High Resolution of the ECG Signal by Polynomial Approximation

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Abstract. *Averaging techniques as temporal averaging and space averaging have been successfully used in many applications for attenuating interference [6], [7], [8], [9], [10]. In this paper we introduce interference removing of the ECG signal by polynomial approximation, with smoothing discrete dependencies, to make up for averaging methods. The method is suitable for low-level signals of the electrical activity of the heart often less than 10µV. Most low-level signals arising from PR, ST and TP segments which can be detected eventually and their physiologic meaning can be appreciated. Of special importance for the diagnostic of the electrical activity of the heart is the activity bundle of His between P and R waveforms. We have established an artificial sine wave to ECG signal between P and R wave. The aim focus is to verify the smoothing method by polynomial approximation if the SNR (signal-to-noise ratio) is negative (i.e. a signal is lower than noise).*

Keywords

Polynomial approximation, electrocardiogram ECG, high resolution of the ECG, low-level signals, interference.

1. Introduction

The electrical activity of myocardial cells establishes small current within the body [5]. This fact leads to potential differences on the surface of the body which can be detected using suitable equipment. The graphic record of these body surface potentials as a function of time is known as the electrocardiogram (see Fig. 2). The ECG signals are acquired by placing electrode directly on the torso, arms and legs.

The specialized cardiac conduction system includes the sinoatrial or the sinus node (SA), anterior, middle and posterior the internodal tracts, the atrioventricular node (AV), the His bundle, the right and the left bundle branches and the Purkinje network, as shown in Fig. 1. The specialized conduction system allows conduction of electrical impulses. The sinoatrial node automaticity is not recorded on the surface ECG. The sinus impulse activates the internodal tracts as well as atrial myocardium. Activation of the atrial myocardium produces the P wave on the surface ECG (shown in Fig. 2). During sinus rhythm the initial part of the P wave represents the right atrial activation and the terminal part of the P wave represents activation of the left atrium with some overlap in the middle.

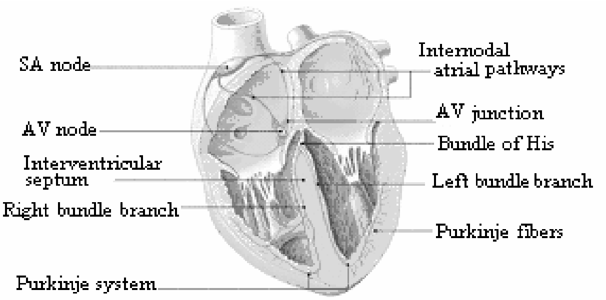


Fig. 1. The specialized cardiac conduction system.

The impulse then depolarizes the AV node, the His bundle, the bundle branches, the Purkinje network and the ventricular myocardium. Propagation of the impulse through the AV node, the His bundle is also not recorded on the surface ECG and occurs during isoelectric PR segment. The Purkinje system efficiently spreads an excitation to muscle cells of the inner wall of the heart. Activation then spreads more slowly from cell to cell through the ventricular myocardium until the entire ventricular muscle mass is depolarized. The excitation begins in the interventricular septum and spreads to the apex, then to the walls of the ventricles and finally to the base of the heart. The QRS complex arises from depolarization of the ventricular muscle. A repolarization is not synchronized from cell to cell as the depolarization and it proceeds in a direction generally opposite to that of the depolarization. The T wave arises from repolarization of the ventricular muscle. The repolarization of the ventricular myocardium is reversed because repolarization of the ventricles spreads conversely as activation of the ventricles from the epicardium toward the endocardium and therefore the T wave is positive (shown in Fig. 2). The U wave that sometimes follows the T wave is second order effect of an uncertain origin and is of little diagnostic significance.

The ECG [4] is generally recorded at a paper speed of 25 mm/sec. At this speed one small square of the ECG paper in the horizontal direction depicts an interval of 40 msec (milisecond) or 0.04 second. In the vertical direction the amplitude of the ECG signals is measured in milivolts is routine (shown in Fig. 2).

