Analysis of Rain Rate Spatial Cross-Correlation Coefficients in the Basque Country Area

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Abstract. *This paper reports on a study on the spatial characteristics of rain rates recorded in the Basque region in northern Spain. To perform this study a network of eighty raingauges spread throughout the region has been used. Statistical parameters such as the spatial cross-correlation coefficient and the space-time cross-correlation coefficient were calculated and their evolution with separation distance studied. It is hoped that information reported could be useful in better understanding characteristics of rain and in developing countermeasures for terrestrial and satellite radio networks operating at frequencies above 10 GHz.*

Keywords

Spatial cross-correlation coefficient, space-time cross-correlation coefficient.

1. Introduction

Rainfall is the main cause of fades in the frequency bands above 10 GHz. To optimize the planning of satellite and terrestrial radio-communication systems operating in these frequency bands and using fade countermeasures such as site diversity, route diversity or on-board resource management techniques, it is important to have a detailed knowledge of the spatial distribution of rain rate. Thus, the need of investigating joint rainfall rates statistics for two or more sites is evident.

The actual phenomenon of interest for radio links is rain induced attenuation, however data on concurrent attenuation levels at different links (e.g. converging links, as for example in site or route diversity configurations) is very difficult to come by while rain data recordings are fairly common, with dense raingauge networks being available, in some cases.

The study of the spatial behavior of rain rate can be carried out at different levels covering different geographical extensions. Thus the study can be performed at local scale analyzing the sizes, shapes, trajectories and speeds of individual rain cells. Rain cell structure information can be gathered from meteorological radars and also from dense networks of raingauges, as in [1], where data from a tightly spaced network of gauges used to monitor and countering possible flooding in the city of Barcelona has been used for the study of the structure of rain at local scale.

Other studies have concentrated on the analysis of much larger areas, e.g. the whole of Italy [2], the UK [3] or Spain [4]. These correspond to a large scale study and the foreseen application in [2] was that of satellite resource management to focus the extra available satellite power on the areas most needing it. Thus, the need for a detailed knowledge of joint probability information.

The novelty of this study is that it looks into the spatial structure of rain for areas in between those referred to in the above paragraphs.

This paper presents a statistical study of the spatial variability of rain rate throughout the Basque region (Comunidad Autónoma Vasca, CAV), in the north of Spain, with a surface of 7,234 km^2.

Average separation between raingauges	**46.2 km**
Maximum separation	**123.0 km**
Minimum separation	**1.4 km**
Standard deviation	**23.7 km**
Median	**43.9 km**
Number of pairs of gauges in terms of their separation	
< 2 km	**2**
< 5 km	**23**
< 10 km	**115**
< 15 km	**253**
> 15 km	**2847**

Tab. 1. Characteristics of the raingauge network used in this study.

In this study, rain rate data from 80 raingauges spread throughout the Basque region have been used. The average distance between gauges is 46.2 km, the minimum separation being 1.4 km, the maximum distance 123.0 km and the surface density of 1.10 per 100 km^2 (Tab. 1). The duration of the recordings, is of the order of 10 years. Fig. 1 shows the locations of the raingauges. An integration time of 10 minutes has been used in this study.

To analyze the spatial structure of rain rate three thresholds have been considered, namely, 2 mm/h, 10 mm/h and 25 mm/h, corresponding approximately to the following exceedance probabilities 1 %, 0.01% and 0001 % for the average year in this particular geographical region.

The study is carried out through the following parameters for all possible pairs of raingauges:

- The spatial cross-correlation coefficient;
- The space-time cross-correlation coefficient.

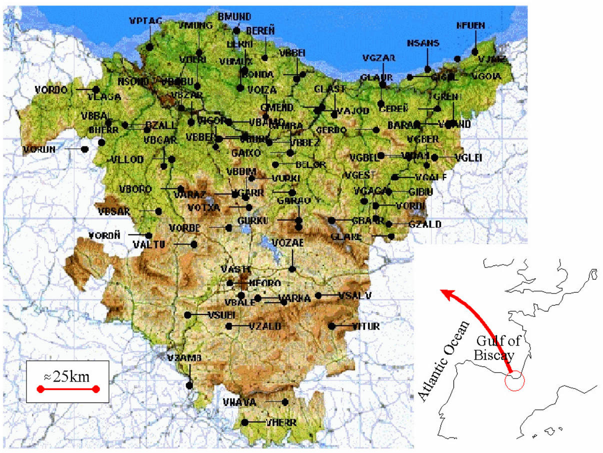


Fig. 1. Location of raingauges in the study area: Basque region.

