A General Model of the Atmospheric Scattering in the Wavelength Interval 300 - 1100nm

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Abstract. *We have presented and developed new theoretic-empirical models of the extinction coefficients of the molecular scattering in the lower, close to the ground troposphere. We have included the indicatrices of back-scattering. The models have been presented using general analytical functions valid for the whole wavelength interval 300-1100 nm and for the whole interval of visibility from 0.1 km up to 50 km. The results have been compared in quantity with the model and experimental data of other authors. The modeling of troposphere scattering is necessary for the analysis and design of all optoelectronic free space systems: atmospheric optical communication systems, location systems for atmospheric research (LIDAR), optical radiometric systems.*

Keywords

Model, extinction, atmosphere, visibility, lidar.

1. Introduction

The atmospheric extinction coefficients $\alpha_S^{aer}(V, \lambda)$ and $\alpha_S^{mol}(\lambda)$ of the aerosol and molecular scattering of the light, as well as the respective indicatrices $J^{aer}(\theta_S, \lambda)$ and $J^{mol}(\theta_S)$, play a major role in the energy balances of the free space optoelectronic systems – the wireless optical communication systems, the laser systems for atmosphere monitoring (LIDAR), the airborne-based optical radiometric systems, etc. (here V is the visibility, λ - the wavelength of the light, θ_S - the scattering angle).

In relation to the coefficient $\alpha_S^{aer}(V, \lambda)$ there are many experimental results [1-9] and model descriptions [4, 5, 7, 10-29]. They have a specific character in that they are valid only for separate value intervals of V, or they have been represented using different expressions, respectively tables (graphs) in the different value intervals of V. For the coefficient $\alpha_S^{mol}(\lambda)$ we have used the well known theoretical results, for example [14], in which the approximate inverse proportionality of λ^4 is theoretically determined, or by introducing $\lambda^{4+\bar{\Sigma}}$, where $\bar{\Sigma} \ll 1$ is the averaged by λ function $\Sigma(\lambda)$.

The indicatrices $J^{aer}(\theta_S, \lambda)$ have been measured [4-6, 8] and modeled [4, 10, 18, 19]. There is a large distribution of the model functions $J^{aer}(\theta_S, \lambda)$ in [10], built on the basis of the representation of function $f(a)$ of the particle size distribution by means of the so-called gamma distribution. Unfortunately, in the monograph mentioned, the indicatrices $J^{aer}(\theta_S)$ of only two λ values from the wave interval we are interested in have been tabulated. As for $J^{mol}(\theta_S)$, the respective theoretical model with respect of the molecule anisotropy is widely represented in the literature, for instance in [14].

2. Problem Formulation

In this paper we are also going to build theoretical experimental models of the atmospheric extinction coefficient $\alpha_{SG}(V, \lambda)$ near the earth's surface and also of the backscattering indicatrice $J^{aer}(\pi, \lambda)$ unlike the already known models, however, here new requirements are set: that the analytical expressions which take part in the models that are being built, be explicit continuous functions of V and λ and also that they be of general validity for the whole interval of V from 0.1 km to 50 km and also for the whole interval of λ from 300 nm to 1100 nm.

We will also build a new model of the coefficient $\alpha_{SG}^{mol}(\lambda)$, in which we will introduce the dependency on λ (rather than averaged by λ) correction of the law λ^{-4}. This is going to be done in the scale factor of $\alpha_{SG}^{mol}(\lambda)$ (not in the exponent) and also in a more practically applicable form.

On the basis of the new model descriptions of $\alpha_{SG}^{aer}(V, \lambda)$, $\alpha_{SG}^{mol}(\lambda)$ and of $J^{aer}(\pi, \lambda)$, by using the formula for $J^{mol}(\pi)$ [14], we are going to derive the essential in quantity for the LIDAR technologies scattering ratio (the ratio of the intensities of the aerosol backscattering to the molecular backscattering) as explicit continuous functions of V and λ in the abovementioned intervals.

3. General Model Descriptions of the Characteristics of Atmospheric Scattering

3.1 Atmospheric Aerosol Extinction Coefficient Near the Earth's Surface

As an initial basis for the building of the model we have used the common and many times tested in practice special case

$$\alpha_{SG}^{aer}(\lambda = 550\,\text{nm}) = \frac{3{,}91}{V} = 3{,}91 V^{-1} \tag{1}$$

($\alpha_{SG}{}^{aer}$[km^{-1}], V[km]), valid only for $\lambda = 550$ nm and $V < 10$ km.

