Broadband Measurement of Complex Permittivity Using Reflection Method and Coaxial Probes

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Abstract. *This paper describes and evaluates a method for determining complex permittivity, and presents results of permittivity measurement of some substances. Complex permittivity of a phantom of biological muscle tissue, of some industrial chemicals and dielectrics is found. A non-destructive and non-invasive method based on reflection coefficient measurement of an open-ended coaxial line attaching the material under test is used. Two coaxial probes are under investigation. Vector measurement of the reflection coefficient on the interface between probes and measured samples is performed with the aid of network analyzer in the frequency range from 300 kHz to 3 GHz. Numerical modeling (FDTD) is compared with measurement. The results indicate that using the coaxial probe with dimensions of N connector the method is suitable in the frequency range approximately from 30 MHz to 1 GHz and using the probe with dimensions of SMA connector in range approximately from 30 MHz to 3 GHz.*

Keywords

Complex permittivity measurement, vector reflection coefficient measurement, coaxial probes.

1. Introduction

The knowledge of dielectric parameters of materials is essential for microwave or radio engineers in analysis and synthesis of devices. Relative permittivity, loss factor and conductivity are input parameters for electromagnetic field modeling and simulations. Although for many materials these parameters could be found in the tables, very often their experimental determination is necessary.

Dielectric properties of biological tissues are determining factors for the dissipation of electromagnetic energy in the human body and therefore they are basic parameters in hyperthermia cancer treatment. Measurement of the dielectric parameters of biological tissues is also a promising method in the medical diagnostics and imaging. Knowledge of the complex permittivity in an area under treatment, i.e. knowledge of the complex permittivity of healthy and tumor tissue, is very important for example in the diagnosing of tumor cell-nests in the human body or in the design of thermo-therapeutic applicators which transform electromagnetic energy into thermal in the pathological tissue [1].

The dielectric properties of chemical substances, of textile or dielectric substrates are important in industrial applications of the microwaves – in the design of systems for microwave drying of textile or microwave heating of liquids.

Permittivity is known from physics or theory of electromagnetic field as

$$\varepsilon = \varepsilon_0 \varepsilon_c \tag{1}$$

where ε_0 is free space permittivity and ε_c is complex relative permittivity (dielectrics are very often lossy). Complex relative permittivity can be given in turn as

$$\varepsilon_c = \varepsilon_r - j\varepsilon_r \tan\delta \tag{2}$$

where ε_r is real part of complex relative permittivity and $\tan\delta$ is loss factor. For purely conductive losses

$$\tan\delta = \frac{\sigma}{\omega\varepsilon_0\varepsilon_r} \tag{3}$$

where σ is the medium conductivity.

2. Complex Permittivity Measurement

Common to all papers in the field of complex permittivity measurement is a more or less extensive tabulation of the dielectric properties of materials selected to illustrate the theoretical deliberations provided by the authors. Nowadays, papers are instigated for example by the need for such information in electromagnetic dosimetry. There is no consensus on the dielectric data yet [2].

The systems using complex permittivity measurement exist, even are commercially available (e.g. Agilent Technologies 85071E analyzer) but the Czech Technical University in Prague is not equipped with such material measurement system. Our measurements are motivated by the need of cheap but accurate determination of the complex permittivity.

There are several methods for measuring the complex permittivity – resonance method, measurement in free space and transmission line method. If we want to use a broadband measurement method that is non-destructive and non-invasive, we should choose a reflection method on an open-ended coaxial line – a method based on reflection coefficient measurement of an open-ended coaxial line attaching the sample of measured dielectric [3].

2.1 Principle of Reflection Method

The reflection method means measurement of reflection coefficient on the interface between two materials, on the open end of the coaxial line (as a detector) and the material under test (MUT, Fig. 1). It is a well-known method for determining the dielectric parameters [4]. This method is based on the fact that the reflection coefficient of an open-ended coaxial line depends on the dielectric parameters of MUT which is attached to it. For calculating the complex permittivity from the measured reflection coefficient it is useful to use an equivalent circuit of an open-ended coaxial line.

