

# Wideband Phase Retrieval Technique from Amplitude-Only Near-Field Data

Sandra COSTANZO, Giuseppe DI MASSA

University of Calabria, 87036 Rende (CS), Italy

costanzo@deis.unical.it, dimassa@deis.unical.it

**Abstract.** *A wideband frequency behavior is demonstrated for a phaseless near-field technique of basically interferometric approach, which uses two identical probes interfering each other through a microstrip circuit and performing amplitude-only near-field measurements on a single scanning surface. The phase retrieval procedure is properly formulated to take into account the frequency dependence without changing neither the microstrip circuit nor the distance between the probes. Numerical simulations on a linear array of elementary sources are presented to validate the theoretical results.*

## Keywords

Near-field methods, phase-retrieval, antenna measurements.

## 1. Introduction

Antenna measurement techniques in the near-field region are established today as an excellent alternative to direct far-field testing, as leading to use compact anechoic chambers of reduced size and cost [1]. In the standard case, the near-field method requires the knowledge of a complex field distribution over a prescribed scanning surface, typically of planar, cylindrical or spherical type, with the far-field obtained as result of a mathematical transformation essentially based on a modal expansion inherent to the particular geometry [2]. Useful transformations directly obtaining the far-field pattern from near-field measurements acquired over new strategic geometries have been recently proposed by the authors [3-5] to significantly reduce the number of required samples and the overall computational effort. A serious drawback affecting complex near-field methods is that an accurate phase evaluation from direct measurements is very difficult to obtain when working at high frequency ranges, so alternative techniques have been suggested in literature to obtain the antenna far-field from amplitude-only near-field measurements. Much of them reduces the far-field characterization to a non-linear estimation problem, based on the minimization of a suitable functional which is defined in terms of at least

two measured near-field amplitudes over two distinct scanning surfaces [6-8]. A serious problem inherent to these functional methods is related to the high occurrence of local minima, limiting their effective implementation. This complexity aspect of functional-based phase-retrieval techniques has been investigated for the planar case in [9], where a modified iterative Fourier technique based on the optimum choice of the initial phase guess is formulated to enhance the efficiency of the phase-reconstruction algorithm. A novel hybrid procedure of basically interferometric type has been proposed by the authors in a recent paper [10]. It suggests the use of two identical probes simultaneously moving on a single near-field surface, whose measurement points interfere each other by means of a simple microstrip circuit. In order to satisfy the Nyquist requirement, the distance  $d_p$  between the probes is fixed in terms of a multiple of  $\lambda/2$  at each measurement frequency  $f$ . Amplitude data collected by the probes are first processed by a simple interferometric formula [10] to obtain a finite number of sets of complex near-field data, with a certain number of unknown phase shifts strictly depending on the fixed distance  $d_p$  between the measuring probes. A minimization procedure based on the adoption of a non redundant field representation [11] is then applied to determine the above unknowns. The original formulation of the phase-retrieval method [10] assumes a distance  $d_p$  between the two probes to be changed accordingly to the measurement frequency  $f$ , which in turns fixes the proper sampling step between adjacent measuring points. So, experimental validations discussed in [10] at different operating frequencies have been performed with a modified test setup at each measurement frequency, uniquely demonstrating the correct wideband behavior of the microstrip circuit which provides amplitude information. In this paper, a generalized phase retrieval procedure valid on a wide frequency range but leaving unchanged the test setup when varying the measurement frequency is properly formulated. A generalized frequency-dependent expression is obtained for the interferometric formula giving the phase shifts between the signals measured by the probes. Furthermore, the minimization procedure [10] used to find the remaining unknowns is itself extended to solve the correct number of phase shifts arising from a distance  $d_p$  between the probes mechanically fixed but changed in terms of wavelength at the operating frequency. Numerical

simulations on a linear array of Huyghens sources are reported to validate the theoretical results.

## 2. Wideband Phase Retrieval Technique

The hybrid procedure proposed in [10] employs two identical probes directly connected to a microstrip circuit (Fig. 1), which provides the necessary amplitude information, namely:

$$|V_1|^2, |V_2|^2, |V_1 + V_2|^2, |V_1 + V_2 \cdot e^{j\beta d}|^2. \quad (1)$$

The terms  $V_1 = |V_1| \cdot \exp(j\varphi_1)$  and  $V_2 = |V_2| \cdot \exp(j\varphi_2)$  in (1) represent the complex output voltages at the probes (Fig. 1), while  $\beta = 2\pi/\lambda$  is the free-space propagation constant at the operating frequency  $f$ , with  $\beta d = \pi/2$  at the central design frequency  $f_0$  to have the proper sum in quadrature [10].