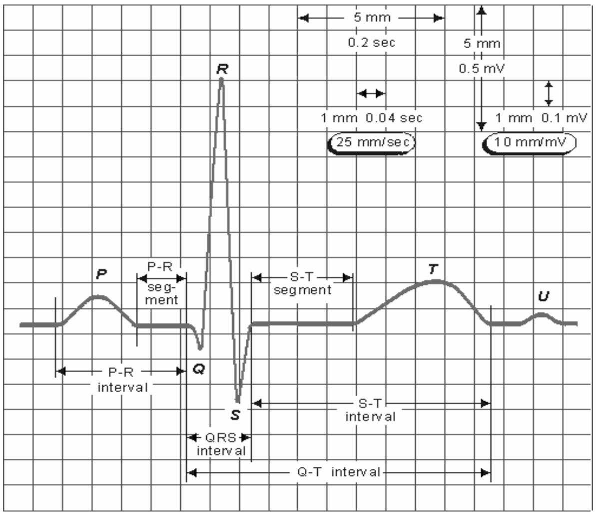


Fig. 2. The standard ECG paper with time intervals and segments.

It is difficult to obtain high-quality electrical signals from biological sources because the signals typically have low level (in the range of μV) and are easily corrupted by interference. The procedure of smoothing by polynomial improves the signal-to-noise ratio (SNR) in low amplitude bioelectrical signal detection and analysis. The His bundle is not recorded by noninvasive methods during isoelectric PR segment, that has been discussed, but using the polynomial approximation we can establish the standard signal measurement of the ECG because we may not increase the amplification of the total gain system on the order $1x10^6$ as has been used to amplify signal for averaging methods.

2. Methods

We use approximation by a polynomial for smoothing measured data and a least square method for the calculation errors (balance) values from smoothing dependencies [1]. We take out $2m+1$ measured values, neighbours of point $x[n]$, where values are $-m$, $-m+1$, -2, -1, 0, 1, 2, m-1, m. We use polynomial of the q-th order across those values.

$$X(n) = \sum_{k=0}^{q} a_k . n^k \tag{1}$$

where $q < 2$, $n = iT$, T is sampling period and $-m \le i \le m$.

We calculate parameters a_k of a polynomial from measured discrete values $x[i]$ and those values introduce points of smoothing curve $X(i)$ and their derivations for $n = i.T = 0$. We want to get the minimum sum square errors of measured values from the approximation polynomial,

$$\sum_{i=-m}^{m} \left(x[i] - X(iT)\right)^2 = \min \tag{2}$$

where $x[i] = x[iT]$ represent measured discrete values and $X(iT)$ are calculated values from polynomial (1) for $n = iT$. We can write system equations in form,

$$x[i] = a_0 + a_1(iT) + a_2(iT)^2 + ... + a_q(iT)^q + e(i) \tag{3}$$

for

$$x[-m] = a_0 + a_1(-mT) + a_2(-mT)^2 + ... + a_q(-mT)^q$$
$$.....$$
$$x[-1] = a_0 + a_1(-1T) + a_2(-1T)^2 + ... + a_q(-1T)^q$$
$$x[0] = a_0$$
$$x[1] = a_0 + a_1(1T) + a_2(1T)^2 + ... + a_q(1T)^q$$
$$....$$
$$x[m] = a_0 + a_1(mT) + a_2(mT)^2 + ... + a_q(mT)^q$$

and the matrix form is

$$[x] = [Q] \cdot [a] + [e] \tag{4}$$

$$[a] = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ .... \\ a_q \end{bmatrix} \quad [x] = \begin{bmatrix} x[-m] \\ .... \\ x[-1] \\ x[0] \\ x[1] \\ ... \\ x[m] \end{bmatrix} \quad [Q] = \begin{bmatrix} 1 & -mT & (-mT)^2 & ... & (-mT)^q \\ ... & & & & \\ 1 & -T & (-T)^2 & ... & (-T)^q \\ 1 & 0 & 0 & ... & 0 \\ 1 & T & T^2 & ... & T^q \\ ... & & & & \\ 1 & mT & (mT)^2 & ... & (mT)^q \end{bmatrix}$$