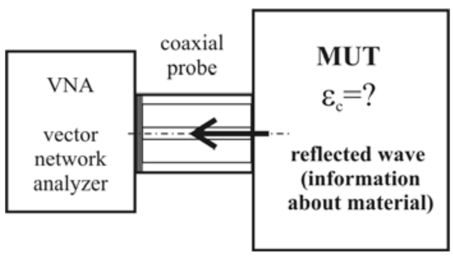

Fig. 1. Illustration to the principle of reflection method.

Fig. 1 shows the complex permittivity measurement from the point of view of theory of electromagnetic field and propagation of the electromagnetic wave on the interface between two materials with different impedances [5]. The probe translates changes in the permittivity of a MUT into changes of the input reflection coefficient of the probe.

The surface of the sample of MUT must be in perfect contact with the probe. The thickness of a measured sample must be at least twice an equivalent penetration depth of the electromagnetic wave *d*. This assures that the waves reflected from the far MUT interface are attenuated approx. -35 dB, which assures that their effect on the measured reflection coefficient is insignificant [4].

$$d = \frac{1}{\omega}\sqrt{\frac{2}{\mu\varepsilon_0\varepsilon_r \tan\delta}} \tag{4}$$

Eq. (4) denotes the dependence of equivalent penetration depth *d* on dielectric parameters ε_r and tan δ and also frequency *f*. Typical values of *d* for distilled water at different frequencies are summarized in Tab. 1.

MUT	*f* (MHz)	2*d* (mm)
distilled water	434	270
	915	61
	2450	17
biological muscle tissue	434	42
	915	28
	2450	14

Tab. 1. Examples of different equivalent penetration depth.

3. Measurement System

A typical measurement system using the reflection method on an open-ended coaxial line consists of the network analyzer, the coaxial probe and software. Our measurements (Fig. 2) were done with the aid of Agilent 6052 network analyzer in the frequency range from 300 kHz to 3 GHz.

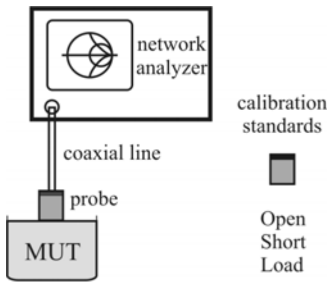

Fig. 2. Complex permittivity measurement.

The coaxial probe (in Fig. 1 and 2) is placed in contact with a MUT. Complex permittivity measurement is very fast and proceeds through three steps. First the calibration of the vector network analyzer is performed. Then the calibration using a reference material (with the known dielectric constant ε_c) is done. And last the reflection coefficient of MUT is measured. The complex permittivity of MUT is evaluated with the aid of PC (in our case using MatLab on PC).

3.1 Measurement Probe

For this measurement method we have adapted the standard N and SMA connectors from which the parts for connecting to a panel were removed. The measurement probe can be described by the equivalent circuit consisting of the fringing capacitance between the inner and outer conductor out of the coaxial structure and radiating conductance which represents propagation losses. These capacitance and conductance are frequency and permittivity dependent and also dependent on the dimensions (inner and outer diameters) of the probe.

Input admittance of the equivalent circuit can be expressed as

$$Y = G_0(\varepsilon_c, \omega) + j\omega C_0(\varepsilon_c, \omega)\,. \tag{5}$$

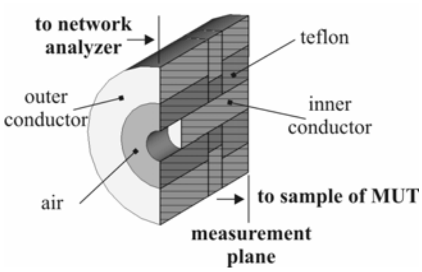

Fig. 3. Model of N connector type probe (inner/outer diameter: N connector type 3/7mm, SMA 1.3/4 mm).

