Robust GPS Satellite Signal Acquisition Using Lifting Wavelet Transform

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Abstract. *A novel GPS satellite signal acquisition scheme that utilizes lifting wavelet to improve acquisition performance is proposed. Acquisition in GPS system is used to calculate the code phase (or shift) and find the pseudo-range, which is used to calculate the position. The performance of a GPS receiver is assessed by its ability to precisely measure the pseudo-range, which depends on noise linked to the signals in the receiver's tracking loops. The level of GPS receiving equipment system noise determines in part how precisely pseudo-range can be measured.*

Our objective, in this paper, is to achieve robust real-time positioning with maximum of accuracy in the presence of noise. Robust positioning describes a positioning system's ability to maintain position data continuity and accuracy through most or all anticipated operational conditions. In order to carry out a robust less complex GPS signals acquisition system and to facilitate its implementation, a substitute algorithm for calculating the convolution by using lifting wavelet decomposition is proposed.

Simulation is used for verifying the performance which shows that the proposed scheme based lifting wavelet transform outperforms both FFT search and signal decimation schemes in the presence of a hostile environment.

Keywords

GPS, acquisition, convolution, FFT, decimation, lifting wavelet.

1. Introduction

The Global Positioning System (GPS) is a worldwide satellite navigation system, which consists of a constellation of 24 satellites and their ground stations. GPS signals are specially coded satellite signals that can be processed in a GPS receiver. When receiving the signals from at least 4 satellites, by measuring the travel time of a signal transmitted from each satellite, a receiver can determine latitude, longitude, altitude, time, and velocity of the receiver [1-3]. For a Standard Positioning Service (SPS), each satellite transmits the signal L1 (1575.42 MHz) which is modulated by the *C/A* (coarse/acquisition) code. *C/A* code is a 1023 chip long pseudo-random noise (PRN) sequence sent at a rate of 1.023 megabits/sec [4].

In the GPS transmitter, information is modulated with a PRN code. A receiver should generate a synchronized copy of the code and multiply it by the received signal. If the local code is perfectly synchronized, then its correlation with the original signal is maximum; otherwise the correlation is very low. Synchronizing a GPS signal with local code is called acquisition. Acquisition in GPS system is used to calculate the code phase (or shift) and find the pseudo-range, which is used to calculate the position. In general, GPS accuracy performance depends on the quality of the pseudo-range and delta pseudo-range measurements as well as the satellite ephemeris data. The most present source of acquisition error is the thermal noise which is due to the noise on the transmission channel. The presence of the noise affects seriously the precision in the measure of the pseudo-range and can provoke the loss of the acquisition process. Measurement noise can introduce 0-10 meters of positional error [4-7].

In the other hand, currently there are various situations when GPS will not operate. The presence of intentional or unintentional jamming and spoofing sources and/or high multi-path environments are examples of these situations [4, 8]. There are various formats of intentional interference that can be tactically deployed against GPS, which includes jamming and spoofing. Jamming signals are easy to produce. Similar to thermal noise, it raises the level of background against which the GPS signal must be detected. Such jammer signal can be produced by just adding some white Gaussian noise N(10, 0.1) with a non-null mean and 0.01 of variance to the received signal as it is shown in Fig.7 from section 7 [9].

Hence, our goal in this paper is to develop a robust GPS signals acquisition system that improve performance and maintain continuity of service over noisy transmission channel. To realize a robust less complex GPS signals acquisition system and to facilitate its implementation, a substitute algorithm for calculating the convolution by using lifting wavelet decomposition [10-13] is proposed.

Wavelet transformation became quite recently an extremely useful tool for all practitioners doing high-performance signal processing and is described in detail elsewhere [14]. In its discrete form, signals are decomposed by so called compact supported base functions, which may be time- and frequency limited. Contrary to the Fourier transformation, this leads to a high precision in time and frequency resolution. An important feature of the wavelet transform is its relatively low complexity. If we achieve the convolution with the FFT algorithm, the complexity is of order $O(n\log_2 n)$. This computing time is longer than the one obtained with the wavelet transform, where the complexity is proportional to $O(n)$ [10, 14]. n represents the signal length. Also, the wavelet transform is of interest for the analysis of the non-stationary signals such as GPS measurements, because it provides an alternative to the classical Fourier transform which assumes stationary signals.

