Chip Interleaving and its Optimization for PAPR Reduction in MC-CDMA

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Abstract. *This paper analyzes the usability of peak to average power ratio (PAPR) reduction in multicarrier code division multiple access (MC-CDMA) by chip interleaving optimization. This means chip position formatting to PAPR minimization. One chip interleaving pattern is used for all users in system (all spreading sequences). Dependency on number of subcarriers and spreading sequence length is simulated. The impact on amplitude histogram is presented and the relation to random interleaving pattern is shown.*

Keywords

MC-CDMA, OFDM, PAPR reduction, chip interleaving.

1. Introduction

In 1993 the hybrid technics combining OFDM and code division multiple access (CDMA) were proposed [3]. One of them is multicarrier code division multiple access (MC-CDMA) which introduces frequency spreading and multiple access. Both, OFDM and MC-CDMA have disadvantages in high peak to average power ratio (PAPR) and require a linear communication link within the dynamic range that fits the distribution of PAPR. Usually there are problems with high power amplifiers (HPA) and analog-to-digital converters (A/D). They are sources of nonlinear distortions. There are many PAPR reduction schemes for OFDM and most of them are useful also for MC-CDMA. For example PTS [4], [5] (partial transmit sequence) and SLM [7], [6] (selective mapping) can be used for OFDM-CDMA PAPR reduction [9]. Both techniques introduce additional complexities and costs in transmission efficiency. Usage of a spreading code in MC-CDMA gives other possibilities to PAPR reduction.

2. System Model

The basic transmitter structure of MC-CDMA is similar to OFDM. Main difference is in spreading symbols across several subcarriers. Each subcarrier transmits a part of spread symbol (called chip). Let the number of subcarriers (N_c) be equal to spreading code (c) length. Then one user signal can be expressed as:

$$x^l(t) = \frac{1}{N_c}\sum_{n=0}^{N_c-1} b^l c_n^l e^{j2\pi f_n t}, \qquad 0 \le t \le T_s, \quad (1)$$

where $x^l(t)$ is the l-th user signal, b^l is the data symbol of the l-th user, c_n^l is the n-th chip of the l-th user and f_n is the frequency of the n-th subcarrier.

2.1 M-modification

The spreading code length has not necessarily to be equal to the number of all subcarriers. Some modifications are presented in [1] for complexity reduction and flexible system design. We will focus on so called M-modification. The total number of subcarriers N_c is divided into M groups with L subcarriers each. For one user (uplink) each group of L subcarriers transmits one symbol spread with the sequence length L. The user transmits M parallel data symbols on all $N_c = M \times L$ subcarriers and M-modification is also called Paralel Data Symbols. Fig. 1 shows an example for $N_c = 16$ and $L = M = 4$. For an arbitrary user, the same spreading sequence is used for all data symbols. Other users transmit on identical subcarriers, but each of them has his own spreading sequence.

2.2 Peak to Average Power Ratio

OFDM and MC-CDMA consist of many modulated subcarriers. This leads to a problem with the peak to average power ratio. If N_c subcarriers are added up coherently (the same symbols on all subcarriers for OFDM or spreading sequence number 1 (all chips are same) for Walsh codes and same data symbols in MC-CDMA), the peak power is N_c times the average power in the case of the baseband signal. The PAPR of signal x_τ, where τ is used to represent both the continuous-time index t and the discrete-time index n, is defined [8] as:

$$PAPR\{x_\tau, \mathcal{T}\} = \frac{\max\limits_{\tau\in\mathcal{T}} |x_\tau|^2}{E\{|x_\tau|^2\}} \quad (2)$$