2. Space-Time Characterization of Rain Rates

As just mentioned, to characterize the space and time variability of rain rate in the Basque region, four statistical parameters have been studied for all possible pairs of raingauges in a dense mesh consisting of a total of 80 spread raingauges throughout the study area.

The space correlation information describes the similarity between the rain rates observed simultaneously at different spatially separated observation points. Three thresholds: 2, 10 and 25 mm/h were considered in this study.

In addition to a purely spatial study, the joint space-time correlation provides further insight into the rain phenomenon giving an idea of the time evolution of the spatial structure of rain fronts. This behavior can be fairly well correlated with synoptic information showing the direction and velocity of the prevailing winds in the region. The chosen parameter for this study has been the space-time cross-correlation coefficient for couples of gauges for the same three rain rate thresholds and several time offsets.

The raingauges used were of the tipping bucket type. The integration times for the various gauges ranged from 1 second to 10 minutes: 47 gauges had an integration time of 1 s (recording of actual bucket tip time) and 10 min the rest.

Typically, in propagation related studies, the recorded data sets are converted to an integration time, T, that reflects the instantaneous rain rate. A commonly used value of T is 1 minute. Several factors have to be taken into consideration when selecting the integration time in this study. For one, a common value of T is advisable and, even though conversion formulas exist to translate time-series with one integration time to another, it was preferred to increase the integration time of the 1 s gauges to the other, larger value used, 10 min. Thus, the common value of T was set to 10 min which still preserves the instantaneous nature of the measurements, at least for the low to moderate intensities.

On the other hand, it is important to consider the usual propagation velocities of meteorological phenomena (e.g. weather fronts). In the area of study, frontal systems tend to travel in the west-east direction with speeds in the order of 20 to 30 km/h. A value of T = 10 min allows the separation of non-simultaneous events at the raingauges closest to each other (i.e. < 5-10 km).

Increasing the value of T would eventually show non-simultaneous events as simultaneous ones. In the studies described in [2] and [4], the integration time was set to 1 hour. This was acceptable for those studies given the larger distances considered, i.e. several tens of kilometers. In [2] it is shown how little difference exists between cross-correlation coefficients calculated with T = 1 h and T = 15 min.

It has been verified from the data set used here that integration times, T, of 1 h have relative little influence on statistical parameters such as the cross-correlation coefficients for the low to medium rain rates for relatively large separations between gauges, i.e. for separations in the order of 30 to 40 km. For the database of this study as a whole, where separations are as small as 1.4 km, an integration time of 10 min was deemed more suitable.

As for the very large rain intensities, an integration time of 10 minutes would not be enough to really capture such higher rates. In any case, those rates would be uncorrelated anyway for separations larger than a few kilometers.

3. Spatial Cross-Correlation Coefficients

The linear cross-correlation coefficient measures the similarity between two variables. This coefficient, ρ, can be calculated by dividing the covariance by the product of the standard deviations of the two variables

$$\rho = \frac{\mathrm{cov}(x, y)}{\sigma_x \sigma_y} ,$$

where cov(x, y) is the covariance of the two variables, σ_x is the standard deviation of x, σ_y is the standard deviation of y, and where

$$\mathrm{cov}(x,y)=\frac{\sum(X_n-\overline{X})(Y_n-\overline{Y})}{(N-1)}=\frac{\sum x_n y_n}{(N-1)},$$

and

$$\sigma_x=\sqrt{\frac{\sum(X_n-\overline{X})^2}{N-1}}=\sqrt{\frac{\sum x_n^2}{N-1}},$$

and

$$\sigma_y=\sqrt{\frac{\sum y_n^2}{N-1}}.$$

To perform the above calculations, the rain-rate time series from the 80 gauges available, converted to an uniform integration time of 10 minutes were used. For each pair of gauges new binary time-series *Xn*, *Yn* are produced according to the selected threshold *Rt*:

$$X_n=\begin{cases}1 & \text{if } R_n \ge R_t\\ 0 & \text{if } R_n < R_t\end{cases},$$

and

$$Y_n=\begin{cases}1 & \text{if } R_n \ge R_t\\ 0 & \text{if } R_n < R_t\end{cases},$$

where n is the 10 min sample interval number, R_n is the rain rate for interval n and R_t is the rain rate threshold for which the cross-correlation coefficient is being calculated.

The intervals, n, making up each pair of time- series, X_n, Y_n, correspond to the 10 min intervals available for both stations. The number of intervals is thus variable for each pair of gauges.

