The Quality Factor of the Folded Cylindrical Helix

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Abstract. *Any electrically small antenna can be impedance matched at any single frequency using a number of well known techniques. Once the small antenna is impedance matched, the primary characteristics of interest are its radiation efficiency, its operating bandwidth and to a lesser extent, its radiation patterns. The bandwidth of the small antenna is often quantified using the antenna's quality factor (Q) since fundamental lower bounds for Q are defined in terms of the antenna's occupied volume. The lower bound on Q, also known as the Chu-limit, is defined in terms of the spherical volume occupied by the antenna. However, many small antenna designs are constrained to fit within volumes other than a sphere. To address this issue, Gustafsson et al derived lower bounds for antennas of arbitrary shape with a specific focus on cylindrical and planar shaped antennas. In this paper we consider the quality factor of the folded cylindrical helix, an antenna design that effectively utilizes the available cylindrical volume. We compare its Q to the Gustaffson limit as a function of length-to-diameter ratio, while maintaining a fixed value of ka, and show that it's Q is at or above Gustafsson's lower bound for cylindrical shaped antennas.*

Keywords

Electrically small antennas, quality factor, dipole.

1. Introduction

This paper is an extended version of a conference paper presented at Eucap 2009. The material that follows has been extracted and edited from [1].

There is a well known lower bound for the quality factor (Q) of an electrically small antenna. This lower bound on Q [2]-[3], Q_{lb}, is often referred to as the Chu-limit. It establishes the theoretical minimum value of Q that can be achieved as a function of the antenna's occupied spherical volume, which is defined by the value of ka, where k is the free space wavenumber $2\pi/\lambda$, and a is the radius of an imaginary sphere circumscribing the maximum dimension of the antenna.

For the purposes of our work in designing electrically small antennas, we consider a small antenna as being one where the value of ka is less than or equal to 0.5. Ultimately, the engineer is concerned with characterizing the operating bandwidth of the small antenna. We often use Q for this purpose since Q and Voltage Standing Wave Ratio (VSWR) bandwidth are inversely related and the existence of a lower bound on Q allows us to quantify how the antenna's bandwidth performs relative to theoretical limits.

The lower bound on Q for the general lossy antenna is given by

$$Q_{lb} = \eta_r \left(\frac{1}{(ka)^3} + \frac{1}{ka} \right) \tag{1}$$

where η_r is the antenna's radiation efficiency. This statement of a lower bound was derived for the electrically small antenna that radiates a single, fundamental TM or TE mode and exhibits a single impedance resonance within its defined VSWR bandwidth.

To achieve a Q that most closely approaches this lower bound, the small antenna must effectively utilize the full extent of the spherical volume defined by the value of ka. The lowest possible Q is achieved when the antenna conductor(s) are placed on the outermost regions of the spherical volume [4] – [6].

In most practical applications, the constraint on the occupied volume of the small antenna is not defined by a spherical shape. Typically, the small antenna must fit within a volume of arbitrary shape or in many cases, a cylindrical or planar shape. In these instances, it is understood that the antenna Q will not approach the lower bound as closely as does the Q of the spherical shaped antenna, where both have the same value of ka. Without an appropriate adjustment in the lower bound of Eq. (1) for differences in antenna shape, the engineer has no measure of how well the arbitrary shaped antenna performances relative to theoretical or practical limits.

Recently, Gustafsson et al derived a lower bound on Q for arbitrary shaped antennas [7], thus providing the antenna engineer with the capability of determining how well the general small antenna performs relative to theoretical limits for the antenna's specific shape. Gustafsson specifically defined the lower bound for the general cylindrical shape [7] - [8], which is a common shape used in small antenna design.

In this paper, we present the design of a folded cylindrical helix with the objective of achieving an impedance match, high radiation efficiency and minimum Q. The Q of the folded cylindrical helix is compared to the Gustafsson limit as a function of its length-to-diameter ratio, while maintaining a fixed value of *ka*. We show that the Q of the folded cylindrical helix varies as a function of length-to-diameter ratio and it closely approaches the Gustafsson limit but does not exceed it.