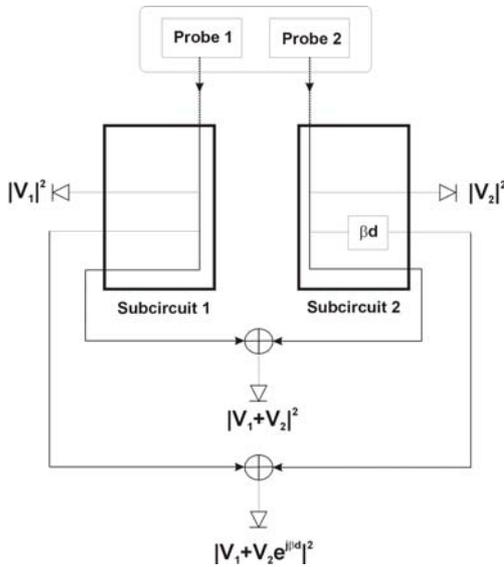


Fig. 1. Block diagram of hybrid probe for near-field phase retrieval.

The exponential factor  $\exp(j\beta d)$  can be expressed in terms of the ratio between the operating (measurement) frequency and the central design frequency  $f_0$  as follows:

$$e^{j\beta d} = e^{j \frac{2\pi}{\lambda_0} \frac{f}{f_0} d} = e^{j \frac{f}{f_0} \beta_0 d} = e^{j \frac{f}{f_0} \frac{\pi}{2}} \quad (2)$$

It is easy to show that amplitude data in (1) can be processed to obtain the phase shift  $\Delta\varphi = \varphi_1 - \varphi_2$  by means of the following generalized interferometric formula:

$$\Delta\varphi = \text{tg}^{-1} \left[ \frac{a - b \cos\left(\frac{f}{f_0} \cdot \frac{\pi}{2}\right)}{b \sin\left(\frac{f}{f_0} \cdot \frac{\pi}{2}\right)} \right] \quad (3)$$

where

$$a = |V_1 + V_2 \cdot e^{j\beta d}|^2 - |V_1|^2 - |V_2|^2, \quad (4)$$

$$b = |V_1 + V_2|^2 - |V_1|^2 - |V_2|^2. \quad (5)$$

The examination of expression (3) leads to exclude from the validity range those frequencies satisfying the equation:

$$\frac{f}{f_0} \cdot \frac{\pi}{2} = n\pi, \quad n = 0, 1, 2, \dots \quad (6)$$

The values  $n = 0$  and  $n = 1$  in (6) fix the frequency interval for which a solution of (3) can be defined, namely:

$$0 < f < 2f_0. \quad (7)$$

If we assume a sampling step equal to  $\lambda_0/2$ , properly imposing the Nyquist requirement at the design frequency  $f_0$ , the Shannon theorem will be satisfied only by those operating (measurement) frequencies greater than the design frequency  $f_0$ . So, the frequency validity range (7) must be restricted as follows:

$$f_0 \leq f < 2f_0. \quad (8)$$

The above interval is clearly satisfactory, as much greater than the operating bandwidth of waveguides, generally used as probes [10].

Let us apply the interferometric formula (3) to a measurement curve of  $2N+1$  points ( $N$  even for simplicity), with a distance  $d_p = i\lambda_0/2$  between the probes ( $i \geq 1 \in N$ ). When performing measurements at a frequency  $f = f_0$ , a number  $i$  of sets of complex near-field data is obtained, with  $(i-1)$  phase shifts determined from the intersection between the set of all fields compatible with measured data and the set of all fields that the Antenna Under Test (AUT) can radiate [10]. When changing the measurement frequency to a value  $f \neq f_0$ , the distance  $d_p$  between the probes can be expressed as (Fig. 2):

$$d_p = i_f \frac{\lambda}{2} + \Delta\lambda \quad (9)$$

where  $i_f \neq i$  is an integer depending on the frequency  $f$ .

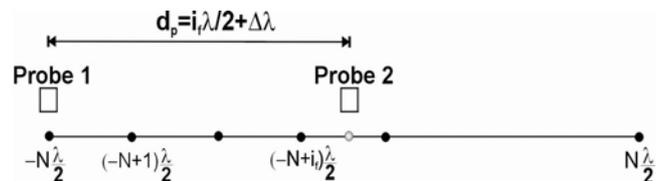


Fig. 2. Probe positions along the acquisition line at a frequency  $f \neq f_0$ .