The efficiency of wavelet depends on the number of terms in the wavelet transform [12]. This aspect will allow us, as it is shown in our paper, to carry out a FFT on 1250 points without degrading the performances of the acquisition system. The wavelet transform application allows to concentrate the energy of the signal in some coefficients, this will make it possible to apply a FFT to a reduced number of points, therefore to simplify the complexity of the acquisition system and to facilitate his implementation.

The remainder of this paper is organized as follows. Decimation and interpolation processes are presented in section 2. The lifting scheme implementation of the wavelet transform is discussed in section 3. Section 4 describes the GPS signal acquisition process, a FFT search algorithm is studied and the signal decimation approach is introduced. The proposed acquisition system based lifting wavelet decomposition is presented in section 5. Computational complexity is given in section 6. Numerical results and comparisons between our algorithm and both FFT search algorithm and signal decimation approach are provided in section 7. Finally, conclusions are drawn in section 8.

2. Decimation and Interpolation

Interpolation and decimation are, respectively, operations used to magnify and reduce sampled signals, usually by an integer factor. Magnification of a sampled signal requires that new values, not present in the signal, be computed and inserted between the existing samples. The new value is estimated from a neighborhood of the samples of the original signal. Similarly, in decimation a new value is calculated from a neighborhood of samples and replaces these values in the minimized signal. Integer factor interpolation and decimation algorithms may be implemented using efficient FIR filters and are therefore relatively fast.

We have found that it can be useful to process data at a different sampling frequency than one given in a data set. The reason of using decimation of the GPS signal is simply to reduce the size of the data set as well as optimize the data set size for the FFT algorithm.

3. Lifting Scheme

Wavelets based on dilations and translations of a mother wavelet are referred to as first generation wavelets or classical wavelets. Second generation wavelets, i.e., wavelets which are not necessarily translations and dilations of one function, are much more flexible and can be used to define wavelet bases for bound intervals, irregular sample grids or even for solving equations or analyzing data on curves or surfaces. Second generation wavelets retain the powerful properties of first generation wavelets, like fast transform, localization and good approximation. Lifting scheme is a rather new method for constructing wavelets. The main difference with the classical constructions is that it is does not rely on the Fourier transform. In this way, lifting can be used to construct second generation wavelets. Lifting scheme [17-20] can in addition, efficiency implement classical wavelet transforms. Existing classical wavelets can be implemented with lifting scheme by factorization them into lifting steps [11].

3.1 The Forward Transform

The basic idea behind the lifting scheme is very simple (see Fig.1); we try to use the correlation in the data to remove redundancy. To this end, we first split the data $x[n]$ (n represents the data element) into two sets (*Split phase*): the odd samples $x_{\text{odd}}[n]$ and the even $x_{\text{even}}[n]$ samples. The even set comprises all the samples with an even index and the odd set contains all the samples with an odd index. Because of the assumed smoothness of the data, we predict that the odd samples have a value that is closely related to their neighboring even samples. We use N even samples to predict the value of a neighboring odd value (*Predict phase*). With a good prediction method, the chance is high that the original odd sample is in the same range as its prediction. We calculate the difference between the odd sample and its prediction and replace the odd sample with this difference. As long as the signal is highly correlated, the newly calculated odd samples will be on the average smaller than the original one and can be represented with fewer bits. The odd half of the signal is now transformed. To transform the other half, we will have to apply the predict step on the even half as well. Because the even half is merely a sub-sampled version of the original signal, it has lost some properties that we might want to preserve. The third step (*Update phase*) updates the even samples using the newly calculated odd samples such that the desired property is preserved. Now the circle is round and we can move to the next level; we apply these three steps repeatedly on the even samples and transform each time half of the even samples, until all samples are transformed. Fig. 1 illustrates these three steps.

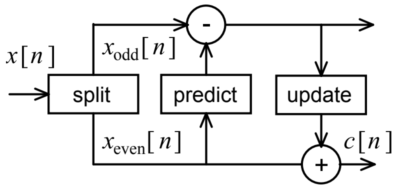


Fig. 1. The lifting scheme, forward transform: Split, Predict and Update phases.