2. Calculation of Antenna Q

The exact Q of an electrically small, tuned or self-resonant antenna is given by [2], [9]

$$Q(\omega_0) = \frac{\omega_0 |W|}{P} \tag{2}$$

where W is internal energy and P is the total power accepted by the antenna, which includes power dissipated in the form of radiation and heat within the antenna structure. ω_0 is the radian frequency ($2\pi f_0$) where the antenna is naturally self-resonant, tuned, or made to be self-resonant. If the tuned small antenna exhibits a single impedance resonance within its defined VSWR bandwidth, its Q can be accurately approximated at any frequency, ω, from its impedance properties using [9]

$$Q(\omega) \approx \frac{\omega}{2R(\omega)} \sqrt{R'(\omega)^2 + \left(X'(\omega) + \frac{|X(\omega)|}{\omega} \right)^2} \tag{3}$$

where $R'(\omega)$ and $X'(\omega)$ are the frequency derivatives of the antenna's feed point resistance and reactance, respectively. Eq. (3) is used here to calculate the Q of the folded cylindrical helix.

3. The Folded Spherical Helix

Recently, there has been an increased interest in developing electrically small antennas for a number of wireless applications. The primary objective is to design small antennas that are impedance matched, exhibit low Q and have high radiation efficiency.

One antenna that was designed with this objective in mind is the folded spherical helix [4] – [5] depicted in Fig. 1. The performance properties of the folded spherical helix are presented here as a reference baseline for the performance of the folded cylindrical helix considered in the next section.

The folded spherical helix exhibits a Q that very closely approaches the lower bound for spherical shaped antennas. To closely approach the lower bound, the folded spherical helix utilizes the entire spherical volume where all of the conductors are wound on the outside of the imaginary spherical shape. Multiple folded arms are used within the structure to both impedance match the antenna and reduce its Q. In the design discussed here, four folded arms are used to match the antenna to a 50Ω characteristic impedance. There is a single feed point in the antenna at the center of one of the short vertical sections of conductor.

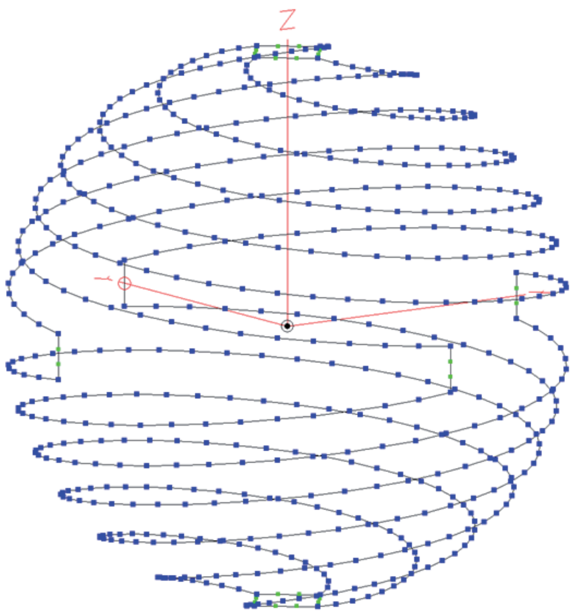

Fig. 1. The 4-arm folded spherical helix antenna.

At a value of $ka = 0.265$, the folded spherical helix is self-resonant, with a total resistance (including both radiation and loss terms) of 47.6 Ω, a radiation efficiency of 97.1% and a Q of 84.64, approximately 1.52 times the lower bound of 55.61. For $ka < 0.5$, this value of Q is consistent with the practical, minimum achievable Q predicted by Thal for spherical wire antennas [6].

4. The Folded Cylindrical Helix

The folded spherical helix discussed in the previous section utilizes the full spherical volume defined by the value of *ka*. This allows the antenna to achieve a Q that very closely approaches the lower bound for spherical shaped antennas. The design techniques and approaches used in designing the folded spherical helix are applied to the design of the folded cylindrical helix with the same performance objectives. As with the folded spherical helix we have the objective of designing a cylindrical shaped antenna that is impedance matched, exhibits high radiation efficiency and a Q that very closely approaches the lower bound.