As a consequence of this, a number  $i_f$  of sets of complex near-field data is obtained, namely:

$$\begin{aligned}
E(s_{(1)}) &= \left[ \varepsilon(s_{(1)}^{(1)}), \varepsilon(s_{(1)}^{(2)}) \cdot e^{j\Delta\phi_{(11)}^{(21)}}, \varepsilon(s_{(1)}^{(3)}) \cdot e^{j\Delta\phi_{(11)}^{(31)}}, \dots \right] \\
E(s_{(2)}) &= \left[ \varepsilon(s_{(2)}^{(1)}), \varepsilon(s_{(2)}^{(2)}) \cdot e^{j\Delta\phi_{(22)}^{(21)}}, \varepsilon(s_{(2)}^{(3)}) \cdot e^{j\Delta\phi_{(22)}^{(31)}}, \dots \right] \cdot e^{j\Delta\phi_{(21)}^{(11)}} \\
&\dots \\
E(s_{(i_f)}) &= \left[ \varepsilon(s_{(i_f)}^{(1)}), \varepsilon(s_{(i_f)}^{(2)}) \cdot e^{j\Delta\phi_{(i_f i_f)}^{(21)}}, \varepsilon(s_{(i_f)}^{(3)}) \cdot e^{j\Delta\phi_{(i_f i_f)}^{(31)}}, \dots \right] \cdot e^{j\Delta\phi_{(i_f 1)}^{(11)}}
\end{aligned} \tag{10}$$

wherein:

$$\begin{aligned}
s_{(1)} &= \left[ -N \frac{\lambda}{2}, (-N+i_f) \frac{\lambda}{2}, (-N+2i_f) \frac{\lambda}{2}, \dots \right] \\
s_{(2)} &= \left[ -(N+1) \frac{\lambda}{2}, (-N+i_f+1) \frac{\lambda}{2}, (-N+2i_f+1) \frac{\lambda}{2}, \dots \right] \\
&\dots \\
s_{(i_f)} &= \left[ -(N+i_f-1) \frac{\lambda}{2}, (-N+2i_f-1) \frac{\lambda}{2}, (-N+3i_f-1) \frac{\lambda}{2}, \dots \right]
\end{aligned} \tag{11}$$

The terms  $\varepsilon(s_{(i)}^{(j)})$  into expressions (10) are known complex quantities [10] obtained after application of the interferometric formula (3), and the notation  $\Delta\phi_{pq}^{mn}$  indicates the remaining unknown phase shifts to be determined, with the term:

$$\Delta\phi_{pq}^{mn} = \varphi_{(p)}^{(m)} - \varphi_{(q)}^{(n)} \tag{12}$$

defined as the difference between the phase values at positions given by the  $m$ -th element of set  $s_{(p)}$  and the  $n$ -th element of set  $s_{(q)}$ .

In the case of an acquisition frequency  $f$  coincident with the central design frequency  $f_0$  of the integrated probe, the number of phase shifts to be determined is strictly related to the distance  $d_p$  between the measuring probes. As a matter of fact, the number of unknowns in the phase retrieval problem is equal to  $i$ , if a distance  $d_p = i\lambda_0/2$  is fixed [10]. When changing the operating frequency, the distance  $d_p$  between the probes is not exactly a multiple of the quantity  $\lambda/2$ , as indicated by equation (9) and it can be easily shown that the number of unknown phase shifts becomes equal to the quantity:

$$(i_f - 1) + (2N + 1 - i_f) = 2N \tag{13}$$

which is independent on the operating frequency  $f$  but exclusively related to the number of measurement points along the observation curve. As a consequence of this, the phase retrieval technique presented in [10] remains valid for all operating frequencies within the range given by (8). Once adopted the generalized notation (10) to express the complex field on the  $2N+1$  sampling points, the greatest lower bound between the set of all reduced fields compatible with the collected data and the set of all reduced fields which the Antenna Under Test (AUT) can radiate is determined as described in [10], with a  $2N$  variable minimization procedure for all operating frequencies  $f$  different from the design frequency  $f_0$ . The phase retrieval technique is obviously repeated on a proper number of scanning curves covering the near-field acquisition surface.