3.2 The Inverse Transform

One of the great advantages of the lifting scheme realization of a wavelet transform is that it decomposes the wavelet filters into extremely simple elementary steps, and each of these steps is easily invertible. As a result, the inverse wavelet transform can always be obtained immediately from the forward transform. The inversion rules are trivial: revert the order of the operations, invert the signs in the lifting steps, and replace the splitting step by a merging step. The steps to be taken for inverse transform are illustrated in Fig. 2.

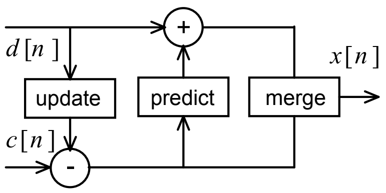


Fig. 2. The lifting scheme, inverse transform: Update, Predict and Merge stages.

4. Signal Acquisition

In a GPS system, when the received signal is multiplied by a synchronized version of the PRN code, the signal is despread. Its power increases over the noise floor. Therefore it appears as a correlation peak. However, in order for a receiver to synchronize to the received signal, an initial estimation of the code and carrier is calculated. The acquisition process is the first process required by the GPS receiver. The acquisition process conducts a three dimensional search. The three elements (or search bins) are the available *C/A* code, the code phase, and the carrier frequency offset. If we assume that a GPS receiver knows which satellite code it is searching for, then a 2-D search is required [1]. One dimension is the code phase in range of 1023 chips. The code phase resolution is half of a code chip. This resolution is required since the correlation peak is considered a true peak only if the code phase is within a half chip. The second search dimension is for the carrier frequency. Its range is ±10 kHz centered at an intermediate frequency (IF) of 1.25 MHz. The resolution of the carrier frequency is typically 667 Hz for 1-ms integration [6]. Searching with 500 Hz steps can be used [21].

Therefore, in a GPS receiver, this search detects the correlation peak and compares it to a certain threshold to determine whether a satellite was detected or not. When a satellite is detected the auto-correlation result provides a rough estimation of the code phase and the carrier frequency. The acquisition process should provide this data. However, if the search process does not locate a peak that passed the detection threshold, a satellite is considered “not-acquired” and the search continues. The search space must cover the full range of uncertainty in the code and Doppler offset. Because the *C/A* code is fairly short (1 ms), typically the range space will include all possible code offset values. The selection of a path through the search space is a function of the vehicle dynamics and requirements for acquisition speed and reliability.

Typically the Doppler is set to the expected value and the code is searched over all possible delays. If this fails, the search is continued in the next Doppler bin, with the sequence of bins alternating above and below the starting Doppler (this is a serial acquisition algorithm principle). Fig. 3. depicts the acquisition uncertainty region.

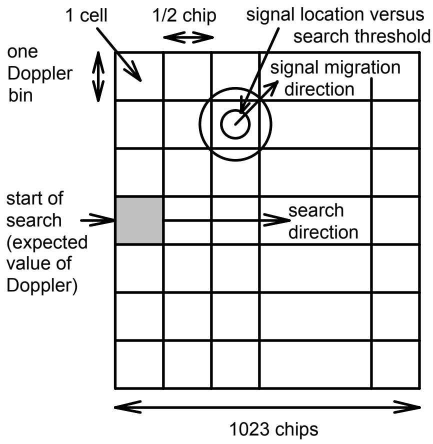


Fig. 3. Acquisition uncertainty region.

4.1 FFT Search Algorithm

The acquisition process time is shortened when the linear search algorithm can be performed as illustrated in Fig. 4 [22]. Here L=5000 samples (since we have a sampling rate of 5 MHz) of the received data $x[n]$, digitized at IF, are correlated with the replica code, *C/A[n+m]*, by circularly shifting the replica code. n represents the n^{th} sample and m represents the number of samples the replicated *C/A* code is phase shifted. This resembles the circular convolution and the correlation may be expressed as

$$R[m] = \sum_{n=1}^{L} x[n].CA[(n+m)_L]\,. \tag{1}$$

The circular convolution is a multiplication in the frequency domain. In fact we can write

$$R[m] = \underbrace{x[n] \otimes CA[-n]}_{\text{Circular convolution}} = F^{-1}\Big(F\big(x[n]\big).F\big(CA[n]\big)^{*}\Big) \tag{2}$$

where the discrete Fourier transform (DFT) and its inverse is used to calculate R. This can be adapted to acquisition of

GPS signals as shown in Fig. 4.