The initial element of our idea to generalize (1) within the interval 300 nm < λ < 1100 nm and within the interval 0.1 km≤ V ≤ 50 km consists in the substitution of the scale factor 3,91 and the exponent -1 with functions of wavelength $A(\lambda)$ and $-Q(\lambda)$ respectively, that is in the development of (1) to

$$\alpha_{SG}^{aer}(V,\lambda) = A(\lambda)\, V^{-Q(\lambda)}. \tag{2}$$

The quantity implementation of the idea to generalize (1) is done by analytical concretization of the functions $A(\lambda)$ and $Q(\lambda)$ in (2). In view to this, the functions in question were subjected to two natural conditions: - their substitution in (2) should coordinate $\alpha_{SG}{}^{aer}$ with the averaged experimental data for a given V published in the literature; - their theoretical relation (by means of (2)) with the function $f(a)$ and with the dependent on λ scattering factor [10, 14] should provide a good compliance between the character of the generalized gamma distribution [10, 19] and the character of the Junge distribution [14,19].

The balanced compliance with these conditions with a current value of λ and for a series of value pairs of V results in a numerical, respectively graphical representation of $A(\lambda)$ and $Q(\lambda)$. The transformation of the discrete numerical results into continuous mathematical functions is done by approximation with the most suitable in this case linear relations between $A(\lambda)$, $Q(\lambda)$ and lnλ. In this way we derive the expressions

$$A(\lambda)\left[km^{-1}\right] = -2.656\ln(\lambda[\mu m]) + 2.499 \tag{3}$$

and

$$Q(\lambda)[-] = 0.199\ln(\lambda[\mu m]) + 1.157 \ . \tag{4}$$

The substitution of (3) and (4) in (2) gives us the necessary model

$$\alpha_{SG}^{aer}(V,\lambda)\left[km^{-1}\right] =$$
$$= \{-2.656\ln(\lambda[\mu m]) + 2.449\} \times V[km]^{-\{0.199\ln(\lambda[\mu m]) + 1.157\}} \tag{5}$$

This model for the extinction coefficient has been graphically represented as a function of λ for two example values of V– for V = 10 km (Fig. 1) and V = 2 km (Fig.2).

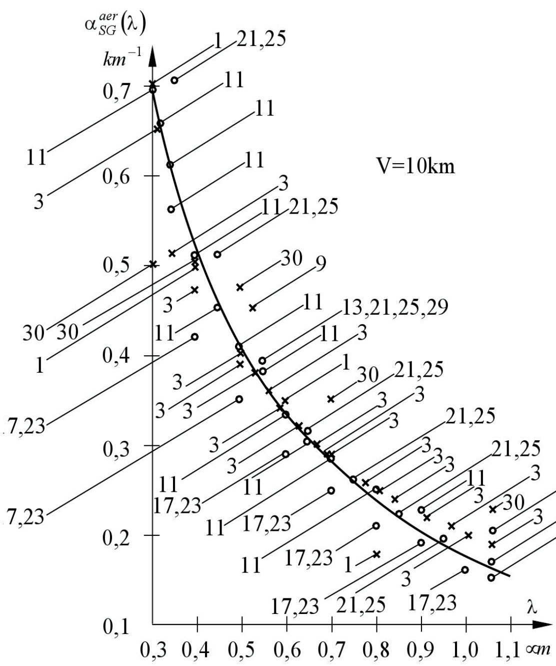

Fig. 1. Model representation of aerosol extinction coefficient when V = 10 km .

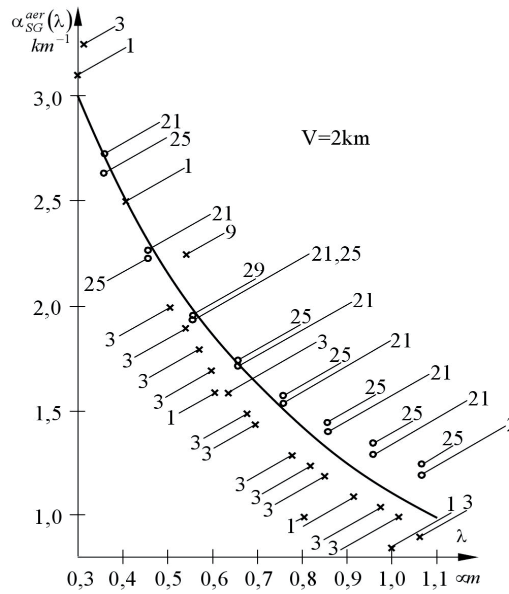

Fig. 2. Model representation of aerosol extinction coefficient when V = 2 km.

For a comparative assessment of the effectiveness of the proposed model, we have drawn the experimental data from the research [1, 3, 9, 30] (with mark "x") and the model results from the works [11, 13, 17, 21, 23, 25, 29] with mark "o") in the same figures. We have added the numbers of the corresponding sources in References to these marks in Fig. 1.