### 3. Numerical Validations

The wideband phase retrieval procedure is numerically validated on a linear array of 21  $y$ -oriented Huyghens sources  $\lambda_0/2$  spaced each other along the  $y$ -axis at the central design frequency  $f_0$ . Amplitude-only near-field is simulated at different operating frequencies on a planar surface  $10\lambda$  away from the AUT. A square grid is assumed as composed of  $121 \times 121$  points along  $x$  and  $y$  axes, with sampling spacings  $\Delta x = \Delta y = \lambda_0/2$  correctly satisfying Shannon's criterion at the design frequency  $f_0$ . A contour plot of the normalized field amplitude for the dominant  $E_y$  component is reported under Figs. 3-5 at different operating frequencies, namely  $f = f_0$  (Fig. 3),  $f = f_0/2$  (Fig. 4) and  $f = 3f_0/2$  (Fig. 5).

The wideband phase retrieval procedure is applied to recover the near-field phase from the simulated amplitude-only near-field data of equation (1). To illustrate the validity of theoretical results discussed in the previous section, a comparison between the exact and the retrieved near-field phase on the cut  $x = 0$  for the dominant  $E_y$  component is reported under Figs. 6-9 at different operating frequencies. As clearly evident, an excellent reconstruction is obtained for the frequency values  $f = f_0$  (Fig. 6),  $f = 3f_0/2$  (Fig. 7) and  $f = 15f_0/8$  (Fig. 8), while a bad result is obtained at  $f = 2f_0$  (Fig. 9), which is the upper limit demonstrated by equation (8) to be out of the frequency validity range.

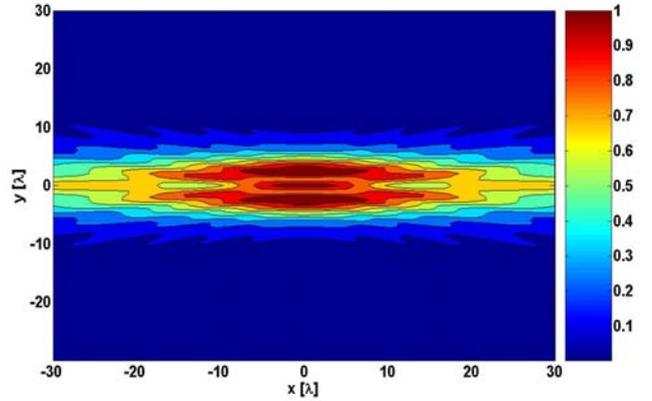


Fig. 3. Normalized near-field amplitude at a frequency  $f = f_0$ .

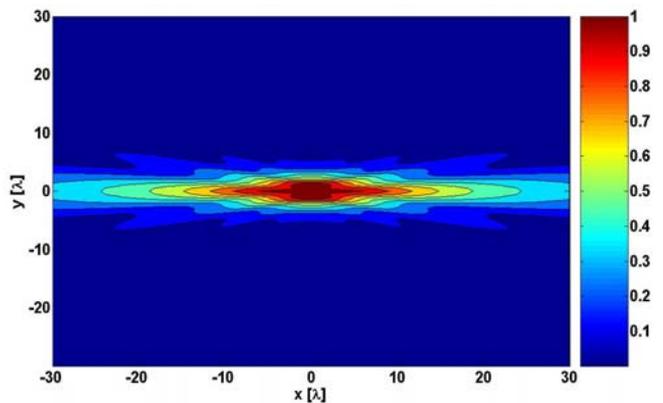


Fig. 4. Normalized near-field amplitude at a frequency  $f = f_0/2$ .

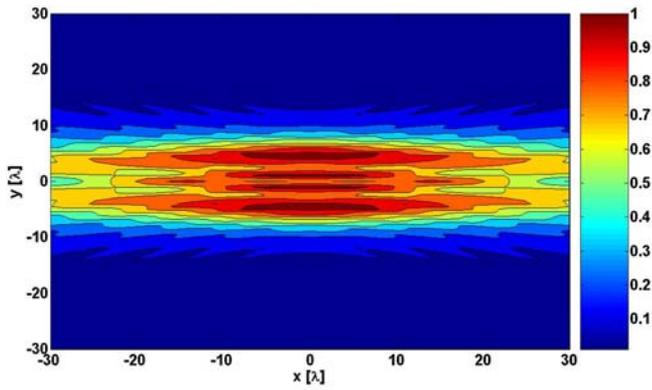


Fig. 5. Normalized near-field amplitude at a frequency  $f = 3f_0/2$ .