It is understood that with the same value of *ka*, the folded cylindrical helix cannot achieve as low a Q as the folded spherical helix. It is also understood that the Chu-limit for spherical shaped antennas is an unrealistic lower bound for cylindrical shaped antennas. The lower bound for cylindrical shaped antennas is higher than the lower bound for spherical shaped antennas as shown by Gustaffson [7] – [8]. In this paper, the Q of the folded cylindrical helix is directly compared to the Gustaffson limit.

To optimize the performance of the folded cylindrical helix, multiple conductors are wound on the outside surface of an imaginary cylinder. In all cases considered here, the dimensions of the antenna (its overall length, overall diameter and conductor length) are set so as to maintain self-resonance at or near the same value of *ka* (0.265) as that of the folded spherical helix presented in the previous section.

With the folded cylindrical helix design, self-resonance is achieved by adjusting the arm length in each of the folded arms. Adjustment of the arm length changes the total self-inductance of the structure, tuning out the inherent self-capacitance associated with the small dipole-like design. Once self-resonance is achieved, the resonant resistance is increased by increasing the number of folded arms within the structure. Since the folded cylindrical helix does not occupy the same overall volume as the folded spherical helix having the same value of *ka*, it will not exhibit as low a Q. The basic configuration of a 4-arm folded cylindrical helix is depicted in Fig. 2.

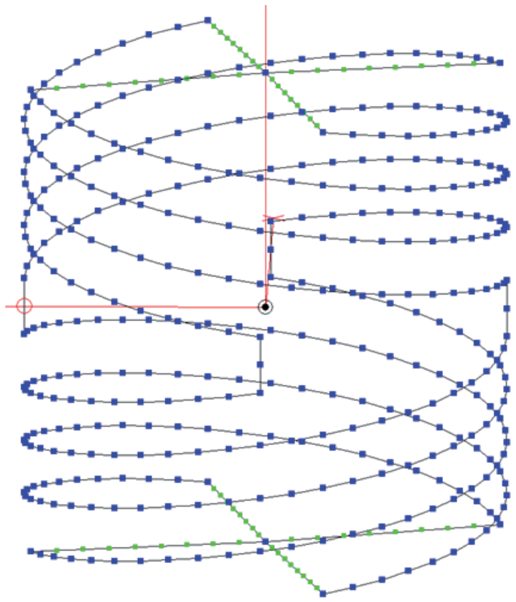

Fig. 2. The 4-arm folded cylindrical helix antenna.

In encompassing the folded cylindrical helix within the same spherical volume (*ka*) as the folded spherical helix, there are a number of length-to-diameter ratios that can be chosen. As expected and quantified by Gustafsson et al, the minimum achievable Q for the antenna will vary as a function of length-to-diameter ratio. Additionally, with this dipole-like antenna design, the resonant resistance is a function of $(l/\lambda)^2$, where l is the overall length of the cylinder. As a result, the resonant radiation resistance and antenna VSWR will also change as a function of length-to-diameter ratio for a fixed number of turns. In this section, a 4-arm folded cylindrical helix is studied and there is no attempt to optimize or implement an impedance match as a function of changing length-to-diameter ratio. The objective is to examine the change in Q as a function of the change in length-to-diameter ratio.

All of the 4-arm folded helices studied are designed to fit within a sphere having an overall diameter of 8.36 cm. They are all designed to be self-resonant near 300 MHz. The conductor diameter used in the NEC simulations is 2.6 mm. Copper loss is not included in the simulations here so that a direct comparison of Q can be made to the Gustaffson limit without having to adjust for differences in radiation efficiency that may occur with differences in length-to-diameter ratio. All of the antennas have high radiation efficiencies, typically in excess of 90 to 95%.