3.2 Atmospheric Molecular Extinction Coefficient Near the Earth's Surface

In compliance with the theory of molecular scattering, the deviation of $\alpha_{SG}^{mol}(\lambda)$ from the law λ^{-4} is due to the relatively weak reduction of the air refractive index n with the increasing of λ and corresponds to

$$\alpha_{SG}^{mol}(\lambda) = M(n(\lambda))\lambda^{-4} . \tag{6}$$

In compliance with the already mentioned expression of this deviation in a more practically applicable form, we use the discrete numerical results represented in [14] for $\alpha_{SG}^{mol}(\lambda)$. We derive $M(\lambda)$ as simple (incomposite) function, i.e. without the clear participation of the dependency of n on λ. After this we approximate the calculated discrete data for $M(\lambda)$ with the continuous function of λ (suitable for this case is the exponential function) and we derive the more appropriate expression of (6):

$$\alpha_{SG}^{mol}(\lambda)\left[\mathrm{km}^{-1}\right] = \frac{0.791.10^{-3}\left(e^{-6.208\lambda[\mu\mathrm{m}]} + 1.312\right)}{\left(\lambda[\mu\mathrm{m}]\right)^4} . \tag{7}$$

The model description (7) of the atmospheric molecular extinction coefficient is graphically represented in Fig.3.

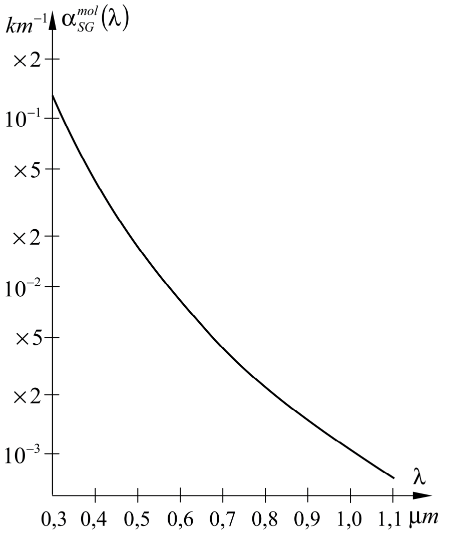

Fig. 3. Model representation of molecular extinction coefficient.

3.3 Aerosol Backscattering Indicatrice

Here we define the aerosol backscattering indicatrice in the way appropriate for the formation of the LIDAR equation – not by the intensity [W/m^2] of the scattered light, but by the scattered optical flux Φ[W]. Now we have

$$J^{aer}(\theta_S,\lambda) = \frac{\Phi_{S,1sr}(\theta_S,\lambda)}{\langle\Phi_{S,1sr}\rangle} . \tag{8}$$

In (8) $\Phi_{S,1\mathrm{sr}}$ is the flux scattered in 1 sr around the direction defined by the angle of scattering θ_S; $\langle\Phi_{S,1\mathrm{sr}}\rangle = \Phi_S/4\pi$ is the averaged flux from all directions, scattered in 1 sr; Φ_S is the total scattered flux. The definition (8) corresponds to the normalization assumed in [10]

$$\frac{1}{4\pi}\int_{4\pi} J^{aer}(\theta_S,\lambda)d\Omega = 1 .$$

The case $\theta_S = \pi$ is of major interest for lidar technologies. For model descriptions we build continuous functions which express the wavelength dependencies of the indicatrices aerosol backscattering for a continental haze (L type) and for a haze near the sea (M type). We mark these models with $J_L^{aer}(\pi, \lambda)$ and $J_M^{aer}(\pi, \lambda)$.

We use the tables for the indicatrices $J_L^{aer}(\theta_S, \lambda)$ and $J_M^{aer}(\theta_S, \lambda)$ for $\theta_S = \pi$, corresponding to the Deirmendjian model [10]. To the values of λ in the wavelength interval in question, for which such tables have been presented, we also add the nearest value $\lambda = 1.19$ μm. The necessary continuous functions are derived by parabolic approximations with argument lnλ. The derived model descriptions are

$$\begin{aligned} &J_L^{aer}(\pi,\lambda) = \\ &= 0.169\{\ln(\lambda[\mu m])\}^2 + 0.0272\{\ln(\lambda[\mu m])\} + 0.114 \end{aligned} \tag{9}$$

and

$$\begin{aligned} &J_M^{aer}(\pi,\lambda) = \\ &= 0.190\{\ln(\lambda[\mu m])\}^2 - 0.162\{\ln(\lambda[\mu m])\} + 0.160. \end{aligned} \tag{10}$$

The models in [10] are tabularly represented. The deviations between them and the formulae (9) and (10) do not exceed 2.5%.