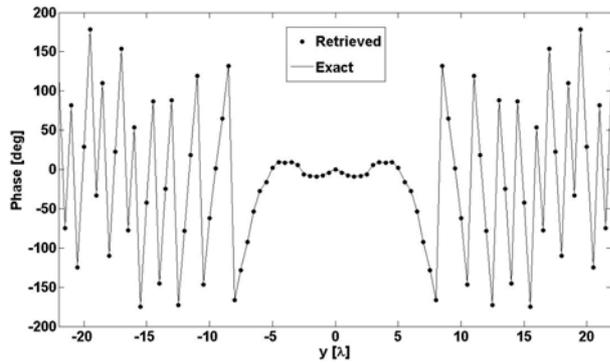


Fig. 6. Comparison between exact and retrieved near-field phase on the cut  $x = 0$  at  $f = f_0$ .

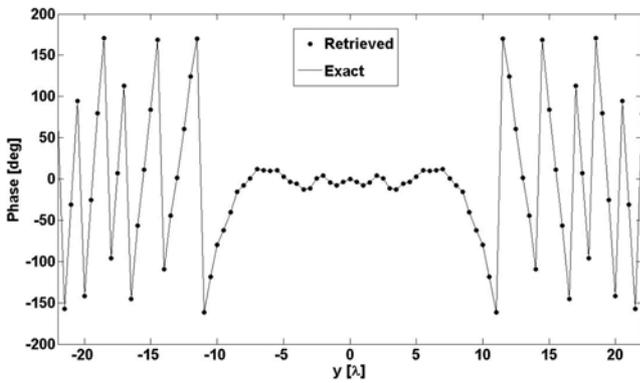


Fig. 7. Comparison between exact and retrieved near-field phase on the cut  $x = 0$  at  $f = 3f_0/2$ .

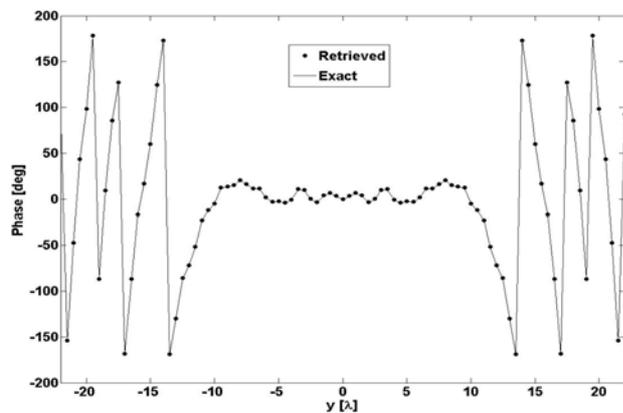


Fig. 8. Comparison between exact and retrieved near-field phase on the cut  $x = 0$  at  $f = 15f_0/8$ .

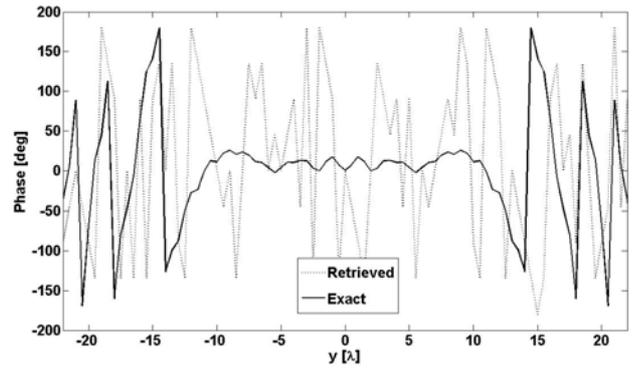


Fig. 9. Comparison between exact and retrieved near-field phase on the cut  $x = 0$  at  $f = 2f_0$ .

To further test the wideband effectiveness of the phase retrieval technique, the far-field pattern radiated by the linear array of 21 dipoles is computed through the standard plane-rectangular near-field to far-field transformation [1], [2]. Complex near-field using both the exact and the retrieved near-field phase is considered for the transformation. In both cases, the same far-field result is obtained, as illustrated under Figs. 10-11, where the successful comparison is shown at two different operating frequencies, namely  $f = f_0/2$  (Fig. 10) and  $f = 3f_0/2$  (Fig. 11).