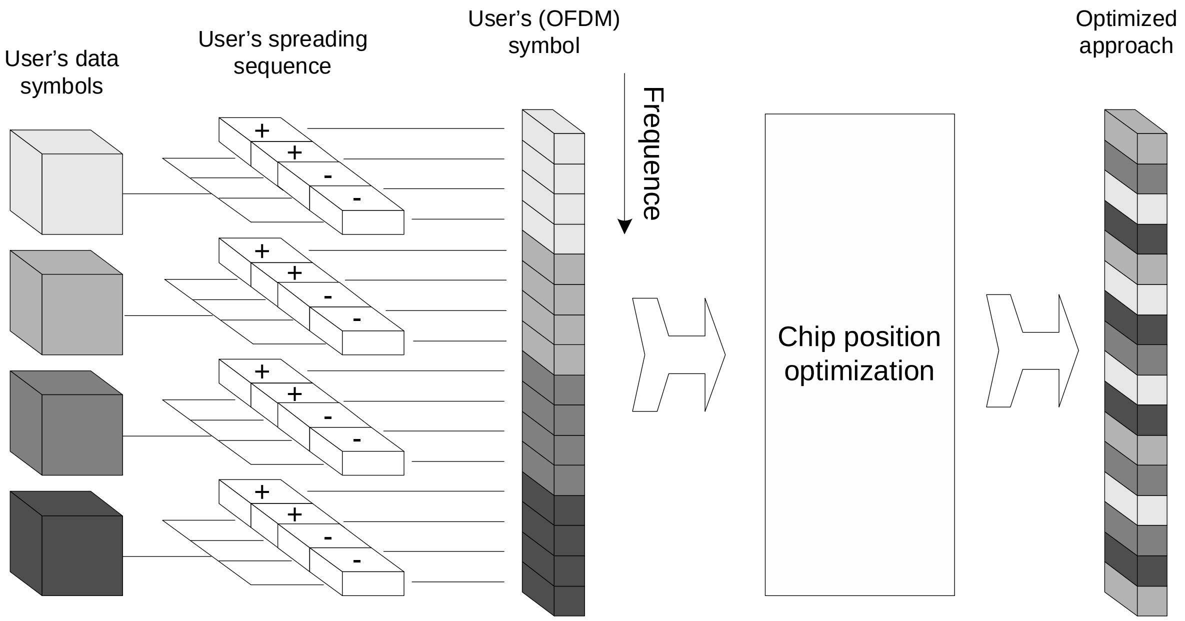

Fig. 1. M-modification of MC-CDMA and its optimization.

where $\max_{\tau\in\mathcal{T}} |x_\tau|^2$ denotes the maximum instantaneous power and $E\{|x_\tau|^2\}$ is the average power of the transmitted symbol in the symbol interval $\mathcal{T}$.

3. Optimization of Chip Interleaving

Using the spreading sequences introduces some kind of redundancy to a system. For example sequence 1 -1 1 will continue with -1 (Walsh spreading sequence lengths 4 and BPSK modulation). OFDM based systems (MC-CDMA included) are represented in time domain by a sum of sinusoids. Now, by changing the position of the chip (chips), the peak value of the sum of sinusoids (representing chip sequence) can be reduced. For no chip interleaving the first chip is on the first subcarrier, the second chip on the second subcarrier, etc. This can be represented by the vector [1 2 3 4 5 … N_c]. A permutation of this vector, for example [3 6 9 13 …], represents the chip interleaving pattern, where the first chip is modulated on the third subcarrier, the second chip on the sixth subcarrier and so on.

The principle of our approach is to find the interleaving pattern (vector permutation) that minimizes PAPR. The chip interleaving pattern must be the same for all users (all spreading sequences) in the system to preserve the orthogonality among them. The number of possible interleaving patterns corresponds with the number of subcarriers in the system (exactly it is factorial of N_c), that makes direct search algorithms improper for systems with more subcarriers. By testing several optimum searching algorithms, optimization with genetic algorithm (GA) provided good results. The fitness function is computed as the mean value of PAPR for all possible combinations of a spreading sequence (user) and data [2].