The possible values of the cross-correlation coefficient are in the [-1,+1] range. A value $\rho = 0$ means complete de-correlation between both variables, so that when one variable takes up a given value there is no influence whatsoever on the value taken up by the other. If $\rho = -1$, it means total inverse dependence so that when one variable is above the threshold the other is with total certainty below the threshold. If $\rho = +1$, then there is total dependence (direct relation). The larger the cross-correlation coefficient the greater their dependence is.

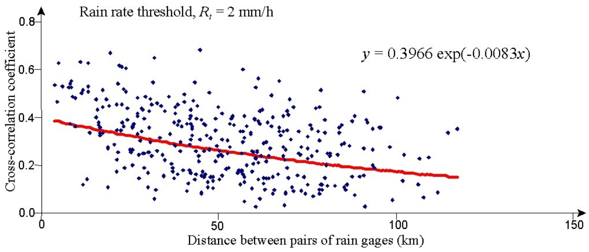
Tab. 2 summarizes the results obtained for the three thresholds considered and Figure 2 shows the computed cross-correlation coefficients as a function of the separation distance between raingauges. One plot is shown for each threshold: low, medium and high rain rate. Also in the Figure exponential fitting curves are shown, the derived formulas were

- For *Rt* = 2 mm/h, $\rho = 0.3966 \exp(-0.0083\, d)$;
- For *Rt* = 10 mm/h, $\rho = 0.2551 \exp(-0.0126\, d)$;
- For *Rt* = 25 mm/h, $\rho = 0.0446 \exp(-0.0212\, d)$;

where d is distance in km.

Threshold, R_t	mean	min.	max.	standard dev.
2 mm/h	0.29871	0.02677	0.68098	0.14111
10 mm/h	0.16476	0.01019	0.58785	0.10299
25 mm/h	0.02735	-0.00264	0.27087	0.03849

Tab. 2. Summary of spatial cross-correlation coefficient results.



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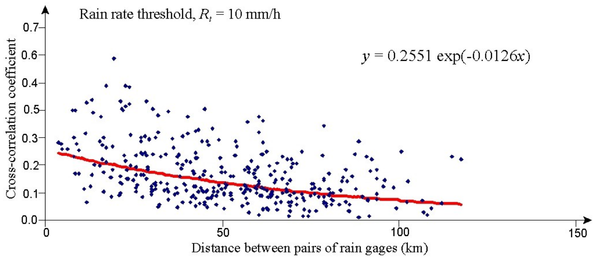

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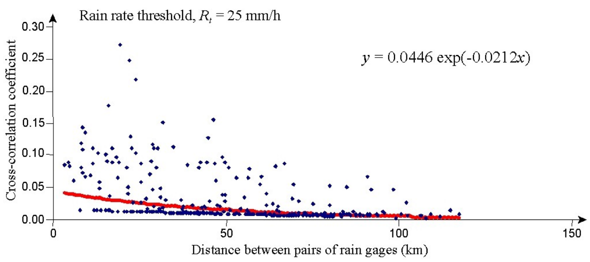

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Fig. 2. Computed cross-correlation coefficients for the 3 rain rate thresholds as a function the distance between gauges.

From Fig. 2 it is clear that the cross-correlation coefficient decreases as the threshold increases. This was to be expected given that the higher the rate the more local the event is. For the lower rates the rain phenomenon tends to be of the widespread type (stratiform). From Figs. 2 and 3 (where results for the three thresholds are plotted together) it can also be observed how the global trend for the cross-correlation coefficient is a decreasing one with the separation distance. For distances larger than 25 km this trend can be fitted to a negative exponential curve.

Comparing these results with the ones presented in earlier studies carried out in the Italy, UK and Spain, similar results can be observed. The more significant difference is that the cross-correlation coefficients are slightly larger, especially for the lower thresholds (low to moderate precipitation). This can be attributed to the smaller distances considered in this study compared to those in the other studies and to the smaller integration time used (10 min v. 1 h). Also, the negative exponential dependence with distance of cross-correlation coefficient matches that reported in [1] and [4] albeit, in this case, it is for separation distances smaller than 100 km.

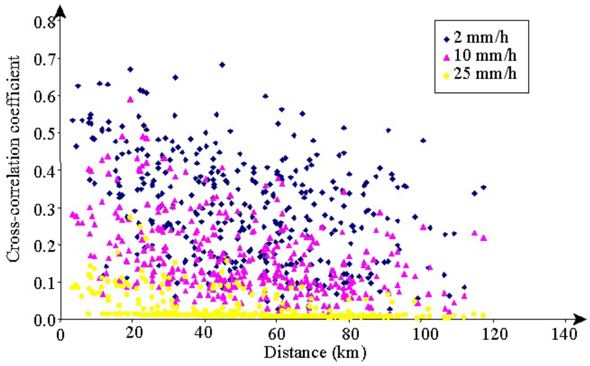


Fig. 3. Cross-correlation coefficient dependence with separation distance.

