Enhanced Coupling Values in Coupled Microstrip Lines using Metamaterials

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Abstract. *In this paper, we show how metamaterials can be used to enhance the coupling values of microstrip directional couplers. Coupling between regular coplanar microstrip lines, in fact, is limited, due to the small ratios between the characteristic impedances of even and odd TEM modes supported by the structure. The broadside configuration or the employment of an overlay are often utilized to overcome this limitation, leading, however, to more bulky components. On the other hand, the employment of metamaterials with a negative real part of the permittivity is able to increase the coupling values, while keeping the profile of the structure very low. A quasi-static model of the structure is developed and physical insights on the operation of the proposed component and on the role of the metamaterial loading are also given. Simple design formulae derived through a conformal mapping technique are presented and validated through proper full wave numerical simulations.*

Keywords

Coupled lines, microstrip lines, metamaterials.

1. Introduction

Coupled microstrip lines are an old subject in the electromagnetic and microwave communities. They have been studied for decades by several researchers and research groups, who added always a little bit more to the understanding of the physical phenomena related to the operation, to the design techniques, and to the ever new possible applications in telecommunication and electronic devices. However, a fundamental limit of coupled microstrip lines resides in the possible coupling values achievable [1].

Recently, we have shown how the employment of proper Epsilon NeGative (ENG) metamaterial slabs [2-7], vertically placed between the two microstrip lines, may be used to dramatically decrease the characteristic impedance of the odd mode, obtaining, thus, very high coupling values. This interesting feature has been predicted using an heuristic interpretation based on very simple physical

models. In [7] we have presented an analytical approach to design the coupled microstrip lines. This approach is based on the employment of a conformal mapping technique in the quasi-static limit approximation. Finally, the analytical method has been validated through full-wave numerical simulations performed through CST Microwave Studio.

In addition, by properly extending the formulation proposed in [1], we show how it is possible to obtain from the same structure depicted in Fig.1 all the possible coupling values (from 0 to 1), just choosing the suitable values of the permittivity of the ENG slab placed between the two microstrip lines. In this way, we are able to design not only coupled microstrip lines with very high coupling values, but also close microstrip lines with a very low crossover. Several applications in the frame of the Electromagnetic Compatibility are foreseen and some of them will be presented at the conferences.

The aim of this paper is two-fold: a) review the previous theoretical investigations on the proposed component developed by our group; b) present some new physical insights on the operation mechanism and the ENG metamaterial effect (e.g. material anisotropy and fabrication issues).

2. The Role of an ENG Metamaterial Thin Slab to Increase the Coupling Coefficient

The first layout we propose here, to overcome the limitation in the coupling values achievable in regular coupled microstrip lines, is shown in Fig. 1. An isotropic and homogenous ENG thin slab is placed between the two strips inside the homogeneous and isotropic dielectric substrate. In the frequency band of operation, the metamaterial slab presents negative values of the real part of permittivity, while the substrate is assumed to be made by a high-index material (i.e. alumina). The negative permittivity of the ENG slab is assumed to be, as absolute value, much smaller than the substrate permittivity.

Since the normal component of the electric displacement is continuous on the interfaces, the choice of a low absolute value of the permittivity for the ENG slab allows

to obtain a rather strong amplitude of the electric field in the points of the slab close to the interfaces.
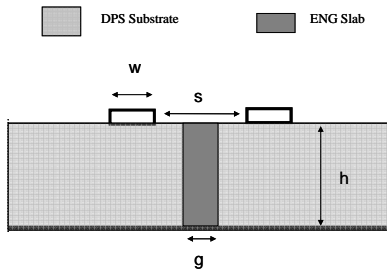

Fig. 1. Coupled microstrip lines with an ENG slab inside the substrate.

