Multibeam Antennas Array Pattern Synthesis Using a Variational Method

Boumediene GUENAD, Sidi Mohamed MERIAH, Fethi Tarik BENDIMERAD

Telecommunications Laboratory, Electronics Dept., Faculty of Engineering, Abou-Bekr Belkaïd University – Tlemcen, P. O. Box 230, Chetouane, 13000 Tlemcen, Algeria

guenadboumediene@yahoo.ca

Abstract. *In this paper a new method is described for multibeam antennas synthesis where both the amplitude and phase of each radiating element is a design variable. The developed optimization method made possible to solve the synthesis problem and to answer all the constraints imposed by the radiation pattern. Two approaches for visualizing satellite antenna radiation patterns are presented. Gain-level contours drawn over a geographical map gives clearest qualitative information. A three-dimensional (3D) surface plot displays the qualitative shape of the radiation pattern more naturally. The simulations results have shown power, precision and speed of the variational method with respect to the constraints imposed on radiation pattern of the of multibeam antennas network.*

Keywords

Multibeam antennas, optimization, radiation pattern synthesis, variational method.

1. Introduction

Multiple-beam antennas are currently being used for direct-broadcast satellites (DBS), personal communication satellites (PCS), military communication satellites, and high-speed Internet applications [1], [2]. These antennas provide mostly contiguous coverage over a specified field of view on Earth by using high-gain multiple spot beams for downlink (satellite-to-ground) and uplink (ground-to-satellite) coverage. The design objectives for the multiple-beam antennas are to maximize the minimum gain over the coverage region, and to maximize the pattern roll-off outside the spot-beam area and to minimize the sidelobe radiation in order to maximize the frequency reuse. A number of design methodologies for multiple-pattern arrays have been described in the literature [3], [4], and [5]. In the present work, we try to find the best excitations coefficients using the variational method, in order to minimize the sidelobes levels and respecting the pattern shape.

2. Multiple Beam Antenna Design

Because several spot beams have to be created simultaneously by the satellite antenna, such antennas are called multiple-beam antenna (MBA) and various MBA designs can be used to create the spot beams. A basic way to design an antenna that produces a shaped beam is to excite the individual ports of a multibeam antenna by superimposing weighted individual spot beams [4], [6], as shown schematically in Fig. 1. A composite beam, often in the shape of a continent, can be produced. An example of such antenna is an array-fed reflector system with a power divides and phasing network that fixes the distribution of the transmitted fields through each horn.

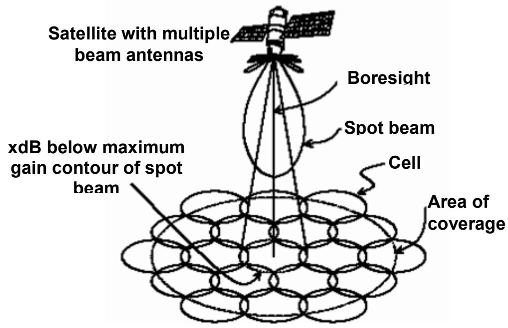


Fig. 1. Cell pattern with cell centers located on triangular grid.

3. Feed Model

The field radiation pattern is expressed initially as a model to represent the main beam [7], [8]:

$$F_1(u,v)=\sum_{n=1}^{N_1} C_n^1 f_n(u,v) \tag{1}$$

where

N_1 is the total number of elements in the array,

$C^1{}_n$ are called the complexes beam excitations. The beam coverage area synthesis specified by constraints for the total radiated field cannot be achieved only by the beam excitations optimization. The optimization is carried out using variational method.

(u,v) space coordinates are defined according to

$$\begin{cases} u = \sin(\theta)\cos(\phi) \\ v = \sin(\theta)\sin(\phi) \end{cases} \tag{2}$$

which are often used to display 2-dimensional radiation patterns. Accordingly it holds, $\theta=\arcsin[\sqrt{(u^2+v^2)}]$, $\varphi=\arctan(v/u)$, further note that for real $\theta\leq\pi/2$ must hold $u^2+v^2\leq 1$.