The solution equations system (3) for parameters $[a]$ in matrix form is

$$[a] = \left[[Q]^T . [Q]\right]^{-1} . [Q]^T . [x] \tag{5}$$

If we solve the matrix system (4) for $m = q = 2$ we can obtain matrix form,

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \frac{1}{70T^4} . \begin{bmatrix} -6T^4 & 24T^4 & 34T^4 & 24T^4 & -6T^4 \\ -14T^3 & -7T^3 & 0 & 7T^3 & 14T^3 \\ 10T^2 & -5T^2 & -10T^2 & -5T^2 & 10T^2 \end{bmatrix} . [x]$$

$$a_0 = x(0T) = \frac{1}{70}(-6(x[-2] + x[2]) + 24(x[-1] + x[1]) + 34x[0]) \tag{6}$$

$$a_1 = x'(0T) = \frac{1}{70T}(14(x[2] - x[-2]) + 7(x[1] - x[-1])) \tag{7}$$

$$a_2 = \frac{x''(0T)}{2} = \frac{1}{70T^2}(10(x[2]) + x[-2]) - 5(x[1] + x[-1]) - 10x[0]) \tag{8}$$

where

$$x(n)_{n=0T} = a_0, \quad x'(n)_{n=0T} = a_1, \quad x''(n)_{n=0T} = 2a_2,$$
$$x^k(n) = k! a_k.$$

The parameters a_0 introduce system values of the approximation signal, the parameters a_1 introduce the first derivation of the approximation signal and so we can get information about maximum and minimum. Parameter a_2 introduces the second derivation of smoothing signal.

Applying z-transform, we get the transfer functions $H(z)$ of the 2nd order polynomial from (6), (7) and (8).

$$H_0(z) = \frac{a_0(z)}{X(z)} = \frac{1}{70}(-6(z^2 + z^{-2}) + 24(z + z^{-1}) + 34), \tag{9}$$

$$H_1(z) = \frac{a_1(z)}{X(z)} = \frac{1}{10T}(2(z^2 - z^{-2}) + (z - z^{-1})) \ , \tag{10}$$

$$H_2(z) = \frac{a_2(z)}{X(z)} = \frac{1}{7T^2}(2(z^2 + z^{-2}) - (z + z^{-1})) \, . \tag{11}$$

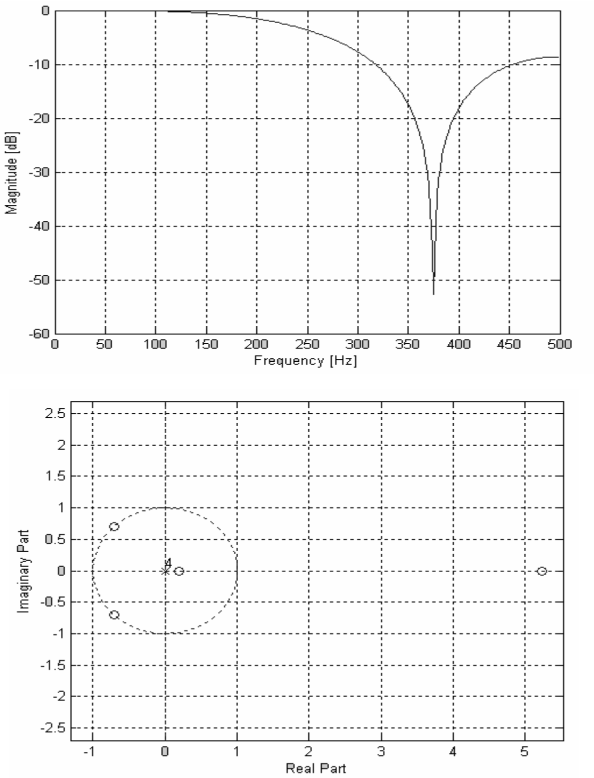
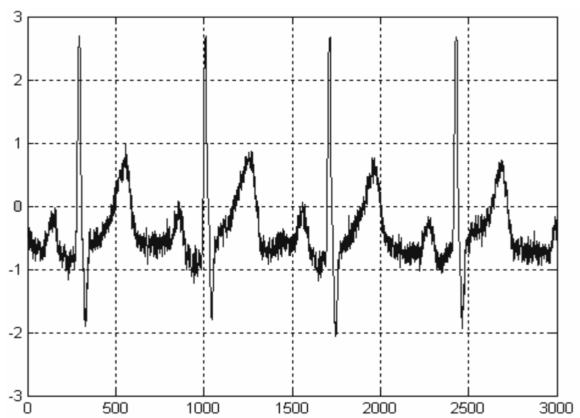


Fig. 3. Magnitude frequency response of the second order polynomial and a plot of the locations of the zeros in the z-plane.