The coaxial probe can be presented as an antenna in lossy medium with the description

$$Y(\varepsilon_c,\omega)=\sqrt{\varepsilon_c}\,Y(1,\omega\sqrt{\varepsilon_c})\,. \tag{6}$$

It means that the admittance of medium with permittivity ε_c at angular frequency ω is the same as an admittance measured in the free space at frequency $\sqrt{\varepsilon_c}$-times higher furthermore multiplied by $\sqrt{\varepsilon_c}$. The input admittance of the coaxial transmission line is then [4]

$$Y=j\omega\varepsilon_c C_0+\sqrt{\varepsilon_c^{\,5}}\,G_0 \tag{7}$$

where Y is the measured admittance of the probe, C_0 and G_0 are constants given by the equivalent circuit of the probe in free space. Admittance Y is related to the measured reflection coefficient S_{11}

$$Y=Y_0\frac{1-S_{11}}{1+S_{11}} \tag{8}$$

where Y_0 = 1/(50 Ω) = 0.02 S is characteristic admittance of the probe.

The method of solving the complex equation (7) consists in splitting it into real and imaginary parts obtaining thus a set of two real nonlinear equations for the two real unknowns, which are either C_0 and G_0 (when calibrating the probe) or the real and imaginary part of complex permittivity ε_c of MUT.

More detailed in steps:

- Splitting of Eq. (7) into real and imaginary parts.
- For obtaining C_0 and G_0, admittance Y for a material with known complex permittivity ε_c (e.g. distilled water, as outlined in Section 5.1) is measured and the set of two equations is solved for the unknowns C_0 and G_0.
- For measurement of complex permittivity, admittance Y of a MUT is measured and the set of the two equations in solved for the unknowns real and imaginary parts of ε_c.

3.2 Calibration Kit for Vector Measurement

The system is one-port network so that the measurement is reduced to only measurement of the input reflection coefficient S_{11}. The calibration method OSL (Open, Short and Load) is performed on the interface between the probe and sample of MUT.

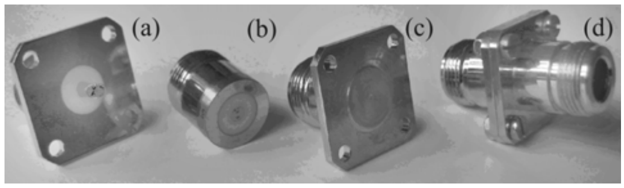

Fig. 4. N connector (a), measurement probe (b) and calibration standards short (c) and open (d).

A coaxial calibration kit was mechanically developed by adapting the panel N (Fig. 4a) and SMA connectors in the same way as the measurement probe (Fig. 4b). The *short* standard (Fig. 4c) is made by the connector which is shorted in the measurement plane by a metal plate. The coaxial *open* standard (Fig. 4d) is created by two connectors and the *load* standard is commonly used 50 Ω termination.

3.3 Step-by-Step Procedure of Measurement

In steps the MUT is measured:

- Calibrate the network analyzer.
- Measure S_{11} for a substance with the known ε_c (distilled water), compute Y using Eq. (8) and, solving Eq. (7) as outlined above, determine the constants C_0 and G_0.
- Measure S_{11} for any desired MUT, compute Y (Eq. 8) and solve Eq. (7) for the unknown real and imaginary part of complex permittivity ε_c. Since the equations are of 5th order in terms of ε_c, care must be taken to choosing a physically correct solution.
- If needed, derive from ε_c any quantities of interest, like relative permittivity ε_r, loss factor tan δ or conductivity σ.

4. Modeling and Simulations

Evaluation of this measurement method involves also numerical calculation and modeling (Fig. 6). Numerical simulation based on a Finite Integration Technique (FIT) is used to calculate the reflection coefficient on the interface between the probe and the sample of MUT [7]. The system that we modeled consists of two parts, i.e. the sensor and the sample of MUT. The modeling is focused on a model with distilled water – the Debye model of distilled water is implemented (Eq. 9). This model will be also used later for the evaluation of measurement accuracy.