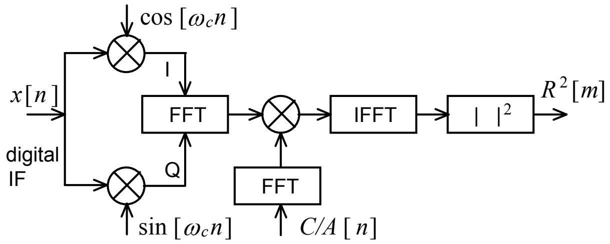


Fig. 4. Non coherent Correlator in frequency domain.

The FFT search algorithm requires the *C/A* code to be up-sampled to 5000 points to match the received GPS signal. Every chip of the *C/A* code is then repeated 4 or 5 times. One suggestion method to remove this redundancy is to use the lifting wavelet scheme where we use only 1250 points instead of 5000 points.

The same approach can be carried out by using signal decimation but the obtained results will be less effective than those obtained by using lifting wavelet as we can see this in numerical and simulation results section (section 7).

5. Proposed Acquisition System Based Lifting Wavelet

In this section, we present our new GPS signals acquisition with use of lifting wavelet decomposition algorithm. We transform the data x and the code *C/A* to the wavelet domain to decrease, the length of signals used to calculate the product, of convolution by FFT. Our algorithm is summarized as follow:

- Analysis: transform the data x to the wavelet coefficients $d[n]$ and scaling function coefficients $c[n]$.
- Thresholding: apply a thresholding function S_τ with a threshold parameter $\tau\neq0$, i.e, $S_\tau(d[n])$.
- FFT processing: apply a non coherent correlation in frequency domain (see Fig. 4).
- Synthesis: reconstruct the version of $R^2[m]$ from the shrunk wavelet coefficients.

One of the great appeal of the wavelet transform is that allows to separate signal from noise by thresholding wavelets coefficients. In our algorithm Donoho's soft thresholding is used [23]

$$S_\tau(x)=\begin{cases} x-\tau\,\mathrm{sgn}(x) & if\,|x|>\tau \\ 0 & if\,|x|\le\tau \end{cases} \tag{3}$$

This shrinks all coefficients towards zero. A threshold parameter τ is related to the variance of the Gaussian noise to the wavelet coefficients.

A block diagram of our GPS signal acquisition system using lifting wavelet decomposition is shown in Fig. 5. Here, NCO represents the Numerical Controller Oscillator.

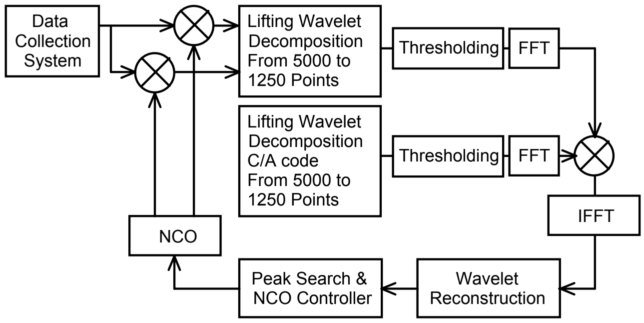


Fig. 5. Acquisition using lifting wavelet decomposition.

6. Computational Complexity

We quantify the complexity by the measure of the number of operations (multiplications and additions) performed by the proposed GPS signal acquisition algorithm using lifting wavelet transform, compared to that of both FFT search algorithm and signal decimation approach.

Tab. 1 summarizes the acquisition time consuming, the number of operations carried out and the correlation ratio (maximum peak to mean of rest peaks ratio) of our proposed algorithm compared to both FFT search algorithm and signal decimation approach.

We can see that the proposed GPS system acquisition is less complex than both FFT search algorithm and signal decimation approach since it requires a small number of operations to carry out the convolution operation. For example, lifting wavelet algorithm using 2500 samples requires around 70 millions operations instead of around 154 millions operations for FFT search algorithm.

In order for sampling theorem to be fulfilled, 2500 samples are used for performing the signal decimation approach. In the other hand, 2500 samples or only 1250 samples can be used for performing lifting wavelet algorithm.