The polynomial is a finite impulse response (FIR) filter with moving window according to the order polynomial. The frequency response of the polynomial is obtained by substituting $z = e^{j\omega T}$ in the expression for $H(z)$, where T is the sampling interval in seconds and ω is the radian frequency ($\omega = 2\pi f$, where f is the frequency in Hz). Note that we may set $T = 1$ and deal with normalized frequency in the range $0 \le \omega \le 2\pi$ or $0 \le f \le 1$, then $f = 1$ or $\omega = 2\pi$ represents the sampling frequency, with lower frequency values being represented as a normalized fraction of the sampling frequency. The frequency responses are given as:



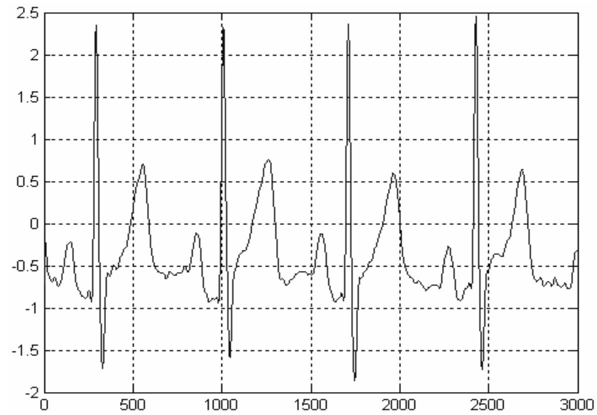


Fig. 4. ECG signal with high frequency noise (upper trace) and the smoothing ECG by polynomial approximation.

$$H_0(j\omega T) = \frac{1}{35}(17 + 24\cos\omega T - 6\cos 2.\omega T) \ , \tag{12}$$

$$H_1(j\omega T) = \frac{j}{5T}(\sin\ \omega T + 2\sin\ \omega T) \ , \tag{13}$$

$$H_2(j\omega T) = \frac{2}{7T^2}(-1 - \cos\omega T + 2\cos 2\omega T) \ . \tag{14}$$

The magnitude frequency response and the pole-zero positions of the approximation polynomial of $H_0(j\omega T)$ is plotted in Fig. 3.

3. Results

We have compared the two methods: the first by polynomial approximation, and the second by moving-average [3] to remove random noise. We evaluated that smoothing by polynomial approximation can be used to remove the high frequency noise and to identify the high-resolution electrocardiogram.

Fig. 4 shows a segment of an ECG signal with high frequency noise and the results of the smoothing ECG signal by the polynomial approximation described above. The noises can origin in the recording system, the instrumentation amplifiers, pickup of ambient electromagnetic signals, in the cables, etc. The illustrated signal has also been corrupted by power-line interference at 60 Hz and its harmonics, which may be considered as a part of high-frequency noise relative to the low-frequency nature of the ECG signal as well [3]. For the test purposes we have used the ECG signal without the signal of His-bundle.

The method is suitable for attenuating interference or random data which sources could be physiological, the instrumentation used, or the environment of the experiment. The polynomial approximation is based on the solution of surrounding point according to size window, which is defined by order polynomial. In our case we use five discrete values of measuring signal, i.e. the second order polynomial. The approximation is rather exact if we have more discrete values of measuring signal, i.e. upper sampling frequency. The smoothing algorithm retains of details of low-level signals but averaging blunts deflections that are individually discrete and sharp.

In Fig. 5 we compared a detail the Q wave with methods by polynomial approximation and moving-average. The QT interval is a useful parameter for quantifying the ECG morphology. For this case, the problem of removing noise from a corrupted signal is better to use of the method by polynomial approximation.