Material model of distilled water was recognized by Debye in 1926 and describes the complex permittivity of distilled water as a function of frequency

$$\varepsilon_c=\varepsilon_r'-j\varepsilon_r''=\varepsilon_\infty+\frac{\varepsilon_s-\varepsilon_\infty}{1+j\omega\tau} \tag{9}$$

where ε_∞ is optical permittivity at high field frequencies, ε_s is static permittivity at low field frequencies and τ is electrical relaxation time (a measure of molecules and dipoles mobility). The values of these parameters for distilled water are: $\varepsilon_\infty = 4.6$, $\varepsilon_s = 78.3$ and $\tau = 8.07$ ps. Graphical representation of Eq. (9) is shown in Fig. 5.

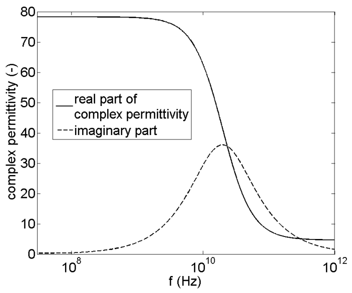

Fig. 5. Debye model of distilled water at T=30°C.

4.1 De-embedding

EM field simulation provides the information about reflection coefficient Γ_x at the reference plane of the excitation port but for calculation of complex permittivity, the measurement technique needs the reflection coefficient ($\Gamma = S_{11}$) on the interface between the probe and the sample of MUT.

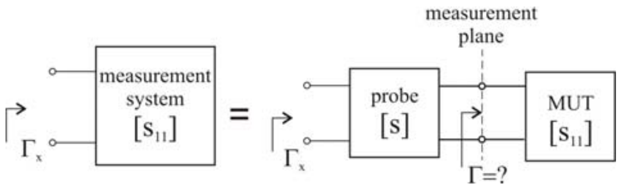

Fig. 6. De-embedding of measurement system.

In addition the measurement probe represents an inhomogeneous transmission line. The shift of reference plane is not possible because of different parts of dielectric materials (air and teflon) inside the N connector. Therefore it is necessary to perform de-embedding procedure for device under test for getting correct phase information of the reflection coefficient.

5. Evaluation of Accuracy

If the Debye model of distilled water is taken as standard, the relative measurement (and simulation) error δ_m (and δ_s) can be defined

$$\delta_m = \frac{|\varepsilon_{rm} - \varepsilon_{rDm}|}{\varepsilon_{rDm}} \tag{10}$$

where ε_{rm} is the real part of complex relative permittivity of distilled water calculated from measured (simulated) reflection coefficient using the equivalent circuit of the probe, and ε_{rDm} is the real part of complex relative permittivity determined by the Debye model (Eq. 9).

This evaluation is verification of self-consistency of the developed relation between the measured reflection coefficient and calculated complex permittivity (Eq. 7). Procedure of this evaluation is:

- Debye model of distilled water is used as the input for electromagnetic field simulator and probe calibration.
- Simulation of reflection coefficient is performed, then de-embedding, the output for calculation is simulated and de-embedded values of S_{11}.
- Using S_{11} and ε_c from the input Debye model, the constants C_0 and G_0 are determined.
- Distilled water is under investigation as a MUT, its complex relative permittivity is calculated (as outlined in 3.1).
- Relative simulation errors δ_s are determined (Eq. 10).

The data from the measurement of reflection coefficient are evaluated as outlined in section 3.1.

f [MHz]	30	434	900	1000
δ_m [%]	0.66	0.80	4.62	5.76
δ_s [%]	0.03	1.33	5.20	6.27

Tab. 2. Relative measurement errors, N-type probe.

Good agreement between the model and measurement with N connector type probe is from approximately 30 MHz to 1 GHz, and between the model and measurement with SMA connector type probe is from approximately 30 MHz to 3 GHz.

f [MHz]	30	434	900	2450
δ_m [%]	0.57	1.33	4.82	20.8
δ_s [%]	0.01	1.04	5.30	27.6

Tab. 3. Relative errors, SMA-type probe.

Relative permittivity is used in calculations of electromagnetic field distribution and is related inversely in the square root in these calculations. It means that the measurement errors from Tab. 2 and 3 are further suppressed. Evaluation of the relative measurement error in case of determination of the imaginary part of complex permittivity is not presented. Distilled water has extremely low values of this imaginary part (lower than 1) and the evaluation is difficult – the relative errors take high values when there is only little difference between the measured (or simulated) and Debye values.