The first attempt at implementing the 4-arm folded cylindrical helix focused on achieving a resonant resistance near 50 Ω. This configuration has an overall length of 6.5 cm and a diameter of 5.26 cm. The length-to-diameter ratio for this configuration is 1.236. Other configurations of the folded cylindrical helix were implemented as a function of changing the length-to-diameter ratio while ensuring that the occupied cylindrical volume fit within the same spherical radius, which was approximately 4.2 cm. For each configuration, the conductor length in each of the four folded arms was adjusted to maintain a resonant frequency as close to 300 MHz as possible.

The quality factor at the resonant frequency was calculated for each configuration using (3). The ratio of the antenna Q to the lower bound (the Chu-limit) of (1) was calculated and compared against the Gustafsson limit for cylindrical shaped antennas [8]. This comparison is presented in Fig. 3. Note that in calculating the Gustaffson limit presented in Fig. 3, it was assumed that the antennas have a *ka* much less than 1, are purely vertically polarized and that the maximum achievable directivity is 1.5. In all cases, the Q of the antenna is above or at the Gustafsson limit.

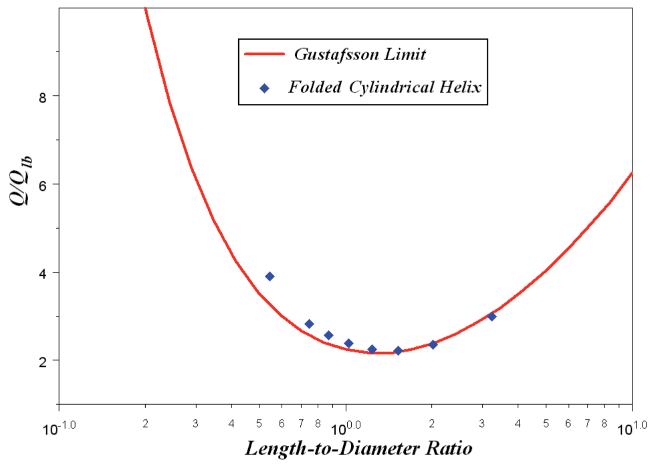

Fig. 3. Comparison of the folded cylindrical helix Q to the Gustafsson limit.

A summary of the antenna's performance properties is presented in Tab. I. Note that the resonant frequencies of the antennas range from 295.4 to 301.5 MHz. This variation in frequency and corresponding *ka* does not affect the Q comparison since the antenna Q is compared relative to the appropriate lower bound. For the antenna having an overall height of 8 cm, resonance could not be achieved. The frequency listed in this case is for the minimum *VSWR*, which was approximately 1.63.

5. Varying the Number of Arms

In the previous section, four folded arms were used in the design of the cylindrical folded helix. Four arms were chosen because the folded cylindrical helix with $ka = 0.26$ having 4-arms has well matched impedance relative to 50 Ω. With fewer arms, the antenna will have a lower resonant resistance and with more arms, the antenna will have a higher resonant resistance. Often, with more than 6-arms, the antenna will not be naturally resonant (near the same value of ka) because of the increased capacitance between each of the arms.

In this section, we consider the performance of the folded cylindrical helix having a length-to-diameter ratio of 1.525 with varying number of folded arms. We consider the antenna's performance with 1, 2, 4 and 6-arms. A comparison of the antenna performance is presented in Tab. 2. A comparison of the antenna's quality factor to the Gustaffson limit is presented in Fig. 4.

Length (cm)	Diameter (cm)	Length-to-Diameter Ratio	Resonant Frequency (MHz)	ka	Resonant Resistance (Ω)	Q	Q/Q_{lb}
4	7.35	0.544	301.1	0.264	23.3	226.9	3.91
5	6.71	0.745	301.5	0.264	37.9	163.2	2.82
5.5	6.31	0.872	295.4	0.259	41.4	157.1	2.56
6	5.84	1.027	298.2	0.262	46.7	142.0	2.38
6.5	5.26	1.236	300.4	0.264	49.9	131.6	2.25
7	4.59	1.525	296.6	0.260	62.5	134.7	2.22
7.5	3.72	2.016	299.9	0.263	78.8	138.0	2.35
8.0	2.47	3.239	299.2	0.262	62.8	177.0	3.0

Tab. 1. Summary of the performance properties of the 4-arm folded cylindrical helix.