If we apply the boundary condition for the electric displacement, in fact, we obtain:

$$E_n^{ENG} = \frac{\varepsilon_{s\,ub}}{\varepsilon_{ENG}} E_n^{sub}$$

and since $|\varepsilon_{sub} / \varepsilon_{ENG}| >> 1$ we get;

$$\left| E_{n_{1,2}}^{ENG} \right| >> \left| E_{n_{1,2}}^{sub} \right|.$$

The main effect of the anomalous permittivity of the ENG slab is, thus, to force in the slab itself an electric field normally directed with respect to the interfaces. From the equivalent circuit model for the quasi-TEM odd mode and for the quasi-TEM even (Fig. 2), it is evident that the presence of the ENG slab affects, mainly, the field distribution of the odd mode, which is the one satisfying the PEC symmetry condition.
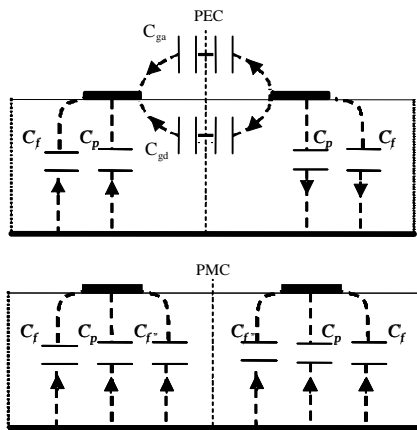

Fig. 2. Equivalent circuit model for the quasi-TEM odd mode (top) with a PEC (Perfect Electric Conductor) and for the quasi-TEM even mode (bottom) with a PMC (Perfect Magnetic Conductor).

The effect of the ENG slab on the odd mode, therefore, can be represented through a further equivalent capacitance C_{eng}, as reported in the new equivalent circuit of the quasi-TEM odd mode presented in Fig. 3.

Assuming the operation of the component in a frequency range where only the two quasi-TEM modes are excited in the structure, the coupling and guiding features of the coupled microstrip lines can be described through

the quasi-static model proposed by Garg and Bahl [8]. The two supported quasi-TEM modes exhibit, respectively, an even (even mode) and an odd (odd mode) electric field distribution. The characteristic impedances and the effective dielectric constants of even and odd modes are defined in [8].

Once the characteristic capacitances are determined, it is possible to obtain the characteristic impedances Z_C^e Z_C^o of the two modes and the coupling coefficient as:

$$K = \frac{Z_c^e / Z_c^o - 1}{Z_c^e / Z_c^o + 1},$$ (1)

It is well evident from formula (1) that high coupling values (i.e. values close to 1) can be obtained only if the characteristic impedances of the two modes differ a lot. Anyway, in practice, the characteristic impedances of the two quasi-TEM modes exhibit rather close values and, thus, the coupling coefficient cannot go beyond – 6/7 dB.
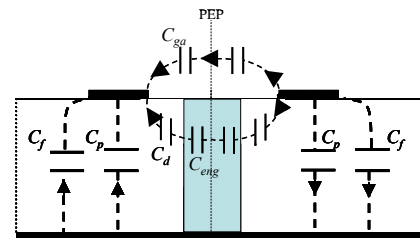

Fig. 3. Equivalent circuit model of the quasi-TEM odd mode with the ENG slab inside the substrate.

Comparing this new circuit model with the one reported in [8], the capacitance C_{gd} is replaced here by a new modified capacitance C_{gd}^{mod}, given by the series of the coupling capacitance of the substrate C_d and the equivalent capacitance of the ENG slab C_{eng}.

The following analytic expression of the modified capacitance is, therefore:

$$C_{gd}^{mod} = \frac{C_d C_{eng}}{C_d + C_{eng}}.$$ (2)

The key point here is that since in the frequency band of operation the permittivity of the slab exhibits negative values, the equivalent capacitance C_{eng} behaves as an equivalent inductance.

Adding the ENG slab allows to modify the value of C_{gd}^{mod} according to (2). Using an ENG material, therefore, the following condition may be imposed:

$$C_d + C_{eng} = 0.$$ (3)

If this particular condition is met, an infinite value of C_{gd}^{mod} occurs and, thus, the coupling reaches its maximum value 1. Condition (3) can be immediately seen as the resonance condition of an LC series circuit.

Working close to the resonance frequency, it is possible to tune the value of C_{gd}^{mod} and, thus, of the coupling coefficient, whatever the air coupling capacitance C_{ga} is.

3. Design Formulas

Today, various design equations for coupled microstrip lines are available in the literature [8]. In particular, many of these analysis use conformal mapping technique; many approximations are assumed and a high accuracy is left in function to have design formulae. In this present work we have modified the Wan's formulas [9] that give an accurate analysis of the coupled microstrip lines by a conformal mapping technique.