4. Space-Time Cross-Correlation

The space-time cross-correlation coefficient allows looking into the impact of the trajectory of rain as well as its velocity of travel. To carry out this study the cross-correlation coefficient has been computed between selected pairs of raingauges (i.e. with different separation distances and relative orientations) for different time lags.

In the Basque region, stratiform is the most common type of rain, specially to the north (Gulf of Biscay). To analyze the space-time correlation characteristics and verify the general west-east prevailing trend in this area, a number of gauges have been picked out along the west-east direction with different separations. These stations were Derio (VDERI), Subijana (VSUBI), Bajo Deba (VAJODE) and Jaizkibel (VJAIZ) as shown in Figs. 1 and 4.

Derio (VDERI) is located close to Bilbao and Jaizkibel (VJAIZ) is already close to the French border and both form a straight line fairly parallel to the Cantabrian See coast in the east-west direction. The Bajo Deba (VAJODE) gauge is halfway between those two and it is almost collinear with them (Fig. 4). These three stations belong to a maritime (Cantabrian-Litoral) climatic zone. To complete the study, the gauge at Subijana (VSUBI) was chosen. This gauge is located at the same meridian as Derio (VDERI) but it belongs to the Semi-Mediterranean climatic zone as it is more toward in-land than the other three. Derio (VDERI) and Subijana (VSUBI) form a straight line in the north-south direction, perpendicular to the Derio (VDERI) - Bajo Deba (VAJODE) - Jaizkibel (VJAIZ) line, Fig. 4.

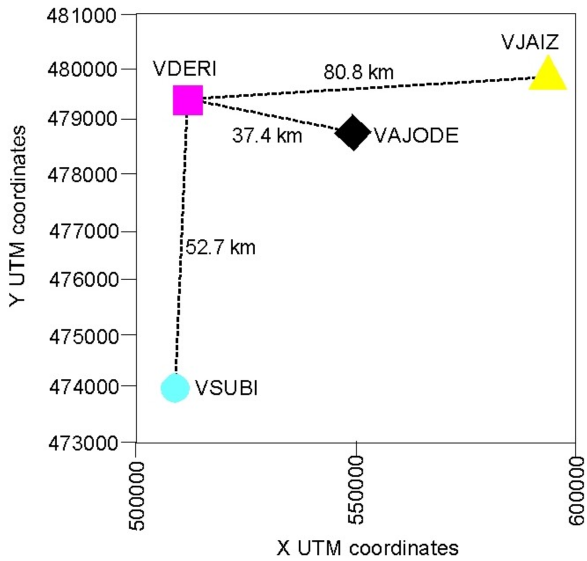
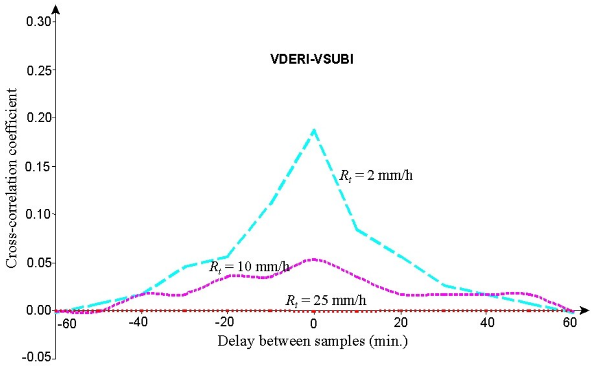


Fig. 4. Relative locations of the raingauges used in the space time correlation study.

For each pair of stations (distances) the space-time cross-correlation coefficients have been computed for a number of time-lags ranging from -60 to +60 minutes. In this study also the lags maximizing the cross-correlation coefficients have been identified. Again, three thresholds: 2, 10 and 25 mm/h have been considered. Fig. 5 shows the cross-correlation coefficients for gauge pairs formed by Derio (VDERI) and the rest of the stations mentioned in Fig. 4.