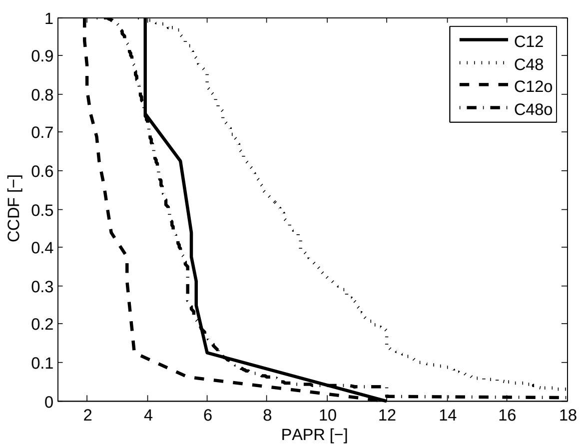

Fig. 2. CCDF of PAPR for different number of subcarriers - 12 subcarriers with (C12o) and without (C12) optimization, and 48 subcarriers with (C48o) and without (C48) optimization).

4. Optimized Results

Through all experiments described in this section, Walsh-Hadamard spreading sequences are considered. Simulations with fixed spreading sequence length (4), BPSK and different numbers of subcarriers are presented in Fig. 2. A complementary cumulative distribution function (CCDF) of PAPR is plotted to compare simulations. Higher values of PAPR can occur with growing number of subcarriers. The CCDF for 48 subcarriers can be reduced to correspond CCDF of 12 subcarriers with usage of chip interleaving optimization. The part, which can't be optimized, is the worst case of PAPR. This case represents the same chips on all subcarriers, where chip interleaving has no effect and PAPR is equal to N_c (for baseband signal). This occurs only with really small probability, so high PAPR values are not plotted in figures to ensure better readability.

Results with the fixed number of subcarriers and the different spreading sequence length are presented in Fig. 3 and 4. Fig. 3 shows CCDF of PAPR for the different spreading sequence length and the number of subcarriers $N_c = 32$ with no optimization and compares it with a pure OFDM. All MC-CDMA systems show worse CCDF of PAPR than pure OFDM (but it should be mentioned that MC-CDMA curves are for 1 user - uplink and pure OFDM does not support multiuser access). PAPR growths with the spreading sequence length. The impact of optimization is shown in Fig. 4. The OFDM CCDF is plotted again for reference. The chip position optimization improves MC-CDMA CCDF so that it tends close to OFDM CCDF. The PAPR improvement is more important in case of long spreading sequences.

An interesting observation is overlapping of CCDF for spreading sequence length 16 and 32. Due to the BPSK modulation, the same chip sequences can be obtained with any data symbol pairs [1 1], [1 -1], [-1 1] or [-1 -1] and the spreading sequence length 16 or with the spreading sequence length 32 and the data symbol 1 or -1.

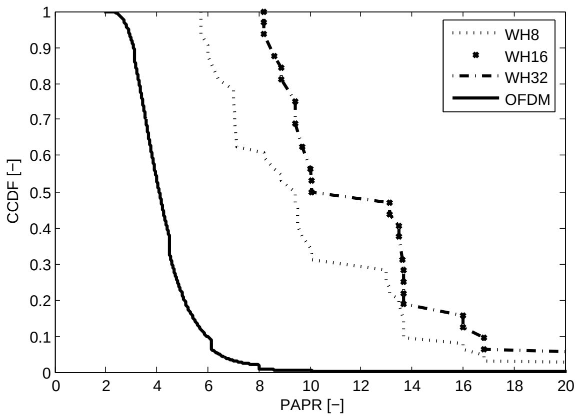

Fig. 3. CCDF of PAPR for no interleaving and spreading sequence (Walsh spreading sequences) length 8 (WH8), 16 (WH16) and 32 (WH32).