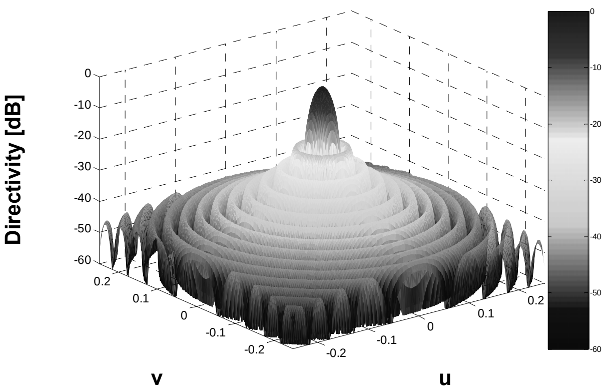
f_n The far field radiation pattern of single aperture multiple-beam antennas using the "basic-feed concept", and is given by:

$$f_n(u,v) = \frac{4}{1+e_0}\left[e_0\frac{J_1(\eta_n)}{\eta_n} + 2(1-e_0)\frac{J_2(\eta_n)}{\eta_n^{\;2}}\right] \tag{3}$$

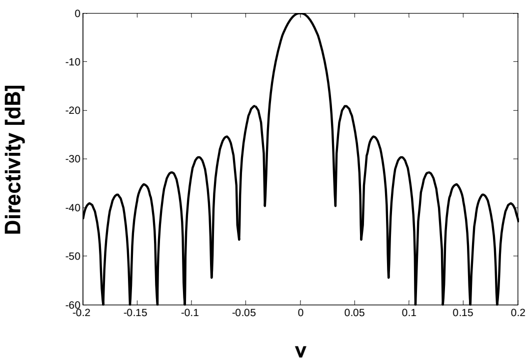
where

$$\eta_n = \frac{\pi D}{\lambda}\sqrt{(u-u_n)^2+(v-v_n)^2} \tag{4}$$

D is the diameter of the circular aperture, J_1: is the definition of a Bessel function of the first kind order 0, J_2: is the definition of a Bessel function of the first kind order 1, λ is the signal wavelength, e_0 is the quantity indicating the electric field value at the edge of the circular aperture.



a)



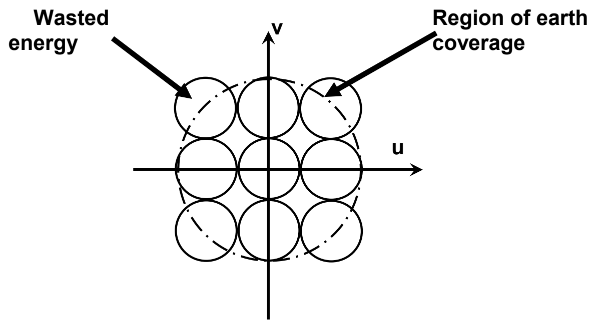
b)

Fig. 2. Radiation pattern of a circular aperture (**a**) 2-dimensional Radiation pattern shown in u-v coordinates,(**b**) Radiation pattern in a cutting plane

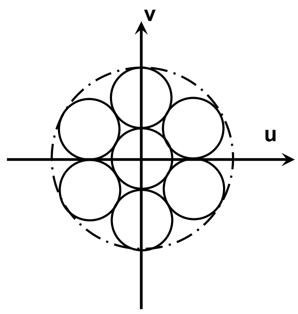
For this simulation $D/\lambda=40$, $e_0=0.707$. A generic radiation pattern introducing important terms describing the various sections of the pattern is shown in Fig. 2.a. In Fig. 2.b, we use the sine space coordinates (u,v).