We consider the transversal section of the structure shown in Fig.1. We assume the following approximations: an equivalent air-dielectric boundary [8], infinitely thin stripes and ideals conductors. Two quasi-TEM modes are supported by the structure and two equivalent geometries have to be considered. Using several times the Schwarz-Christoffel transformations, it is possible to transform the geometries of the two modes in new geometries that are parallel plates capacitors filled with air, dielectric and ENG (Fig.4).

The permittivity and the characteristic impedances of the odd and even modes can be calculated using the formula of the parallel plates capacitor. Then the even and odd modes capacitances can be calculated by the series and the parallel among the equivalent capacitances of the air, dielectric and ENG rectangular regions. The mode characteristic impedances and the effective dielectric permittivities, thus, can be calculated through the following expressions:

$$Z_c^{e/o} = \frac{120\pi}{\sqrt{C_{eq}^{e/o}}} \left(\frac{L^{e/o}}{H^{e/o}} \right)^{\frac{3}{2}},$$

$$\varepsilon_{eq}^{e/o} = C_{eq}^{e/o} \frac{H^{e/o}}{L^{e/o}}$$

(4)

where $C_{eq}^{e/o}$, $L^{e/o}$ and $H^{e/o}$ are the capacitances, widths and heights of the equivalent capacitors of the even and odd modes.
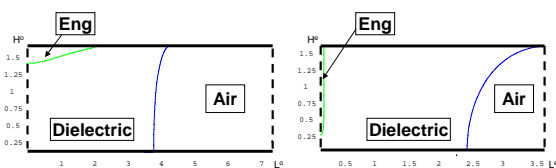

Fig. 4. Equivalent circuit model of the quasi-TEM odd mode with the ENG slab inside the substrate.

In the case of even mode, the presence of ENG leaf doesn't involve remarkable changes respect to standard case [8]: the ENG region is in parallel to the substrate and air, and so, because the ENG region is thin and his permittivity is very small, the equivalent capacitance of the structure can be calculated, essentially, through the Wan's formula [9].

On the other hand in the odd case the influence of ENG leaf is relevant: there are two contributions, a parallel and, especially, a series contribution. We know by the previous section that the ENG leaf can cause a resonant phenomenon due to the series of the ENG capacitance with the dielectric substrate capacitance. We can transform, thus, the ENG region in an equivalent area in series with the dielectric substrate.

It can be shown that the odd mode total capacitance is given by the series of the ENG leaf C_{eng} and dielectric substrate C_{sub} capacitances in parallel with the air one C_{air}

$$C_{eq}^o = C_{air}^o + \frac{C_{sub}^o C_{eng}^o}{C_{sub}^o + C_{eng}^o}.$$

(5)

A design formula has been derived where the permittivity of the ENG slab can be expressed as a function of the coupling coefficient K, the geometrical dimensions $h_o(h,w,s)$, $l_o(h,w,s)$, $l_{o,eng}(\varepsilon_{sub},h,w,s)$, the substrate permittivity ε_{sub} and the thickness of the ENG slab d, as:

$$\varepsilon_{eng} = -\varepsilon_{sub} \frac{d}{h\,h_o}\left(1 + \frac{N}{D_1 - D_2}\right)$$

(6)

where:

- $N = \varepsilon_{sub} l_o l_{o,eng} (K-1)^2 (Z_c^e)^2$,

- $D_1 = (120\pi)^2 (K+1)^2 h_o^2$,

- $D_2 = l_o[l_o + l_{o,eng}(\varepsilon_{sub}-1)](K-1)^2 (Z_c^e)^2$,

- $Z_c^e(\varepsilon_{sub},h,w,s)$ is the even characteristic impedance calculated with the Wan's formulas,

- $l_{o,eng}(h_{o,eng})$ is the equivalent width (height) of the ENG substrate, determined through a proper extension of Wheeler's approach [9], [10],

- $l_o(h)$ is the equivalent width (height) of the isotropic substrate, determined through a proper extension of Wheeler's approach [9, 10].