5. Amplitude Histogram and Random Interleaving

To better understanding a principle of this method, the amplitude histogram can be plotted. The amplitudes in OFDM have Rayleigh distribution. The distribution for one

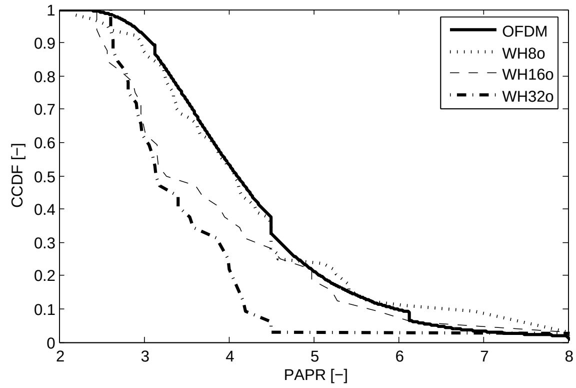

Fig. 4. CCDF of PAPR for optimized interleaving and spreading sequence (Walsh spreading sequences) length 8 (WH8o), 16 (WH16o) and 32 (WH32o).

user and MC-CDMA system is not even close to Rayleigh, it is more like exponential distribution and produces higher PAPR values (Fig. 5).

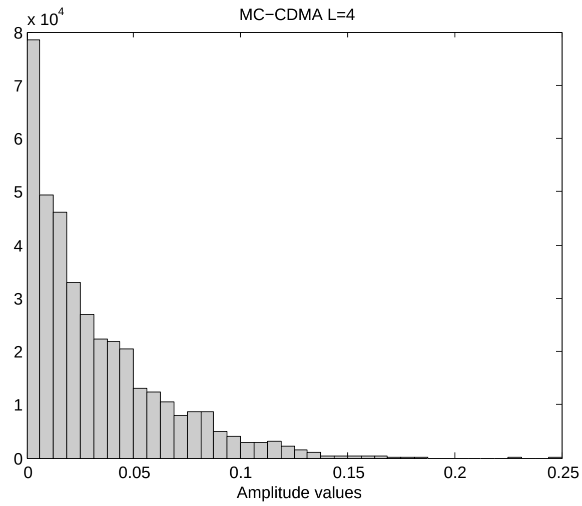

Fig. 5. Amplitude histogram for MC-CDMA, 32 subc. L=4, BPSK.

The amplitude histogram is close to Rayleigh distribution after application of optimized interleaving pattern (Fig. 6) and PAPR values are close to values for OFDM too.

The values modulating subcarriers are random symbols for OFDM (assuming random data), but are highly correlated for MC-CDMA (Walsh-Hadamars sequences). It can be assumed, that random interleaver will provide the same results as optimized one. The results with random interleaver are close to the results with optimized interleaving (Fig. 7). Not all random interleaving patterns present optimal results (especially low L). The optimization algorithm can also introduce additional parameters (like maximization interchip distance).

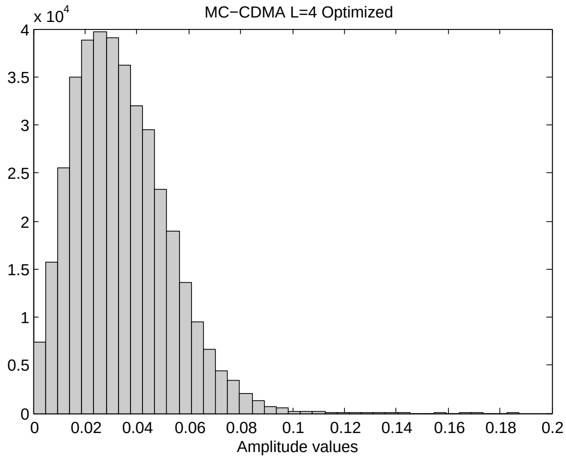

Fig. 6. Amplitude histogram for MC-CDMA, 32 subc. L=4, BPSK with optimized interleaving.

6. Conclusion

The above mentioned PAPR reduction can be used for uplink in M-modification MC-CDMA. Time-consuming GA is performed only during system planning. The resulted interleaving pattern is the same for all users in the system to preserve orthogonality among each other. The main advantage of this approach is low complexity (modified interleaving only), no performance degradation, and no need of side information. Although the worst case of PAPR is still present in an optimized approach, the CCDF is improved. If the clipping or nonlinear distortion is present, the system performance degradation is reduced.