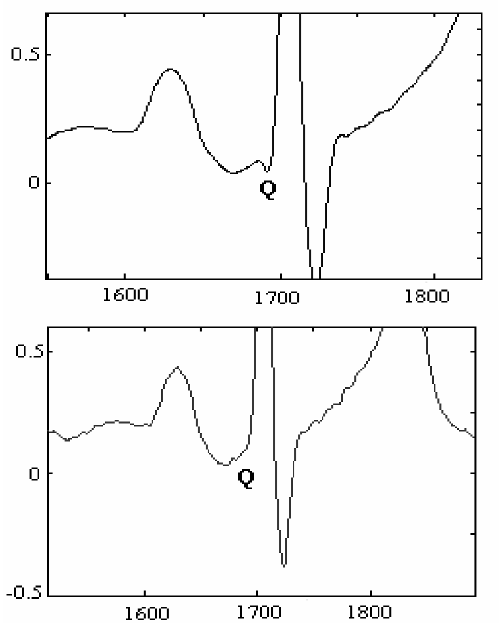


Fig. 5. Detail Q wave of the polynomial approximation (upper trace) and moving average (lower trace).

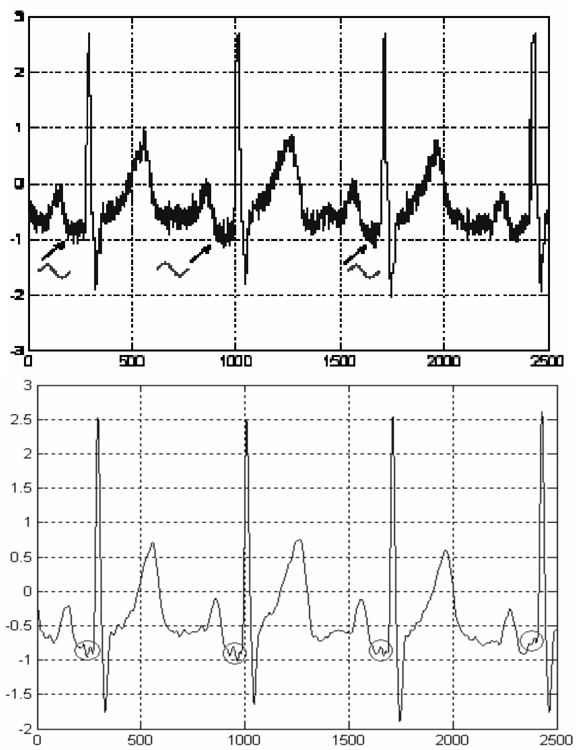


Fig. 6. An ECG signal with noise and superimposed sinus (upper trace) and results of the smoothing by polynomial approximation (lower trace).

Our initial concern was to remove noise and to find signal superimposed in noise. We established the sinusoidal signal with amplitude lower than noise between P and R. waves in the first, second and third period of the ECG signal and in the fourth period is not sinusoidal signal to compare results, shown in Fig. 6. (upper trace). That situation creates artificial form establishment activity of the His bundle if the signal-to-noise ratio (SNR) is -6 dB about. This signal represents the depolarization of the His bundle (H). The discrete signal (H) can be seen in the P-R segment between the activity of the atria (A) and the ventricles (V). It may be used to obtain an accurate measurement of the H-V interval, and thus to define the instant at which cardiac trigger signal passes down the bundle of His.

Fig. 6. lower trace illustrates the smoothing ECG with marking of analyzed places. Those details are plotted with display boundary of superimposed sinusoidal signal in Fig. 7. We can see that the approximation is different in every period but where the sinusoidal signal is superimposed, the results are very similar. The analyzed interval in every period has a length of 45 ms which represents the activation of the His bundle so we can detect continuously in single heart of cycle "beat by beat" whether those signals are recording or not. The signal of this activity is in the range of less than 10 μV and this interval is about 45 ms, these conditions represent the activity of the His bundle. Every small failure makes distortion in the recording of low-level signals therefore we must prepare fairly every measurement. In the usual circumstance of recording in hospital settings, this level may be acceptable below the noise level, so we created similar conditions as described before.

We can see spectral analysis of signal before and after smoothing by polynomial approximation in figure 8. The signal is corrupted by power-line interference at 60 Hz and its 3-rd harmonic 180 Hz, which is very strong. The algorithm of the polynomial of the second order is just suitable for frequency sample 1000 Hz and so we obtained very good results after smoothing of repeated cycles. The frequency components of the signal are eliminated step by step by repeating smoothing.

