On the Reduced Complexity Interleaving Method For OFDM PAPR Reduction

Roman MARŠÁLEK

Dept. of Radio Electronics, Brno University of Technology, Purkyňova 118, 612 00 Brno, Czech Republic

marsaler@feec.vutbr.cz

Abstract. *Many methods for OFDM Peak to Average Power Ratio reduction have been proposed during approximately last ten years. There are nowadays many research efforts on PAPR reduction methods with reduced computational complexity. The method presented in this paper is based on adaptive symbol selection principle, with several replicas of signal created using set of interleavers incorporated inside an IFFT block at OFDM transmitter. This paper also discuss some practical aspects of this method – influence of zero padding and pilot positions.*

Keywords

OFDM, PAPR reduction, multicarrier modulations.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is widely used in contemporary communication systems for its good robustness in mutipath environment. One of the drawbacks of the OFDM transmitters is high Peak to Average Power Ratio (PAPR) of OFDM signals. The maximal value of the PAPR growths with logarithm of number of subcarriers. The PAPR of the OFDM signal $x(t)$ is defined as:

$$\mathrm{PAPR}\{x(t)\} = \frac{\max\left\{|x(t)|^2\right\}}{E\left\{|x(t)|^2\right\}} \quad . \tag{1}$$

High PAPR of signal in connection with nonlinear power amplifier results in undesirable spectral emissions into adjacent channels and in in-band distortion manifested in BER degradation. To suppress these effects many methods for PAPR reduction have been proposed [1], [3].

The cost paid for the OFDM system with reduced PAPR is often the reduced throughput (reserving of some subcarriers for peak reduction signal [1] or for necessary side information [3]) and inevitably also the increased complexity of the transmitter. The current research efforts are concentrated also in methods with reduced computational complexity [4], [5].

For evaluation of PAPR reduction methods, statistical characterization of OFDM signal PAPR using Complementary Cumulative Distribution Function – CCDF is used (probability that PAPR exceeds given threshold γ).

In [2], we have proposed a principle of reduced complexity method based on adaptive symbol selection [3] with replicas of signal obtained using several interleavers. This paper will discuss some practical aspects of proposed principle-influence of interpolation using zero padding and influence of the pilot sequence position.

2. Basic Principles of Interleaving for PAPR Reduction

In the adaptive symbol selection scheme [3], P signals containing the same information are created from the input data and the variant with the smallest PAPR is selected for transmission. It is possible to create these variants using P–1 interleavers followed by constellation mapping and Inverse Fast Fourier Transform (IFFT) as shown in Fig.1. Note that a side information about the path with lowest PAPR has to be sent over channel and that a computation of each IFFT of length N using radix-2 algorithm requires $(N/2)\log_2 N$ complex multiplications [6].

Receiver for such system needs to calculate FFT of received signal and perform de-interleaving operation with the de-interleaver according the received side information.

3. Principle of Complexity Reduction

The basic idea of complexity reduction we have proposed in [2] is based on integration of interleavers inside IFFT block. Exploiting some property of FFT results in no need of calculation of whole IFFT's for each signal path.

Following description will be based on a radix-2 decimation in time (DIT) algorithm. Radix algorithms exploits the property that the calculation of N-point FFT can be translated into calculation of two FFT's of even-ordered and odd-ordered samples [6]:

$$X(k) = G_{even}(k) + W_N^k G_{odd}(k) \tag{2}$$

where G are the length $N/2$ FFT of even and odd samples of input time-domain sequence $x(k)$ and

$$W_N^k = \exp\left(-j2\pi / N\right) .$$

This procedure can be repeated until the basic 2-point butterfly. If we interchange G_{even} and G_{odd} in equation (2) we can easily obtain an alternative to original with interchanged odd and even inputs:

$$X_1(k) = G_{odd}(k) + W_N^k G_{even}(k) \tag{3}$$

with only small additional complexity (multiplication by W_N^k and addition of both elements).