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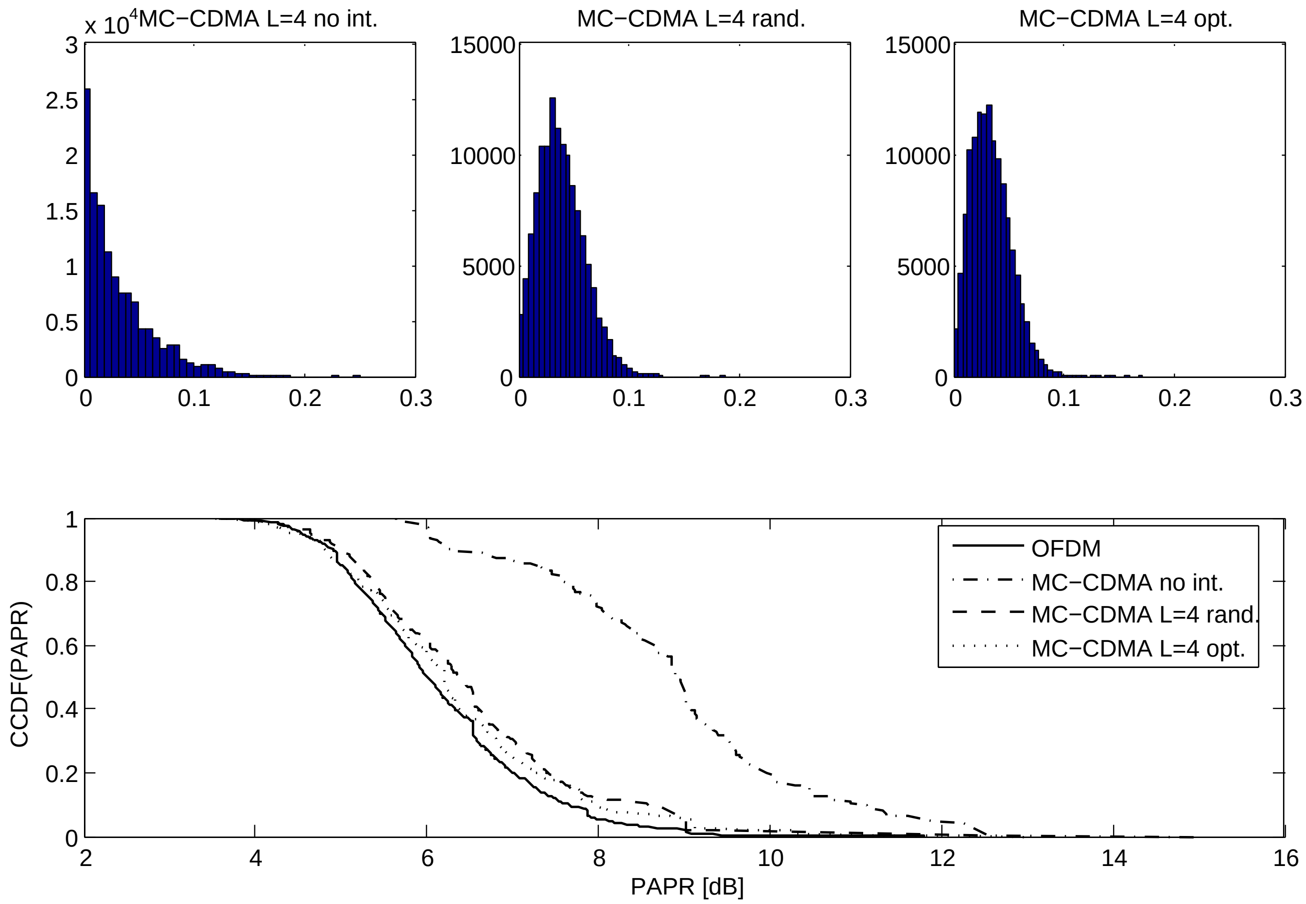

Fig. 7. Comparison of not interleaved, optimized and random interleaving for one user MC-CDMA with 32 subc., L=4 and BPSK modulation.