4. A Typical MBA Configuration

Before proceeding to the synthesis of the optimum nulling radiation pattern, we first determine a suitable sampling interval for earth-coverage. To accomplish this, we utilize the geometrical aid introduced by Dion and Ricardi [9] using a rectangular region R for the sampling interval results in a rectangular set of beams illuminating the earth as shown in Fig. 3.a. Much of the energy is wasted for the diagonal beam out toward the earth the edge. In fact, Dion and Ricardi have shown that a much better earth-coverage gain is realized if the beams are positioned hexagonally as shown in Fig.3.b [10].



a)



b)

Fig. 3. **a**) Rectangular beam positioning. **b**) Hexagonal beam positioning.

5. Optimization Method

The classical problems of antenna systems synthesis for a given complex weights (amplitude and phase) of the field pattern are the most studied. As a rule, the problems are linear in character in various statements and their solutions are unique. However, the increased requirements on shaped beams and integral characteristics of antenna systems yield new and more complicated statements of the problems. In particular, no conditions are imposed on the phase of the antenna pattern in many practical applications.

This leads to a new class of synthesis problems (not yet sufficiently studied) where the desired pattern is given in field amplitude or power. In mathematical terms, these synthesis problems are nonlinear. The solution quantity estimation is given in the past by [11], [12], and recently by [13].

5.1 Optimization Criterion

Then the synthesis problem consists of finding an aperture shape with amplitude-phase field distribution such that the magnitude of the pattern approximates as well as possible. Let the desired amplitude pattern $F_1(u,v)$ be given in a certain area and equal to zero outside (see Fig. 4).

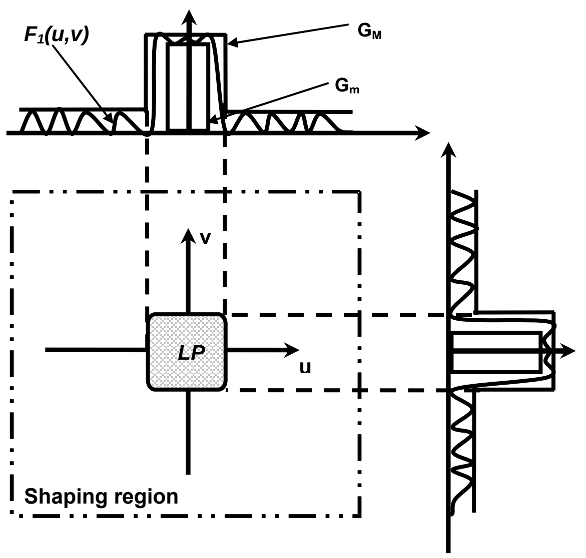


Fig. 4. Desired pattern shape.

A different cost function can be implemented if the principle requirement is to minimize the sidelobe with respect to either the main beam peak or the swath edge. In this case, a ratio cost function can be defined by:

$$J = \int_{LP} L(G_M, G_m, F_1, p) \cdot du \cdot dv + \int_{LS} \left|F_1(u,v)\right|^2 \cdot du \cdot dv \tag{5}$$

where

LP,LS represents respectively the coverage-area of the minimum and maximum shaping region (Fig. 4).

$L(G_M, G_m, F_1, p)$ is the function measuring the variation between the calculated and the desired pattern and is given by:

$$L(G_M, G_m, F_1, p) = \frac{K(u,v) + \left|K(u,v)\right|}{2} \cdot p(u,v) \tag{6}$$

$$K(u,v) = \left(G_M(u,v) - \left|F_1(u,v)\right|\right) \cdot \left(G_m(u,v) - \left|F_1(u,v)\right|\right) \tag{7}$$

G_M and G_m represent respectively the minimum and maximum shaping region (Fig. 4) to obtain the desired pattern, the cost function given in equation (5) will be minimized by the variational method, which is described in the following section.

$p(u, v)$ is a weight function, in what continuation one will pose $p(u, v)=1$.