	Acquisition time [sec]	Number of operations carried out (x 10^6)	Correlation ratio
FFT search (5000 samples)	8,29	140,478505	734,3424
Decimation (2500 samples)	9,89	153,932942	652,1254
Lifting Wavelet (2500 samples)	8,12	69,906025	1249,4
Lifting Wavelet (1250 samples)	8,07	37,387244	1033,0

Tab. 1. Acquisition performance comparison with PC processor-750 MHz (the case of N(0, 0.1).

Tab. 2 gives a summary of the acquisition performance comparison results of the proposed lifting wavelet algorithm using only 1250 samples and the decimation one using 2500 samples in the case of noisy signal (we add some white Gaussian noise N(10, 0.1) with a non-null mean and 0.01 of variance to the received signal).

	Acquisition time [sec]	**Number of operations carried out (x 10^6)**	**Correlation ratio**
Decimation (2500 samples)	**9,94**	**153,932942**	**190,0847**
Lifting Wavelet (1250 samples)	**8,13**	**37,387244**	**736,0284**

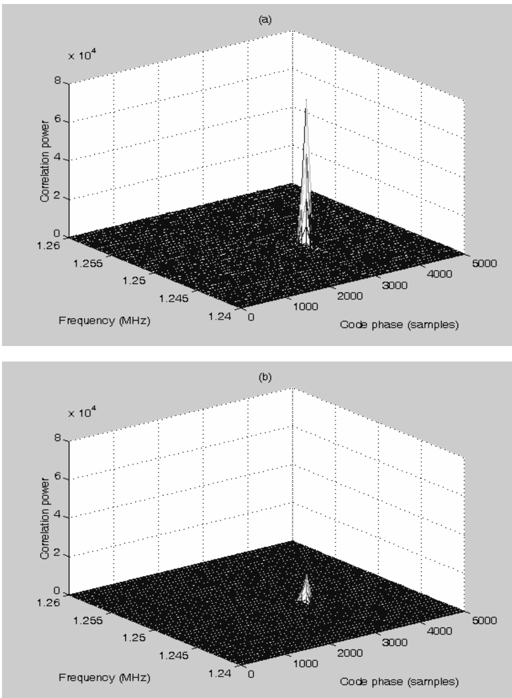
Tab. 2. Acquisition performance comparison with PC processor-750 MHz (the case of N(10, 0.1).

The proposed GPS system acquisition shows better results. It is less complex. Also the correlation ratio of the lifting wavelet algorithm is higher than the one of the decimation approach. As a result the detection of the correlation peak becomes easier when using the proposed algorithm.

7. Numerical and Simulation Results

We will examine here the computation of a convolution by using the lifting wavelet transform in order to carry out a robust less complex GPS signals acquisition system and to facilitate its implementation.

In our simulation, 3 methods use the same Doppler shift set to -4 kHz and the same code phase set to 3000 samples. The GPS signal to noise ratio is fixed to –19 dB.



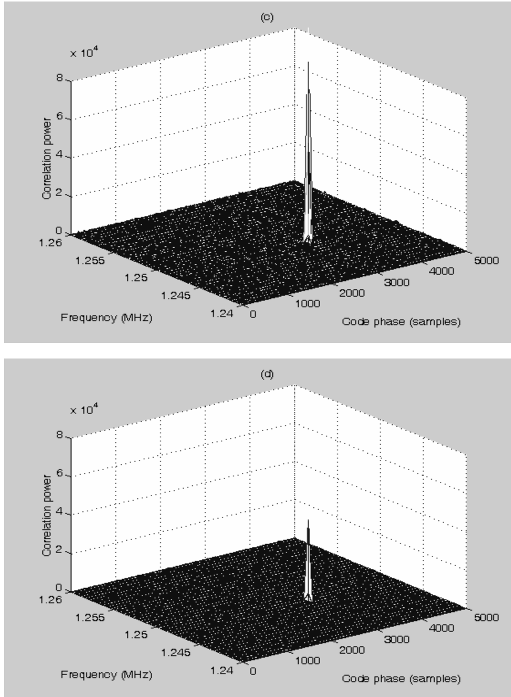
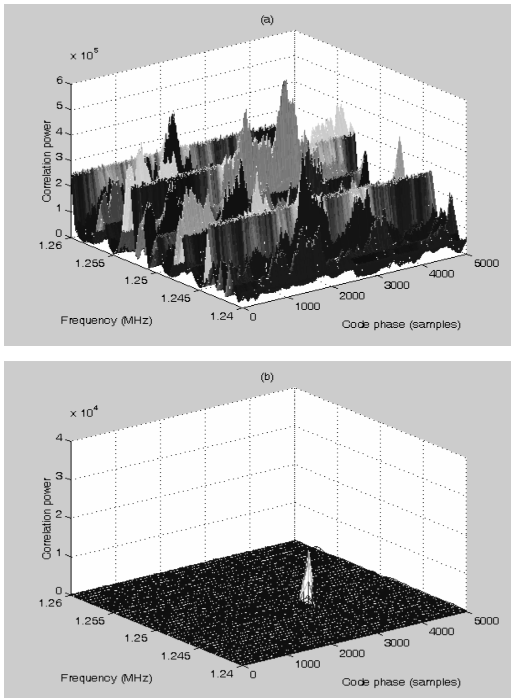