Number of Arms	Resonant Frequency (MHz)	ka	Resonant Resistance (Ω)	Q	Q/Q_{lb}
1	297.5	0.261	2.44	183.98	3.06
2	301.6	0.264	12.74	142.16	2.45
4	296.6	0.260	62.5	134.7	2.22
6*	294.5	0.258	103.5	130.61	2.11
*The 6-arm folded cylindrical helix in not self resonant and has an impedance of 103.5 – j155.3 Ω.					

Tab. 2. Summary of the performance properties of the multiple arm folded cylindrical helix. The folded cylindrical helices with 1, 2, 4, and 6-arms are compared.

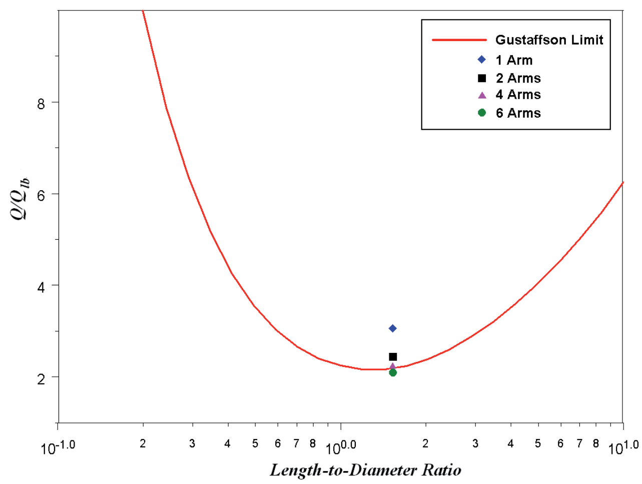

Fig. 4. Comparison of the folded cylindrical helix Q to the Gustafsson limit as a function of number of arms.

As seen from Tab. 2, the resonant resistance increases and the Q decreases as the number of folded arms within the cylindrical helix increases. However, the antennas Q remains at or very near the Gustaffson limit. The point where the ratio of Q/Q_{lb} is less than the Gustaffson limit is attributed to the assumptions made in defining the limit, particularly the fact that Gustaffson defines the Chu-limit as $1/(ka)^3$ rather than the exact formula of Eq. (1).

6. Validation of NEC Simulations

The results presented for the folded cylindrical helix in the previous sections were based on simulations performed using the Numerical Electromagnetics Code NEC. The validity of using NEC to simulate the performance properties the folded spherical helix has been previously demonstrated in [4] and [10]. Here, we validate the NEC simulations using a Microwave Studio (CST) model of the 4-arm folded cylindrical helix having an overall height of 6.5 cm, as shown in Fig .5.

A comparison of the feed point impedance of both models is presented in Fig. 6. Copper loss in included here in both models. As is evident from Fig. 6, there is excellent correlation in the predicted performance from each simulation tool. The predicted resonant frequency and resonant resistance using NEC are 300.4 MHz and 51.2 Ω, respectively. The predicted resonant frequency and resonant resistance using Microwave Studio are 298.6 MHz and 57.1 Ω, respectively. This is consistent with similar comparisons for the folded spherical helix.

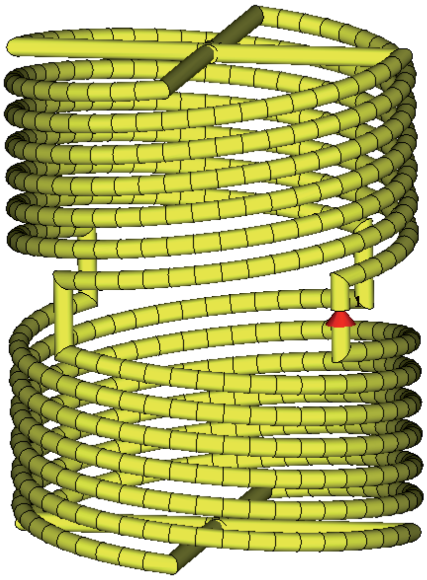

Fig. 5. Depiction of the Microwave Studio model of the cylindrical folded helix having an overall height of 6.5 cm.