In the maximum coverage-area gain ($K(u,v)+|K(u,v)|$) is used on optimization function criterion, this is better adapted to our optimization problem. We would like to cancel the criterion when the gauge is satisfied. The figure below shows the two functions on:

$$\begin{cases} h_1(x) = 2 \cdot \left(G_m - |x|\right) \cdot \left(G_M - |x|\right) \\ h_2(x) = \left(G_m - |x|\right) \cdot \left(G_M - |x|\right) + \left|\left(G_m - |x|\right) \cdot \left(G_M - |x|\right)\right| \\ \text{for example } G_M = 2, G_m = 1 \end{cases} \tag{8}$$

The figure shows well that the function $h_2(x)$ is null for $1 \le x \le 2$, who corresponds to a zero value, so the gauge is satisfied and a positive cost is obtained when the gauge is not satisfied.

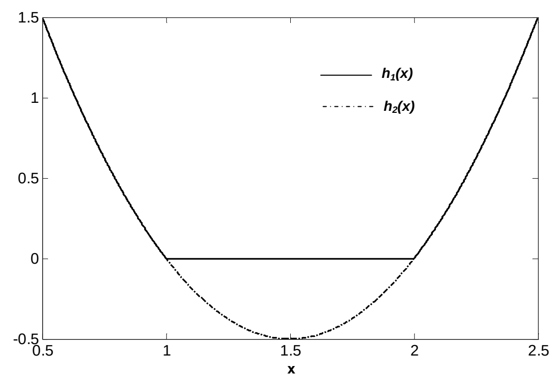


Fig. 5. Comparison between 2 K and K+|K|.

5.2 Variational Synthesis Problem Formulation

The basic philosophy of variational method in creating a slight variation of excitation, this variation act of the criterion of optimization:

$$C_n^1 \to C_n^1 + \delta C_n^1 \tag{9}$$

and

$$J \to J + \delta J\,. \tag{10}$$

Then (5) can be rewritten in the form:

$$J + \delta J_1 = \int_{LP} \Gamma \cdot \mathrm{X}\, d\Omega + \int_{LS} \left|F_1 + \delta F_1\right| d\Omega \tag{11}$$

where

$$\Gamma = \frac{\left(G_M - \left|F_1 + \delta F_1\right|\right)\left(G_m - \left|F_1 + \delta F_1\right|\right)}{2}, \tag{12}$$

$$\mathrm{X} = \frac{\left|\left(G_M - \left|F_1 + \delta F_1\right|\right)\left(G_m - \left|F_1 + \delta F_1\right|\right)\right|}{2}. \tag{13}$$

|F1+δF1| may be approximated by

$$\left|F_1+\delta F_1\right|^2 \approx \left|F_1\right|^2 + 2\Re\left(F_1\delta F_1^*\right) \tag{14}$$

where an asterisk superscript indicates the complex conjugate. Using equations (12), (13) and (14), equation (11) can be rewritten in the following form (15)

$$\delta J = -\int_{LP}\Re\left(F_1\delta F_1^*\right)\left(\frac{G_M-G_m}{2|F_1|}-1\right)\left(\frac{|K(u,v)|}{K(u,v)}+1\right)d\Omega + 2\int_{LP+LS}\Re\left(F_1\delta F_1^*\right)d\Omega\,.$$

Equation (15) can be more conveniently written in the form

$$\delta J = -\int_{LP}2\Re\left(F_1\delta F_1^*\right)\left\{\left[\frac{G_M-G_m}{2|F_1|}-1\right][1+\tau]\frac{p}{2}+1\right\}d\Omega + \int_{LP+LS}2\Re\left(F_1\delta F_1^*\right)d\Omega \tag{16}$$

and

$$\tau = \frac{\left|\left(G_M-|F_1|\right)\left(G_m-|F_1|\right)\right|}{\left(G_M-|F_1|\right)\left(G_m-|F_1|\right)}\,. \tag{17}$$