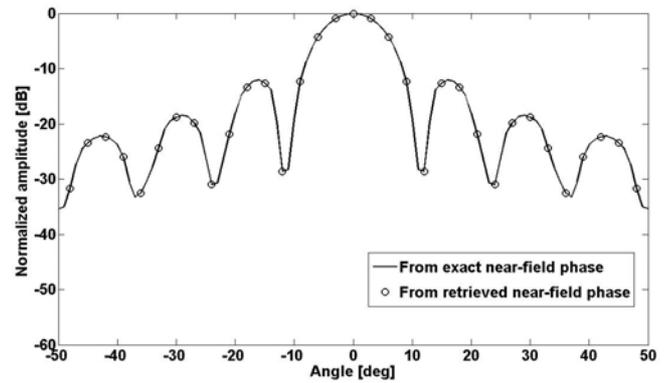


Fig. 10. Far-field pattern ( $E_\theta$  at  $\phi = 90^\circ$ ) of 21 element array: comparison between results from exact and retrieved near-field phase at  $f = f_0/2$ .

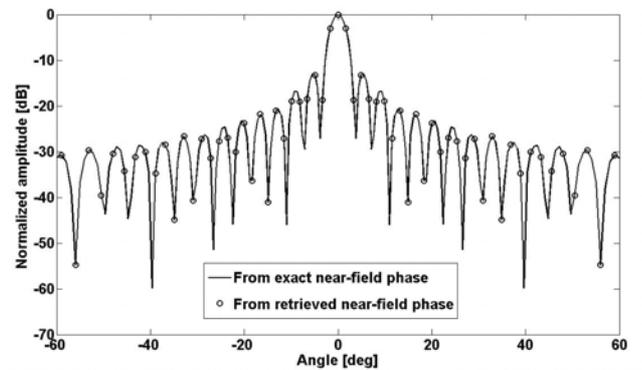


Fig. 11. Far-field pattern ( $E_\theta$  at  $\phi = 90^\circ$ ) of 21 element array: comparison between results from exact and retrieved near-field phase at  $f = 3f_0/2$ .

## 4. Conclusions

A wideband formulation is presented for a phase retrieval technique combining a basically interferometric approach with a minimization procedure used to determine the unknown phase shifts arising from the amplitude-only acquisition scheme. The broadband behavior is theoretically demonstrated through the generalized formulation, with a fixed test setup not requiring to change the distance between the probes at the various measurement frequencies. This feature leads to a fast multifrequency acquisition which guarantees the accurate phase retrieval on a wide frequency range. Successful numerical validations are presented on a linear array of elementary Huyghens sources.

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## About Authors

**Sandra COSTANZO** received the Laurea degree (summa cum laude) in Computer Engineering from the University of Calabria and the Ph.D. degree in Electronic Engineering from the University of Reggio Calabria in 1996 and 2000, respectively. Since 1996 she has been with the Research Group in Applied Electromagnetics at the University of Calabria. Currently, she is an Assistant Professor with the Faculty of Engineering, University of Calabria, Cosenza, Italy. Her research interests are focused on near-field far-field techniques, antenna measurement techniques, numerical methods in electromagnetic scattering, millimeter-wave antennas, antenna analysis and synthesis. Dr. Costanzo received the "Telecom" prize for the Best Laurea Thesis in 1996. In 2001 she was the Conference Chairman of Antenna Measurement Techniques Session at the IEEE AP-S International Symposium.

**Giuseppe DI MASSA** was born in Barano d'Ischia (Na), Italy, in 1948. He received the Laurea degree as Doctor in Electronic Engineering from the University of Naples in 1973. From 1978 to 1979 he was professor of Antennas at the University of Naples. In 1980 he joined the University of Calabria as professor of Electromagnetic Waves. Since 1985 he served as an Associate Professor and since 1994 as a Full Professor at the same University. From 1985 to 1986 he was a Scientific Associate at CERN, Geneva. In 1988 he was Visiting Professor at Brookhaven National Laboratory in Long Island (NY) USA. From 1997 to 2002 he was the Dean of the Department of Elettronica, Informatica and Sistemistica and the President of Programming Committee at the University of Calabria. From 2002 to 2007 he was WP leader in the Network of Excellence ACE 'Antenna Centre of Excellence' of European Commission. At present he is the chairman of the Course in Telecommunication Engineering at the University of Calabria, in the executive board of Consorzio Nazionale Interuniversitario delle Telecomunicazioni (CNIT), the Italian Delegate in the European COST IC0603 "Antenna Systems & Sensors for Information Society Technologies". His main research interests are focused on applied computational electromagnetic, microstrip antennas, microwave integrated circuits, Gaussian beam solutions, millimeter wave antennas, near field measurements.