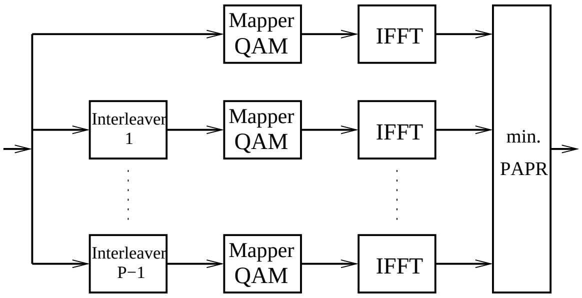

Fig. 1. Adaptive Symbol Selection principle using interleaving.

These 2 signal alternatives (P=2) can be used as a 2-path adaptive symbol selection based PAPR reduction device. We can obtain more alternatives for PAPR reduction if we do interchanging on shorter IFFT sequences (IFFT of length $N/4$, $N/8$, …). This can be illustrated in Fig. 2, where 64-point FFT is split into 8 blocks of 8-point FFT or into 16 blocks of 4-point FFT.

If no additional restrictions are posed, it is possible to interchange the blocks outputs with each other (for example output of block No. I can be interchanged with any of blocks from II to VIII, etc.). Using different interchangings it is possible to obtain several signal alternatives for PAPR reduction similarly to Fig. 1. Only the first IFFT has to be calculated entirely. All the other alternatives can make use of the IFFT subblocks already calculated for the first IFFT. Note finally that the IFFT inputs are in bit-reversed order that will be crucial for following discussion.

4. Effect of Oversampling Using Zero Padding

In [2], we have considered the use of this method for reduction of OFDM signal created according IEEE 802.11a signal [7]. The important property of many systems based on OFDM (including 802.11a) is that a part of the subcarriers is set to zero in order to limit frequency spectrum of rf signal. It can be well seen in Fig. 2 (null subcarriers are marked wit 'Z' symbol). As the positions of these zeros have to be kept, zeros limit the number of possibilities for interchanging. It is not for example possible to interchange the output of blocks VII and VIII as there is no zero at