The radiation pattern variation is given by

$$\delta F_1 = \sum_{n=1}^{N_1}\delta C_n^1 f_n(u,v)\,. \tag{18}$$

Substituting equation (16) to (14), we obtain

$$\frac{\delta J}{2} = \Re\left\{\sum_{n=1}^{N_1}\delta C_n^1\zeta_1\right\} \tag{19}$$

where

$$\zeta_1 = \int_{LP+LS}F_1 f_n^* d\Omega - \int_{LP}\left[\left[\frac{G_M+G_m}{2|F_1|}-1\right][1+\tau]\frac{p}{2}+1\right]d\Omega\,. \tag{20}$$

J is stationary for a set of excitation coefficients (C_n^1)

$$\delta J = 0 \quad ; \forall \delta C_n^1, \tag{21}$$

$$\zeta_1 = 0\,. \tag{22}$$

At the final step, we have

$$\int_{LP+LS}F_1 f_n^* d\Omega = \int_{LS}\left[\left[\frac{G_M+G_m}{2|F_1|}-1\right][1+\tau]\frac{p}{2}+1\right]d\Omega\,. \tag{23}$$

with $n = 1,2,\ldots,N_1$

Equation (23) can be written in matrix notation

$$\begin{bmatrix} \int_{LP+LS} f_1 f_1^* d\Omega & \int_{LP+LS} f_2 f_1^* d\Omega & \cdots & \int_{LP+LS} f_{N_1} f_1^* d\Omega \\ \int_{LP+LS} f_1 f_2^* d\Omega & \int_{LP+LS} f_2 f_2^* d\Omega & \cdots & \int_{LP+LS} f_{N_1} f_2^* d\Omega \\ \vdots & \vdots & \cdots & \vdots \\ \int_{LP+LS} f_1 f_{N_1-1}^* d\Omega & \int_{LP+LS} f_2 f_{N_1-1}^* d\Omega & \cdots & \int_{LP+LS} f_{N_1} f_{N_1-1}^* d\Omega \\ \int_{LP+LS} f_1 f_{N_1}^* d\Omega & \int_{LP+LS} f_2 f_{N_1}^* d\Omega & \cdots & \int_{LP+LS} f_{N_1} f_{N_1}^* d\Omega \end{bmatrix} \cdot \begin{bmatrix} C_1^1 \\ C_2^1 \\ \vdots \\ C_{N_1}^1 \end{bmatrix} = \begin{bmatrix} \int_{LS} F_1 f_1^* \left\{\left[\frac{G_M+G_m}{2|F_1|}-1\right][1+\tau]\frac{p}{2}+1\right\} \\ \int_{LS} F_1 f_2^* \left\{\left[\frac{G_M+G_m}{2|F_1|}-1\right][1+\tau]\frac{p}{2}+1\right\} \\ \vdots \\ \int_{LS} F_1 f_{N_1}^* \left\{\left[\frac{G_M+G_m}{2|F_1|}-1\right][1+\tau]\frac{p}{2}+1\right\} \end{bmatrix} \tag{24}$$

This iterating system is solved from initial value (C_n^1) as shown in Fig. 6.

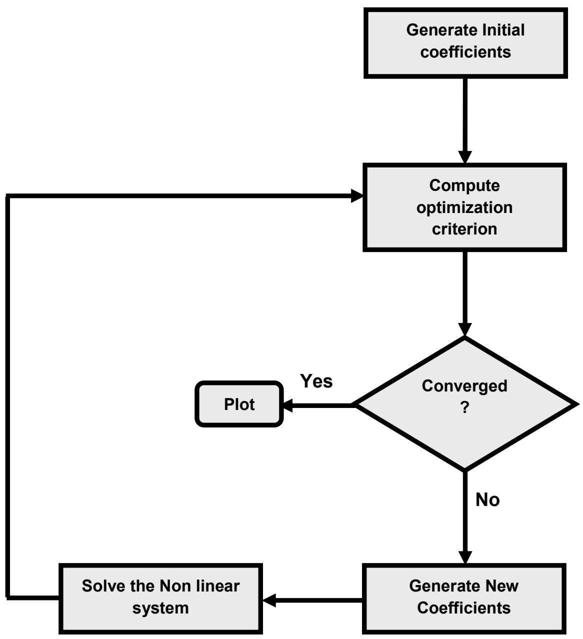


Fig. 6. Basic flow of the variational optimization algorithm.