Fig. 6. Correlation matrix for satellite 19 with N(0, 0.1). **(a)** with FFT search (5000 points), **(b)** with signal decimation (from 5000 to 2500 points), **(c)** with lifting wavelet decomposition (from 5000 to 2500 points) and **(d)** with lifting wavelet decomposition (from 5000 to 1250 points).

Fig. 6, presents a correlation matrix when satellite 19 is searched for (in our Matlab simulation) in the presence of the additive white Gaussian noise N(0, 0.1) with zero mean and 0.01 of variance.

We can state (Fig. 6) our method gives a more accurate peak. Furthermore, performing lifting wavelet algorithm with only 1250 samples (see Fig. 6(d)) gives a better



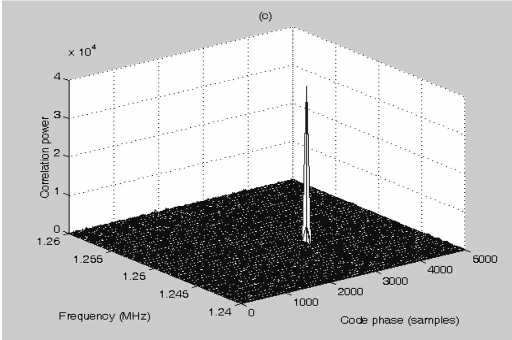


Fig. 7. Correlation matrix for satellite 19 with N(10, 0.1). **(a)** with FFT search (5000 points), **(b)** with signal decimation (from 5000 to 2500 points) and **(c)** with lifting wavelet decomposition (from 5000 to 1250 points).

correlation peak than the one obtained when using signal decimation approach with 2500 samples (see Fig. 6(b)).

Fig. 7, presents the case of noisy signal N(10, 0.1). We can see this signal (i.e, N(10,0.1)) as an intentional interference which is easy to produce and can cause a perfectly functioning receiver to stop working completely.

From this figure, we note the good performance of the proposed method (see Fig. 7(c)), it still works and the FFT method fails to detect the signal (see Fig. 7(a)).

To avoid a dysfunction of the FFT use, we should have zero in the output of the non coherent correlator (Fig. 4) if there is no GPS signal present in the input. Remarking the output $R[m]$ is different from zero, if we inject a constant signal in the input of the non coherent Correlator. Hence, the presence of a constant signal, in the GPS signal, introduces a new frequency component in the frequency domain which causes the dysfunction of the FFT. On the other hand, in the case of our proposed acquisition system shown in Fig. 5, the use of wavelets permits to filter this frequency component and get the good performances.

Also we can see that, the correlation peak of the proposed algorithm (see Fig. 7(c)) is more powerful than the one of the signal decimation approach (see Fig. 7(b)).

8. Conclusions

This paper presented a design for robust and less complex GPS satellite signal acquisition system using fast lifting wavelet decomposition. We focused on the lifting scheme implementation of the wavelet transform. The lifting scheme uses three simple steps to calculate the wavelet coefficients, namely the Split, Predict and Update phase. We discussed these three steps and explained how the inverse transform can easily be calculated in a similar way.

The acquisition performance of the proposed algorithm was evaluated and compared with that of both FFT search algorithm and signal decimation approach. It is found that the proposed method significantly outperforms both FFT and signal decimation methods in the presence of a hostile environment.

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