It can be observed how the maximum values of all gauge couples were obtained for the Derio (VDERI) - Bajo Deba (VAJODE) pair, while the minimum values were observed for the Derio (VDERI) - Subijana (VSUBI) pair; this occured even though they are located at a closer distance than the Derio (VDERI) - Jaizkibel (VJAIZ) pair. However, it must be borne in mind that Derio (VDERI) and Subijana (VSUBI) belong to different climatic zones and several small mountain ranges separate these two gauges. Thus, one straightforward conclusion is that, it is not only the distance that influences the cross-correlation coefficient value. Other aspects like uniformity in climatic zones (both gauges belong or not to the same climatic zones) and the general trend of weather fronts are also important as shown next.

For the Derio (VDERI) - Subijana (VSUBI) couple, the maximum values are obtained for samples with 0 min lags and these show a sharp fall with respect to prior and ensuing samples. Furthermore, the decay slope is gentler for the larger rain rates, albeit the fall is from a much lower value. These results confirm the fact that, for the same meridian, no significant correlation values exist for time lags other than 0 minutes since the direction of travel of weather fronts is typically perpendicular to the north-south direction.



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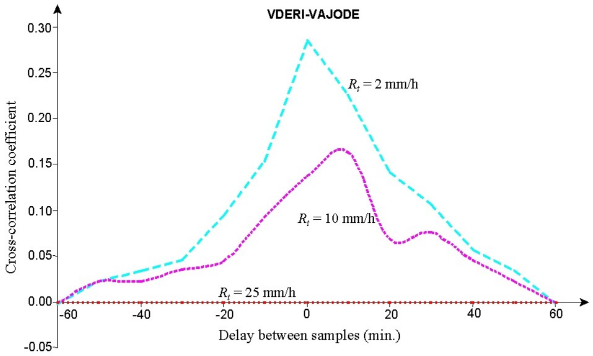

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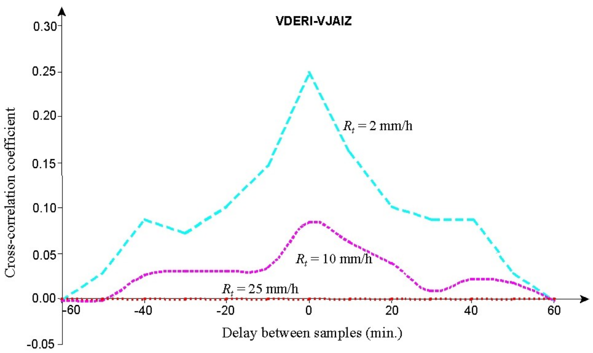

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Fig. 5. Cross-correlation coefficients for pair of raingauges.

In the case of the Derio (VDERI) - Bajo Deba (VAJODE) couple, for R_t = 2 mm/h the maximum cross-correlation coefficient values are obtained for 0-lag samples except that the values are much larger and, still for positive lags, large values are observed. For the medium rate, R_t = 10 mm/h, the maximum is observed at the 10 minute lag. As the time offset increases, the coefficient decreases even though a relative maximum is also observed at the 30 minute lag. The 10 minute maximum reflects the propagation delay of rain fronts as they travel in the west-east direction toward the Bajo Deba (VAJODE) gauge. The smaller maximum at the 30 min lag can be due to a new front coming in from the west after the one showing up at the 10 min lag. For the larger intensities, i.e. R_t = 25 mm/h, zero cross-correlation coefficient values were found, thus reflecting the local character of strong rain episodes.

The maximum values for the Derio (VDERI) – Jaizkibel (VJAIZ) couple are observed for 0-lag samples. The variations of the cross-correlation coefficient are gentler than for the other two gauge pairs analyzed. The slope is even gentler for the positive lags. There exists a low relative maximum for R_t = 10 mm/h at the 40 minute lag, which corresponds to the propagation velocity of rain fronts of moderate to high intensity.

It can be observed for the two pairs of gauges on the west-east direction and for an intensity of 10 mm/h, how the maximum obtained for a 10 min lag at 37.4 km appears some time later (40 min lag) at 80.8 km on the second gauge. This could be explained by the likely scatter in direction of travel of weather fronts with increasing distances, and hence, the relative higher value of the maximum at 37.4 km compared to the one at 80.8 km.

5. Summary

In this paper a study of the spatial characteristics of rain rate in the Basque region (north of Spain) has been presented. This study has followed previous ones covering longer distances up to 1000 km. This paper concentrated on the region of the shorter distances separating the observation points ranging from 1.4 km to over 100 km.

Statistical parameters such as the spatial cross-correlation coefficient and the space-time cross-correlation coefficient have been calculated and their evolution with the separation distance studied.

It is hoped that the information reported here could be useful in better understanding the characteristics of rain, in developing countermeasures for terrestrial and satellite radio networks operating at frequencies above 10 GHz.

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