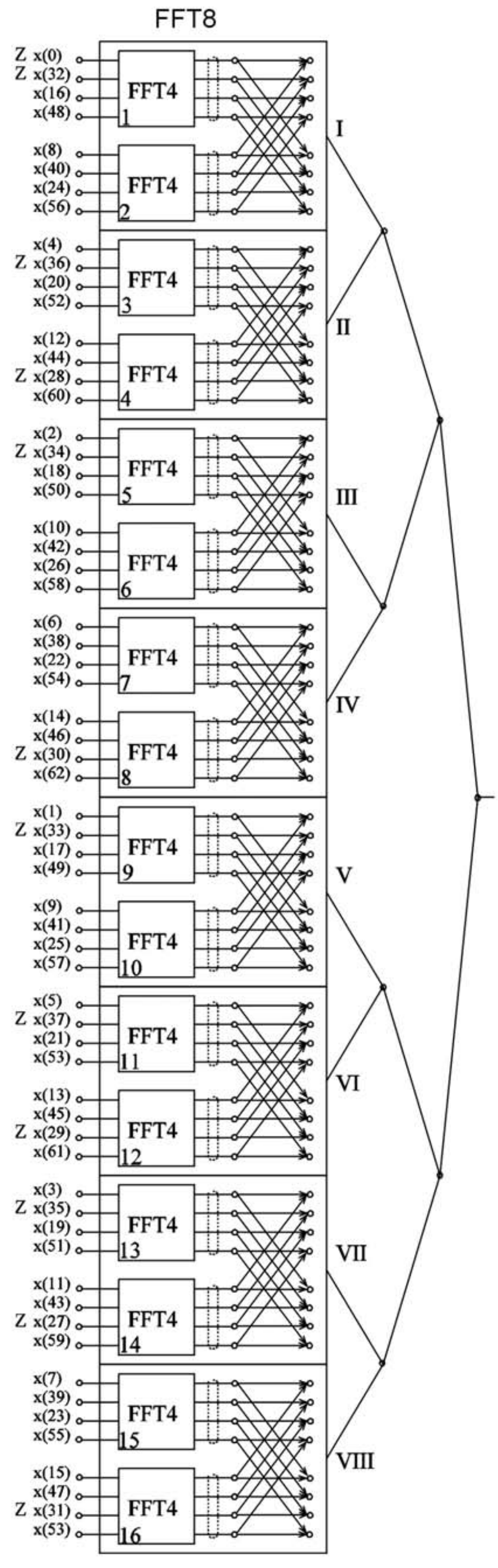

Fig. 2. Null subcarriers in 802.11a.

x(39) and it would result in emissions at frequency corresponding to input x(35). As the consequence, the number of variants for PAPR reduction is reduced. On the other side it is possible to interchange output of block VII and VI as both have zeros at the same frequency positions.

It is well known that inserting LN zeros in frequency domain representation followed by IFFT (zero padding) can be used for time domain signal interpolation by factor of L+1. In baseband, these zeros are inserted in the middle of frequency domain sequence. Zero padding is also often used in OFDM.

Insertion of zeros in the middle of frequency domain symbol sequence at the input of OFDM modulator is advantageous for the proposed complexity reduction method due to the bit reversation of input sequence in radix-2 IFFT algorithm (we have used algorithm with bit reversed inputs, [6]). It will be evident from following example. Suppose now the number of data subcarriers N_{data}=32 and 2-time interpolation (inserting of 32 zeros). The resulting IFFT length will be 64, similarly to Fig. 2. This zero padding is shown in Fig. 3. The zeros are inserted in the middle of input sequence – the zero sequence starts at subcarrier 16 and ends at subcarrier 47. If we now take into account the bit reversal order of IFFT input sequence, the zeros will be at positions according Fig. 4.

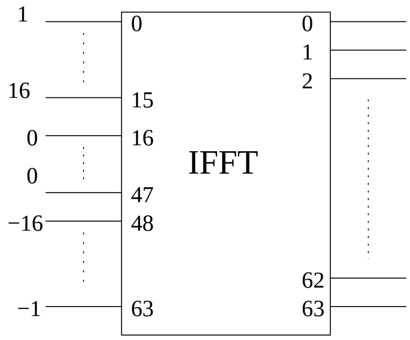

Fig.3. Illustration of zero padding at IFFT input.

Zeros are now placed in the middle of 4-point FFT blocks and their positions are thus same for all 4-point, 8-point, 16-point or 32-point FFT blocks. As the consequence, we can arbitrarily interchange the blocks with each other.

5. Effect on pilot positions

Pilot subcarriers are special subcarriers in OFDM system reserved in order to make a detection robust against frequency offset and phase noise [7]. Pseudorandom sequences are usually transmitted as pilots. Position of pilots, if are specified in standard, have to be kept and must be preserved from interchanging. In example of IEEE 802.11a, the subcarriers –21, –7, 7, 21 are reserved for pilots that corresponds to IFFT inputs No. 43, 57, 7, 21 (Fig. 5). IFFT blocks with these inputs have to be also kept in their positions and can not be interchanged with others. The way how to overcome the problem with pilot positions