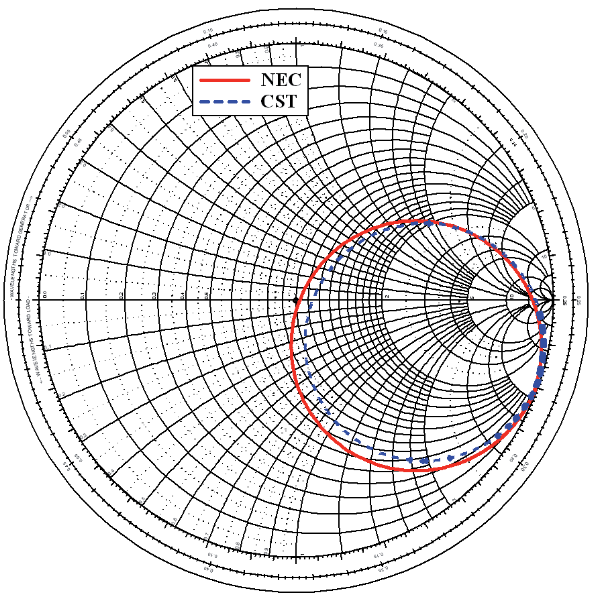

Fig. 6. Comparison of the feed point impedance of the folded cylindrical folded helix simulated using NEC and CST's Microwave Studio.

7. Discussion

Achieving a Q that most closely approaches the lower bound requires that the small antenna utilize the full spherical volume defined by the value of ka. Antennas that occupy a cylindrical volume will exhibit a higher Q than the spherical shaped antenna having the same ka. Gustafsson et al recently developed a limit predicting the minimum achievable Q for the cylindrical shaped antenna as a function of length-to-diameter ratio. Here, a series of resonant cylindrical folded helix antennas were developed occupying the same overall spherical value, having nearly the same value of ka, but having different length-to-diameter ratios. Their Q*'s* were compared relative to the Chu limit and in all instances, their Q*'s* were at or above the Gustafsson limit.

References

[1] BEST, S. R. A comparison of the folded cylindrical helix Q to the Gustafsson limit. In *Eucap 2009*, Berlin (Germany), March 2009.

[2] CHU, L. J. Physical limitations on omni-directional antennas. *J. Appl. Phys.,* 1948, vol. 9, pp. 1163-1175.

[3] MCLEAN, J. S. A re-examination of the fundamental limits on the radiation Q of electrically small antennas. *IEEE Trans. Antennas Propagat.*, May 1996, vol. 44, pp. 672-676.

[4] BEST, S. R. The radiation properties of electrically small folded spherical helix antennas. *IEEE Trans. Antennas Propagat.*, Apr 2004, vol. 52, no. 4, pp. 953-960.

[5] BEST, S. R. Low Q electrically small linear and elliptical polarized spherical dipole antennas. *IEEE Trans. Antennas Propagat.*, Mar 2005, vol. 53, no. 3, pp. 1047-1053.

[6] THAL, H. L. New radiation Q limits for spherical wire antennas. *IEEE Trans. Antennas Propagat.*, Oct 2006, vol. 54, no. 10, pp. 2757-2763.

[7] GUSTAFSSON, M., SOHL, C., KRISTENSSON, G. *Physical Limitations on Antennas of Arbitrary Shape.* Lund University Report: LUTEDX/(TEAT-7153)/1-36/(2007), July 2007.

[8] GUSTAFSSON, M. *Private Communication*.

[9] YAGHJIAN, A. D., BEST, S. R. Impedance, bandwidth and Q of antennas. *IEEE Trans. Antennas and Propagat.*, Apr 2005, vol. 53, no. 4, pp. 1298-1324.

[10] BEST, S. R. A low Q electrically small (TE mode) dipole. *IEEE AWPL.*, vol. 8, pp. 572-575, 2008.

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