4. Numerical Results

The coupled microstrip layout presented in the previous sections allows to obtain high coupling values, up to unity value, which is the theoretical limit corresponding to the satisfaction of the resonance condition.

Once the coupling value K and the thickness of the ENG slab are chosen, the needed values of the permittivity may be straightforwardly determined both analytically through (6) or and graphically through Fig.5.

For example, let us consider the directional coupler depicted in Fig.6. If we want to obtain a power divider (i.e. a $-3dB$ coupling value) by using a thickness of 1.1. mm for the ENG slab, the relative permittivity would be equal to -0.7, as it can be straightforwardly obtained by applying formula (6). The theoretical results, coming from the analytical model proposed in the previous section, have been verified using the commercial code CST Microwave Studio® (Fig.7).
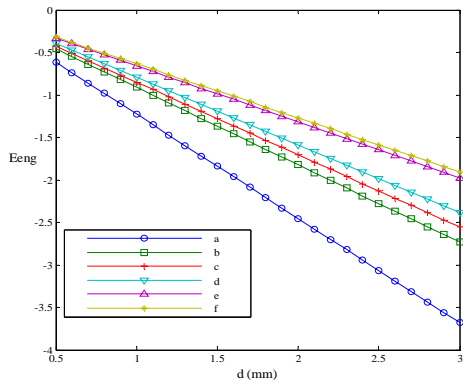
Fig. 5. Permittivity ε_{eng} versus the ENG thickness d (mm) calculated by the design formula for some geometries: **a)** $s=5$, $w=4$, $h=10$, $\varepsilon_{sub}=17.2$, **b)** $s=3$, $w=7$, $h=8$, $\varepsilon_{sub}=10.2$, **c)** $s=5$, $w=7.5$, $h=8$, $\varepsilon_{sub}=9.57$, **d)** $s=3$, $w=8$, $h=9$, $\varepsilon_{sub}=9.95$, **e)** $s=3.4$, $w=9.8$, $h=10$, $\varepsilon_{sub}=9.2$, **f)** $s=3$, $w=9.8$, $h=10$, $\varepsilon_{sub}=8.8$.
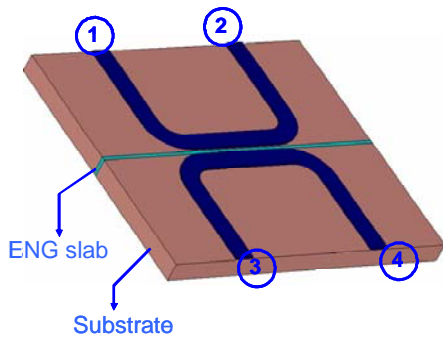

Fig. 6. Sketch of the proposed directional coupler. The geometrical and electrical parameters are: $d = 1.1$ mm, $s = 3.2$ mm, $w = 9.8$ mm, $h = 10$ mm, $\varepsilon_{sub} = 8.9$.
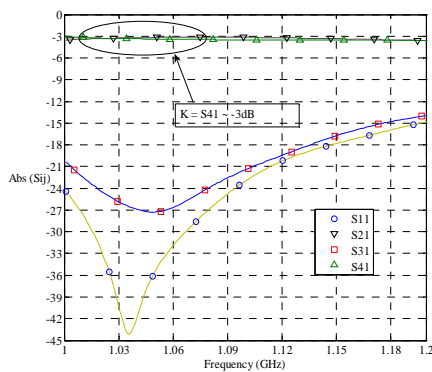

Fig. 7. Amplitude of the scattering parameters for the coupler of Fig. 6.

The ENG slab has been considered as made by an isotropic material described through the Drude dispersion law, with the plasma frequency at 1.4 GHz. This material exhibits the needed value of the permittivity equal to -0.70 at the frequency 1.035 GHz. The full-wave numerical simulations confirm the expected behavior ($|s_{41}|$ represents the coupling value).

Due to the material dispersion, the coupling is rather flat over a certain frequency range (1-1.11 GHz), while

both the matching at the input port and the coupler isolation are rather good.