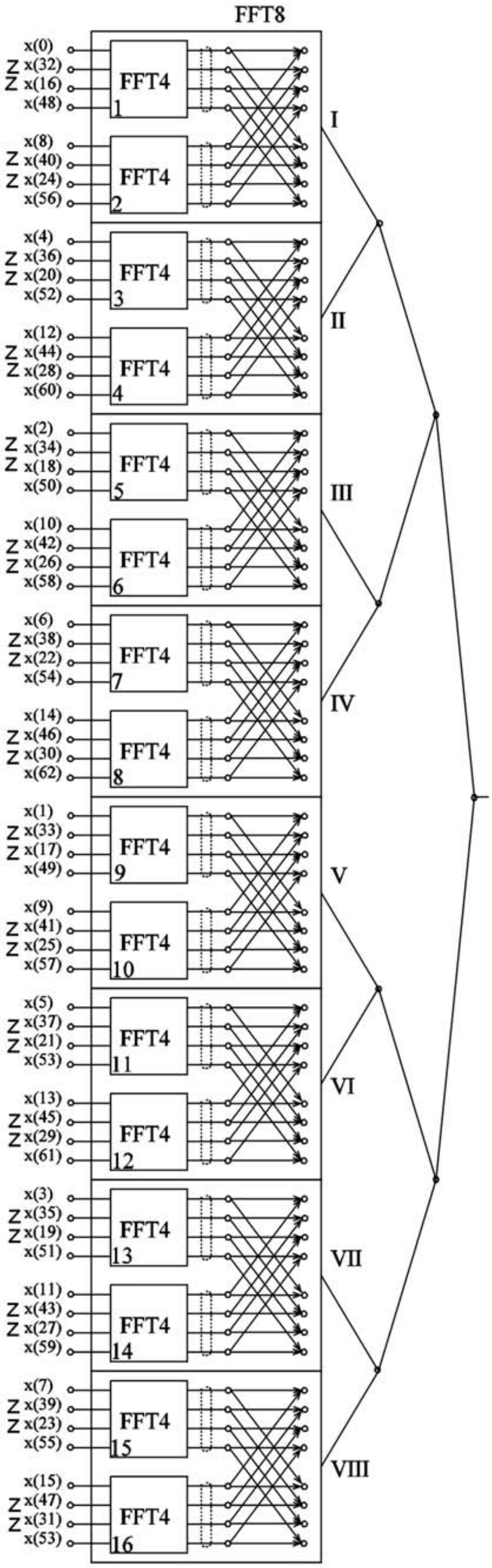

Fig.4. Positions of zeros with oversampling.

is to redesign a set of pilot subcarriers (if it is not of course already specified by standard).

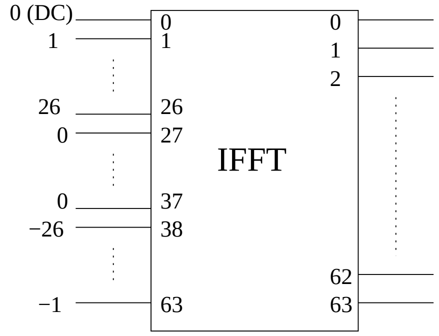

Fig. 5. IFFT mapping according to IEEE 802.11a standard.

The possible solution is to design pilot positions in such a manner that all the pilots will be at the input of least IFFT blocks as possible (ideally 1). Moreover as the zero frequency subcarrier (X(0)) is usually set to zero it is desirable that all the pilots will be placed at the input of block I (8-point) or 1 (4-point).

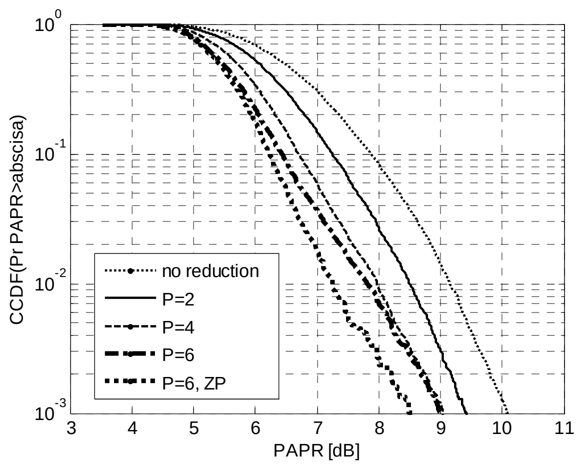

Fig. 6. CCDF of proposed method.