4. Discussion and Conclusion

The signals, which we analyzed, can be corrupted by noise arising from a number of sources, including skeletal muscular activity (involved, for example breathing), the electrode-skin interface, interference induced by environmental electric and magnetic fields and electronic system noise. In this case the ECG is corrupted by high frequency noise which we can consider as random data. Because the approximation polynomial representatives symmetrical distribution of multiplying constants and random data representing a random phenomenon, the results are also different but if we can set up of weights of the polynomial the results are rather exact. The case is extreme and it is not possible to process the noise data of the ECG but we wanted to demonstrate the smoothing ECG by polynomial and

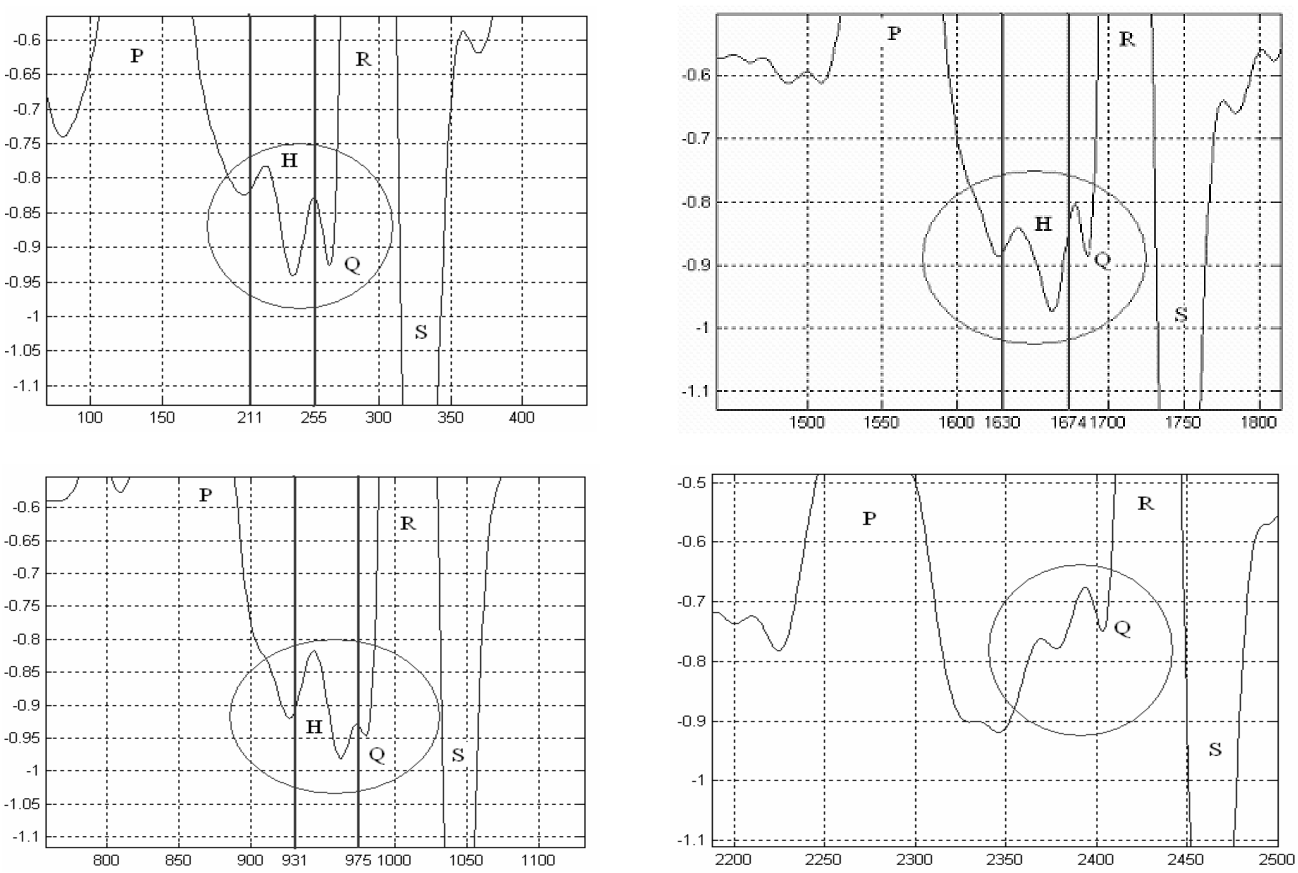


Fig. 7. Details of the smoothing ECG, first, second and third period of the ECG with sinusoidal signal and the smoothing ECG in fourth period of the ECG without sinusoidal signal.

