression

This task is shown in Fig. 8, where the *db4* wavelet is used for the analysis at levels 1 to 64.

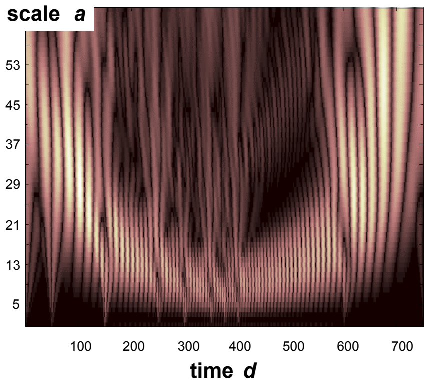

Fig. 8. Application of the Daubechies 4 (*db4*) CWT to a radar pulse where bpsk direct sequence spread spectrum is used in conjunction with linear FM chirp pulse compression. The levels 1 to 5 detect the phase changes of the pn-sequence while the levels 6 to 64 reveal the frequency modulation in the pulse.

The linear FM chirp pulse compression in the pulse is expressed by the equation [5]:

$$s_c(t) = \sqrt{2} \cdot A_c \cdot \cos\left(2\pi f_c t + \gamma \cdot t^2\right), \tag{25}$$

where γ is the input frequency slope. For the first half of the pulse a positive γ is used, while for the second half of the pulse an equal magnitude negative slope γ is used. So the pulse is simulated to contain a linear frequency transition from a minimum to a maximum value, and back to the minimum value. This frequency modulated waveform is further multiplied by the BPSK spreading waveform represented by the polar random binary wave $c(t)$ as before, to give:

$$s_t(t) = \sqrt{2} \cdot A_c \cdot c(t) \cdot \cos\left(2\pi f_c t + \gamma \cdot t^2\right) \tag{26}$$

The scales 1 to 5 of the signal's CWT clearly contain peaky coefficients corresponding to the changes of sign of the chipping sequence. The scales 6 to 64 reveal the frequency transition existing in the signal. This transition, although it is linear, does not appear to have linear shape to the CWT image. On the contrary, it has exponential shape. This happens because the CWT filter bank is not uniform, but the center frequency of the filter and its bandwidth are logarithmically increased if the scale a is decreased. The CWT in this case is very revealing and exposes both types of the modulation applied to the pulse.

In the case that the spreading code and the pulse compression technique are found, an eavesdropper can effectively identify the target, and further use the information obtained to transmit a highly effective jamming signal. However, since in radar systems the transmitter and receiver both belong to the same apparatus, the pulse compression technique and the chipping sequence used can be changed in a pulse-to-pulse basis. In that case the information obtained from a certain cycle of the radar operation will be useless to the next cycle. So for the radar case, the only effective jamming method still remains fast repeater jamming based on a memory loop with good operational characteristics [16].

7. Conclusion

Both the DWT and the CWT constitute a valuable tool than can be efficiently used to the detection of the features of digitally modulated direct-sequence spread spectrum communication signals. In all cases the CWT provides a scalogram that is characteristic of the modulation used. This scalogram localizes in time and scale (frequency) those changes that can be efficiently used to identify the different modulation schemes (data and spreading) that characterize each particular signal. The phase changes that appear to the waveforms of these signals contain the essential information of the change of sign of their corresponding chipping sequence. Where the rectangular pulse shaping function was used, the consistency between the peaks of the wavelet transform at high levels of analysis and the phase changes, allows them to be detected. The more abrupt the phase change, the easier it is to detect. An increase to the sampling frequency devotes more points to the change of phase areas and facilitates their detection. The use of a wavelet with increased number of vanishing moments also permits the detection across more levels of analysis. The *db4* wavelet was proved to provide the best results in all cases. In the case of a noisy signal, the detection of the phase changes is still possible by using the outputs at not only one but at several scales.

When the raised cosine pulse shaping function is used by the chipping sequence, the detection of the phase changes is performed by using a 'haar' wavelet and by considering the local minima of the envelope of the detail coefficients at high scales.

The method is not considerable usefulness in the case that a CDMA direct sequence spread spectrum signal is encountered. The result of its application is limited to the extraction of simple parameters of the transmitter's modem e.g. the chip duration, the type of spread spectrum modulation used (BPSK or QPSK) and the type of the pulse shaping function (rectangular or raised cosine).

References

[1] PETERSON, R. L., ZIEMER, R. E., BORTH, D. E. *Introduction to Spread Spectrum Communications*. Englewood Cliffs, NJ: Prentice–Hall, 1995.

[2] LEE, J. S., MILLER, L. E. *CDMA Systems Engineering Handbook*. Boston: Artech House Publishers, 1998.

[3] SIMON, M., OMURA, J., SCHOLTZ, R., LEVITT, B. *Spread Spectrum Communications Handbook*. New York: McGraw–Hill, 1994.

[4] SKOLNIC, M. I. *Introduction to Radar Systems*. 2nd ed. New York: McGraw–Hill, 1980.

[5] SKOLNIC, M. I. *Radar Handbook*. 2nd ed. New York: McGraw–Hill, 1990.

[6] VITERBI, A. J. *Principles of Spread Spectrum Communications*. Massachusets: Addison–Wesley, 1995.

[7] MALLAT, S. *A Wavelet: Tour of Signal Processing*. 2nd ed. San Diego: Academic Press, 1999.

[8] BURRUS, S. et. al. *Introduction to Wavelets and Wavelet Transforms: A Primer*. New Jersey: Prentice Hall, 1998.

[9] RAO, R. M., BOPARDIKAR, A. S. *Wavelet Transforms: Introduction to Theory and Applications*. Massachusetts: Addison Wesley, 1998.

[10] BHOURI, N. H., COCHRAN, D. Multiresolution time – frequency techniques for spread spectrum demodulation and jamming, signals, systems and computers. In *Record of the Twenty-Sixth Asilomar Conference*. 1992, p. 105 – 107.

[11] MISITI, M., MISITI, Y., OPPENHEIM, G., POGGI, J. M. *Wavelet Toolbox For Use with Matlab*. Natick, MA: The Mathworks, Inc., 1997.

[12] HAYKIN, S., *Communication Systems*. 3rd ed. New York: John Wiley & Sons, Inc. 1994.

[13] HO, K. C., PROKOPIW, W., CHAN, Y. T. Modulation identification of digital signals by the wavelet transform, radar, sonar and navigation. *IEE Proceedings*. 2000, vol. 147, no. 4 , p. 169 - 176.

[14] STRANG, G., NGUYEN, T. *Wavelets and Filter Banks*. Wellesley MA: Wellesley–Cambridge Press, 1997.

[15] PROAKIS, J. G., SALEHI, M. *Communication Systems Engineering*. New Jersey: Prentice–Hall International Editions, 1995.

[16] SCHLEDER, D. C. *Introduction to Electronic Warfare*. Norwood, MA: Artech House, 1986.

[17] PROAKIS, J. G., MANOLAKIS, D. G. *Digital Signal Processing, Principles, Algorithms and Applications*. 3rd ed. New Jersey: Prentice–Hall, 1998.

[18] PROAKIS, J. G. *Digital Signal Processing Using Matlab*. Boston: PWS Publishing Company, 1998.

[19] PROAKIS, J. G. *Digital Communications*. 3rd ed. New York: McGraw–Hill, 1995.

[20] PROAKIS, J. G., SALEHI, M. *Contemporary Communication Systems Using Matlab*. Boston: PWS Publishing Company, 1998.

[21] AKANSU, A. N., HADDAD, R. A. *Multiresolutional Signal Decomposition*. 2nd ed. San Diego: Academic Press, 2001.