6. Results

Relative permittivity of lossy materials is a heavily frequency-dependent quantity. Because of decreasing ability of particles to follow fast changes of electrical field, the relative permittivity decreases with the increasing frequency. The frequencies in following tables are chosen as interesting from industrial, scientific and medical point of view.

6.1 Agar

The phantom is a tissue-equivalent material, in this case an equivalent of biological muscle tissue. An agar phantom (agar gelatine) is the most used phantom in the testing of thermotherapy applicators, the use of phantoms is significant in the measurement of impedance matching and Specific Absorption Rate (SAR).

f [MHz]	ε_r [-]	$\tan\delta$ [-]	σ [S/m]
434	57.5 (56.9)	0.31 (0.59)	0.45 (0.81)
915	55.9 (54.9)	0.29 (0.34)	0.85 (0.95)
1800	52.7 (53.5)	0.27 (0.25)	1.42 (1.34)
2450	51.1 (52.7)	0.23 (0.24)	1.63 (1.74)

Tab. 4. Dielectric parameters of agar phantom. The values of relative permittivity, loss factor and conductivity in brackets are valid for the biological muscle tissue and are shown for comparison [1].

Agar phantom is relatively good equivalent of biological muscle tissue. There is good agreement in the relative permittivity but in the loss factor or conductivity there is a difference mainly at lower frequencies. It appears that agar has lower water content and is not so lossy as muscle tissue.

6.2 Pentacarbonyl

Microwave substrates and components are distributed with catalogue sheets but in some cases or different frequency applications, where the information about dielectric parameters is missing, it is possible to determine the complex permittivity experimentally by the reflection method.

Pentacarbonyl is usually used in high frequency technology in form of a wedge for insertion into the waveguide. As a substance it has 97.5% of iron. Other constituents are oxygen, nitrogen and carbon.

f [MHz]	ε_r [-]	$\tan\delta$ [-]	σ [S/m]
434	15.1	0.042	0.015
900	14.9	0.040	0.027
1800	13.9	0.046	0.064
2450	12.8	0.057	0.099

Tab. 5. Dielectric parameters of pentacarbonyl (no data for comparison were available).

6.3 Industrial Chemicals

The elimination of acids and Mixed Waste Descaling Bathes (MWDBs, WDB+Chloride) with the aid of microwaves is preferable to classical chemical processes. The main advantages of microwave heating are ecological and low cost aspects.

Substance	ε_r [-]	$\tan\delta$ [-]	σ [S/m]
MWDB	92	0.061	0.765
WDBCl	98	0.056	0.748

Tab. 6. Dielectric parameters of industrial chemicals – waste descaling bathes, measured at f = 2.45 GHz (no data for comparison were available).

7. Limitations

The interface between the measurement probe and the sample of biological tissue's phantom represents an impedance jump (Fig.1). Dielectrics have sometimes extremely high permittivity values. At low frequencies their permittivity can be more than 100 and the value of the loss factor more than 0.1. An accurate evaluation is very difficult because the reflection coefficient is close to 1. This means that only a very small part of the incident energy penetrates into the sample.

At higher frequencies the higher order modes at the aperture of probe are present. No rigorous description exists, in this paper approximate solution is available (Eq. 7). If the probe is radiating at higher frequencies (wavelength is comparable with the probe's dimensions), radiating conductance G must be taken in account.

8. Conclusion

The complex permittivity determination based on reflection coefficient measurement is suitable for the determination of dielectric parameters of materials in wide band. This method was described from the viewpoint of theory of electromagnetic field and the coaxial probes were described with the equivalent circuit as an antenna in lossy medium respecting the radiation effects at higher frequencies. Some materials were measured and where it was possible the comparison between measurement (modeling) and values from tables was carried out. If the evaluation should be complete, the accuracy had to be specified. The results obtained indicate that the accuracy may be sufficient for most of practical applications.

Acknowledgments

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