6. Synthesis Results

We will display the synthesis result of a space antenna radiating 19 elementary beams, the whole beam is laid out on a triangular mesh grid (beams center), the phase center spacing between elementary beams is fixed at ΔS=0.030, the diameter of the circular aperture is equal to ΔS. In this work only directly radiating hexagonal arrays comprising array elements with circular aperture are considered (Fig. 7).

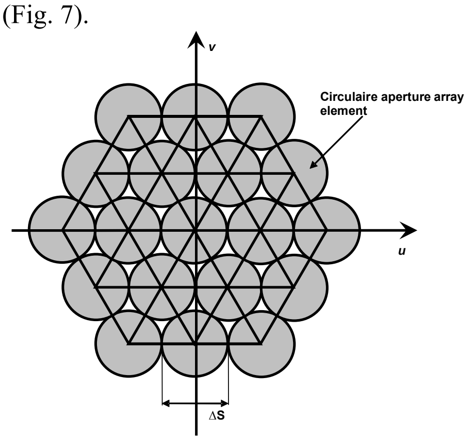


Fig. 7. Hexagonal array geometry where the phase centers of the array elements are located on a regular triangular grid.

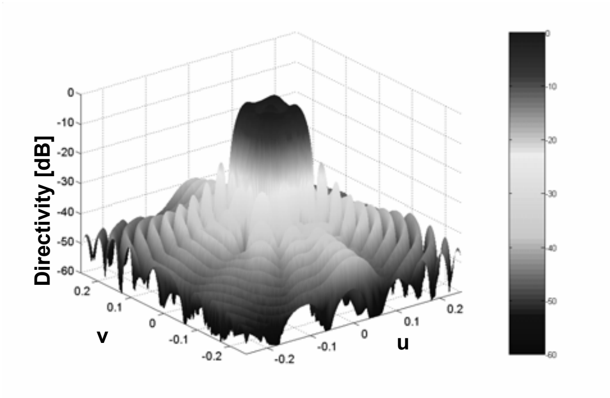
To produce the array pattern with a large main beam width defined in the sector u∈[-0.2, 0.2] and v∈[-0.2, 0.2], the G_M and G_m are given below

$$G_M = \begin{cases} 0 \text{ dB} & \text{for } -0.075 \le u \le 0.075 \\ -25 \text{ dB} & \text{elsewhere} \end{cases}$$

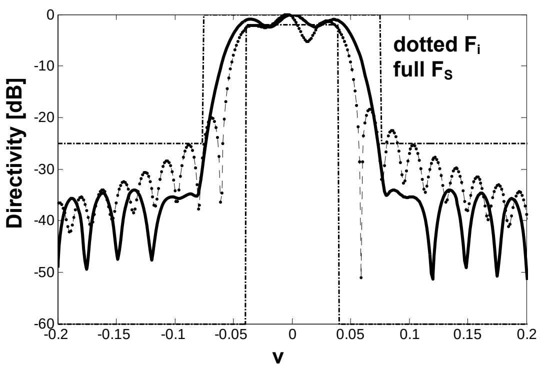
and

$$G_m = \begin{cases} 0 \text{ dB} & \text{for } -0.045 \le u \le 0.045 \\ -60 \text{ dB} & \text{elsewhere} \end{cases}$$

As it can be seen from Fig. 8.a, the variational method is capable to synthesize multibeam antenna in order to produce shaped beam patterns with a good performance both in the shaped region and in the sidelobe region.



a)