We will consider the case similar to 802.11a to find out the ideal pilot positions. Let's also preserve the same number of pilots (4). For the reasons of interchanging it is desirable that the pilots will be placed on subcarriers -24,-8,8,24 that corresponds to IFFT inputs 40,56,8,24. As shown in Fig. 2, all these subcarriers are thus at the input of IFFT block I together with zero DC subcarrier and the pilots do not reduce a number of possible IFFT blocks for interchanging.

6. Simulations and Results

In Fig. 6 the CCDF of described method is shown for different number of 'interleavers' and 32 data subcarriers with oversampling using 32 zeros. *P* denotes the number of interleavers and is evident that the performance is improving with higher *P*. To illustrate the effect of zero padding and thus increased number of possibilities for IFFT interchanging, two cases have been considered. In the first one ($P = 2$, $P = 4$, $P = 6$) the IFFT blocks were interchanged to respect zero positions according to Fig. 2 (for comparison and having in mind that there is no need to do this). In the second case (P=6 ZP) no restriction on interchanging has been done as zero positions according to Fig. 4 have been expected.

The number of required complex multiplications for proposed method was estimated in [2] as

$$N_{mpy,c} = \left(\tfrac{N}{2}\log_2 N\right) P - \left(\tfrac{N}{2}\log_2 M\right)\left(P-1\right) ,$$

where M is the length of elementary IFFT blocks (in the cases presented above in the paper M was equal to 8 or 4).

7. Conclusions

We have presented a reduced complexity adaptive symbol selection PAPR reduction method. In [2] it was shown that this is advantageous only for small values of *P* (max. 4) and increasing *P* has no advantage, because of the need to preserve the zero subcarriers. We have shown in this paper that this is not true for the case of signal oversampling using zero padding and that in this case performance of the method can be further improved. A problem of pilot subcarriers positions has also been discussed. As in the most cases of similar methods, this method also requires sending side information about the interchanging used for transmitted signal with lowest PAPR. The way to remove this requirement can be subject of further work.

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References

[1] TELLADO, J. *Multicarrier Modulation with Low PAR Applications to DSL and Wireless*, Kluwer Academic Publishers, 2000.

[2] MARSALEK, R., FEDRA, Z. Reduced complexity adaptive symbol selection method for OFDM PAPR reduction. In *Proceedings of 7th Nordic Conference NORSIG 2006*, June 2006, Reykjavik, Iceland.

[3] OCHIAI, H., IMAI, H. Performance of the deliberate clipping with adaptive symbol selection for strictly band-limited OFDM systems. *IEEE Journal on Selected Areas in Communications*, 2000, vol. 18, no. 11, p. 2270-2277.

[4] HAN, S.H., LEE, J.H. PAPR Reduction of OFDM signals using a reduced complexity PTS technique. *IEEE Signal Processing Letters*, 2004, vol. 11, no. 11, p. 887-890.

[5] LIM, D.-W., NO, J.-S., LIM, CH.-W, CHUNG, H. A new SLM OFDM scheme with low complexity for PAPR reduction. *IEEE Signal Processing Letters*, 2005, vol. 12, no. 2, p. 93-96.

[6] INGLE, V.K., PROAKIS, J.G. *Digital Signal Processing using MATLAB*, Brooks/ Cole publishing, 2000.

[7] IEEE Std 802.11a-1999(R2003), Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications.

[8] VAN NEE, R., PRASAD, R. *OFDM for Wireless Multimedia Communications*, Artech House Publishers, 2000.

About Authors...

Roman MARŠÁLEK was born in Brno, Czech Republic in 1976. He graduated at Brno University of Technology in 1999 and received the Ph.D. equivalent degree (Docteur) in 2003 from Université de Marne la Vallée, France. He is currently assistant professor at Institute of Radio Electronics, Brno University of Technology. His research interests are in wireless communications theory and applied digital signal processing.

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