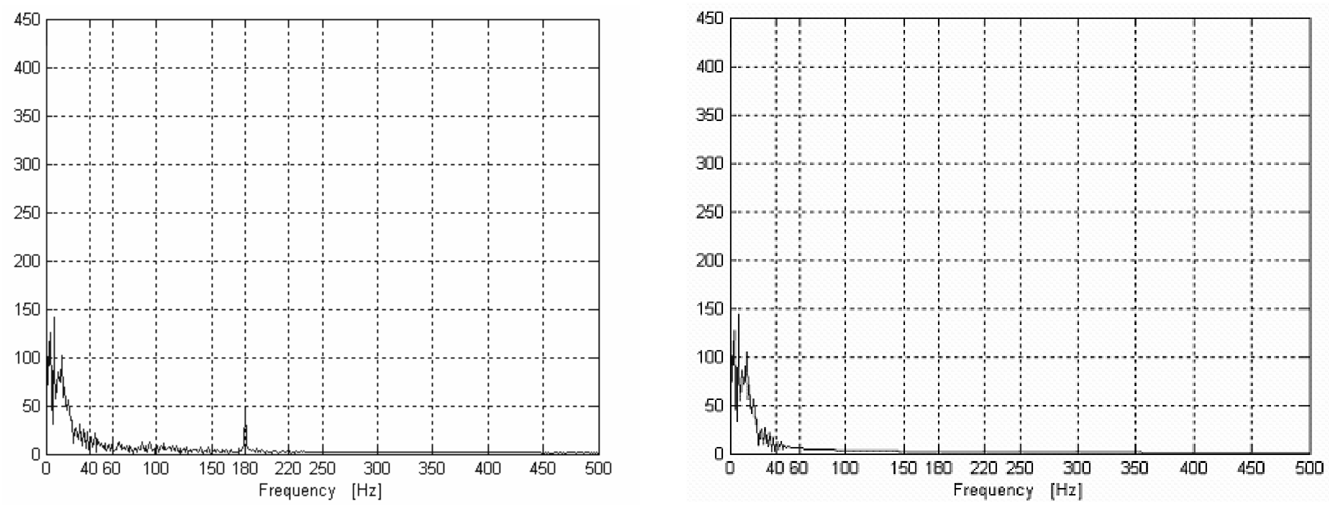


Fig. 8. Spectral analysis of the ECG signal before and after smoothing by the second order of the polynomial approximation.

to identify the high resolution ECG. We see that the method can be used for analysis of the ECG signal, heart of the cycles, "beat by beat". The value of the results obtained from the processing system thus depends both on parameters of the system itself and on the exact nature of noise.

The averaging blunts deflections that we can see in Fig. 5, therefore the method process by approximation polynomial has attribute to remain of importance Q wave of the ECG signal. The QT interval includes ventricular activation and repolarization. The overall QT interval depends on the heart rate. Rate-corrected QT intervals are generally expressed as QT_c intervals, which can be calculated from the formula K/RR where K is equal to 0.397 for male and 0.415 for female.

The standard ECG does not provide the deflection of the His bundle. The amplitude of these specialized cardiac conduction system signals of the His bundle is not sufficient for them to be resolved in a normal ECG. Typical gains of that signal between 5x 10^5 and 2x10^6 are employed. By approximation polynomial we have not used those gains which amplify also the interference of the signal.

Our goal has been to analyze low–level activity of the ECG signal on every-beat of the heart cycle basis as apart of the surface ECG. The described method is suitable to extract the low-level signals from random data. The method is also appropriate for the processing data by program for other signals as that is in the ECG. The technique of smoothing needs several repetitions to obtain quality results, which enable us to change weights of a polynomial and to achieve accurate results.

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