The results of Fig. 7 can be compared with those ones of Fig. 8, where the amplitude of the scattering parameters are reported when the ENG slab is removed. From this comparison, it is clear that the introduction of the ENG slab allows to obtain a coupling value not obtainable in standard conditions.
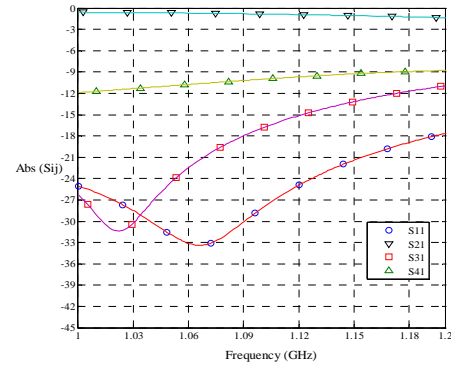

Fig. 8. Scattering parameters of the coupler of Fig. 6 without the ENG slab.

The design formulas have been tested through several full wave simulations and it has been obtained a relative error always less than -10% (Tab. 1).

w(mm)/s(mm)/h(mm) /ε_{sub}	d(mm)	ε_{neg} (design formula)	ε_{neg} (CST)	Relative error(%)
9.8/3.4/10/9.2	0.8	-0.53	-0.51	3.779
	1.3	-0.855	-0.797	7.277
	2.5	-1.645	-1.663	1.082
	3.0	-1.974	-1.915	3.08
9.8/3.0/10/8.8	0.8	-0.507	-0.497	2.012
	1.3	-0.825	-0.772	6.865
	2.5	-1.586	-1.501	5.662
	2.8	-1.766	-1.61	9.689

Tab. 1. Comparison the results obtained by the design formula and CST Microwave Studio.

5. Conclusions

In this paper, the analysis and the design of coupled microstrip lines loaded with an ENG slab have been presented. The presence of the metamaterial allows to increase the coupling values well beyond the values obtainable through traditional materials. New analytical formulas to analyze the behavior of the proposed device have been derived through the use of the conformal mapping tech-

nique. Some examples of directional couplers have been presented, showing a very good agreement between the CST full-wave simulations and the analytical model we propose.

References

[1] POZAR, D. M. *Microwave Engineering*. 2nd Edition. New York: J. Wiley and Sons, 1997.

[2] ZIOLKOWSKI, R. W., ENGHETA, N. (guest editors), *IEEE Transactions on Antennas and Propagation*. Special Issue on Metamaterials, 2003, vol. AP-51, no. 10.

[3] LANDAU, L., LIFSCHITZ, E. M. *Electrodynamics of Continuous Media*. Elsevier Ed., 1984.

[4] KILLIPS, D., BARBA, P., DESTER, G., BOGLE, A., KEMPEL, L. Simulation of Double Negative (DNG) composite materials using non-periodic constitutive materials. In *Proceedings of the Third Workshop on Metamaterials and Special Materials for Electromagnetic Applications and TLC*, Rome (Italy), 2006.

[5] BILOTTI, F., ALÙ, A., ENGHETA, N., VEGNI, L. Metamaterial complementary pairs for antenna size reduction. In *Proceedings of the Loughborough Antennas & Propagation Conference (LAPC 2006)*. Loughborough (UK), 2006.

[6] BILOTTI, F., ALÙ, A., ENGHETA, N., VEGNI, L. Metamaterial covers over a small aperture. I*EEE Transactions on Antennas and Propagation*, 2006, vol. AP-54, no. 6, p.1632–1643.

[7] LAURO, S. E., BILOTTI, F., TOSCANO, A., VEGNI, L. Metamaterials as complex dielectrics in the design of high-speed integrated circuits. In *Proceedings of the Third Workshop on Metamaterials and Special Materials for Electromagnetic Applications and TLC*, Rome (Italy), 2006.

[8] GARG, R., BAHL, I. J. Characteristics of coupled microstrip lines. *IEEE Transactions of Microwave Theory and Techniques*, 1979, vol. MTT-27, no. 7.

[9] WAN, C. Analytically and accurately and determined quasi-static parameters of coupled microstrip lines. *IEEE Transactions on Microwave Theory and Techniques*, 1996, vol. 44, no 1.

[10] WHEELER, H. A. Transmission-line properties of parallel strips separated by a dielectric sheet. *IEEE Transactions on Microwave Theory and Techniques*, 1965, vol. 13, p. 172–185.

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