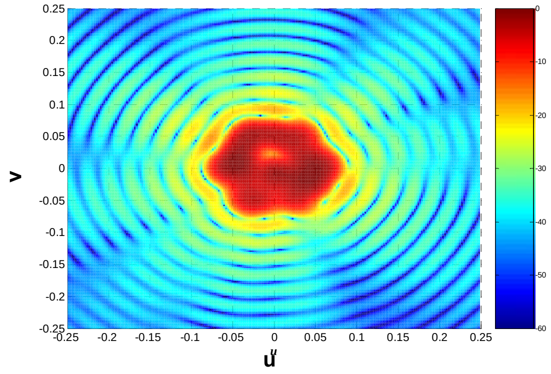


b)

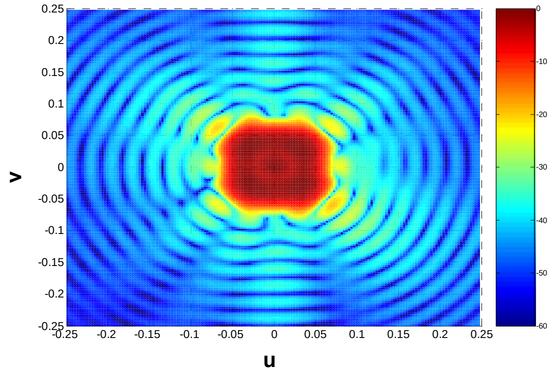
Fig. 8. The radiation pattern obtained by the variational method using 19 elements: **a**) Three-dimensional radiation pattern shown in *u*-*v* coordinates, **b**) Radiation pattern in a cutting plane (F_S synthetized pattern, F_i initial pattern).

Fig. 9.a and 9.b show initial directivity isocontours and optimized directivity isocontours. We clearly notice that after optimization, the totality of energy is emitted in the useful zone and a considerable attenuation of energy apart from this zone.

Finally, the elements amplitude and phase obtained by the variational method to produce the optimum patterns are shown in Fig. 10.

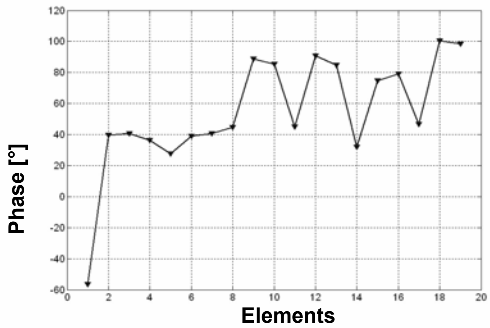


a)

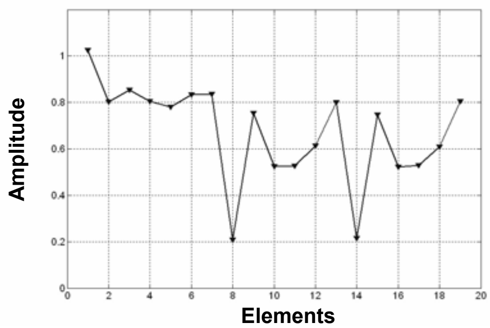


b)

Fig. 9. The radiation pattern obtained using 19 elements: **a**) Initial directivity isocontours. **b**) Optimized directivity isocontours.



a)



b)

Fig. 10. Array optimized parameters of **a**) optimized amplitude, **b**) optimized phase of excitation.

The computer program described earlier was written to implement the optimization technique. The evaluation function number is fixed to 300 and ε to 0.001. This is sufficient to obtain satisfactory patterns and a desired performance. The flexibility of this method and the speed of its computing time (15 mn on P4 2.80 Go CPU and 250 MB as memory) allowed us to simulate other configurations.

7. Conclusion

In this paper, a robust and flexible optimization technique based on the variational method is successfully used to determine the elements excitation of multibeam antenna arrays for the shaped-beam patterns synthesis. Advantages of this technique showed the power, the precision and the speed of the variational method with respect to the all constraints imposed on the radiation pattern. The representation iso-contours of the field radiated by the antenna on the cover shows clearly that we have an optimal distribution of the field radiated towards the zone to cover after optimization. The design of shaped beam satellite antennas provides an example of the utility of this algorithm.

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