3.4 Molecular Backscattering Indicatrice

The theoretical results for $J^{mol}(\theta_S)$ have been widely discussed in detail in the literature. We have applied the respective formula which (by taking into account the molecule anisotropy) is [14]

$$J^{mol}(\theta_S) = 0.7629\left(1 + 0.9324\cos^2\theta_S\right). \tag{11}$$

For $\theta_S = \pi$ formula (11) gives

$$J^{mol}(\pi) = 1.474 . \tag{12}$$

3.5 Ratio of the Aerosol and Molecular Backscattering Coefficients

This ratio is very important for the solution of a series of specific atmosphere dynamics problems with the help of lidar equipment. We define it as

$$\beta_{SG}(V,\lambda) = \frac{\alpha_{SG}^{aer}(V,\lambda)J^{aer}(\pi,\lambda)}{\alpha_{SG}^{mol}(\lambda)J^{mol}(\pi)} . \tag{13}$$

The quantities on the right side of (13) have already been defined by the model expressions (5), (7), (9), (10) and (12). In Fig. 4 the model relations $\beta_{SG}(V=10\text{ km}, \lambda)$ and $\beta_{SG}(V=2\text{ km}, \lambda)$ are represented for an aerosol haze of the type M.

4. Conclusions

Fig. 1 and Fig. 2 show that the lines which graphically represent the model description (5) of the wavelength dependency of the coefficient $\alpha_{SG}{}^{aer}$ with parameter V, are mean lines of the coordinate system areas $(\alpha_{SG}{}^{aer},\lambda)$, formed by the significant dispersion of the results derived by other authors in the interval $0.3\ \mu m \le \lambda \le 1.1\ \mu m$. Apart from this, in Fig. 1 and Fig. 2 we can see that the lines in question also express the "balance" between the experimental and model data of these authors.

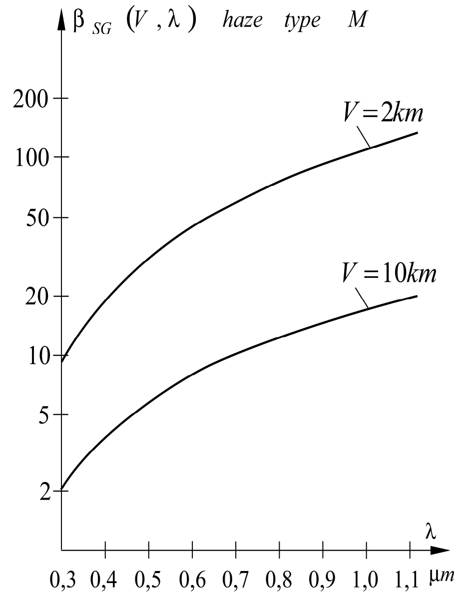

Fig. 4. Model relations $\beta_{SG}(V=10\text{ km}, \lambda)$ and $\beta_{SG}(V=2\text{ km}, \lambda)$ for an aerosol haze of the type M.

The model description (7) of the coefficient $\alpha_{SG}{}^{mol}(\lambda)$ in the interval $0.3\ \mu m \le \lambda \le 1.1\ \mu m$, also graphically presented in Fig. 3, is much more appropriate for practical calculations than the theoretical expression of $\alpha_{SG}{}^{mol}(\lambda)$, because the application of the latter demands knowledge of the wavelength dependency of the air refractive index. What is more, the numerical comparison of the theoretical expression with the model (7) shows that the relative difference between them does not exceed 0.9%.

The expressions (9) and (10) are model descriptions of the indicatrices $J^{aer}(\pi, \lambda)$ in the interval $0.3\ \mu m \le \lambda \le 1.1\ \mu m$, represented by continuous functions. We have not encountered such descriptions in the literature that we are familiar with. During the formation of (9) and (10) on the basis of the tables in [10], we have made an extrapolation of the latter within a very small border interval because of the restricted quantity of data on the wavelength interval in question. The assessment of the possible relative error Δ during this extrapolation by the angle variations of the tangents in the area of the border shows that the probability for $\Delta > 4\%$ is so small that it can be neglected.

As we can see in Fig. 4, the ratio $\beta_{SG}(V, \lambda)$ of the backscattering from the atmosphere aerosol and the atmosphere molecules (13) varies too strongly depending on λ in the interval 0.3 μm to1.1 μm, and also depending on V. In the wavelength interval shown, this ratio can exceed 1 by two or more decades, especially for small values of V (in a relatively turbid atmosphere). In the area of the "blue" border of this wavelength interval, however, the values of $\beta_{SG}(V, \lambda)$, in the most common aerosol states of the troposphere, are not so great – they vary, for example, between 2 and 6. This is due to the $\alpha_{SG}{}^{mol}(\lambda)\sim\lambda^{-4}$ and also to the fact that around this border $J^{mol}(\pi)$ essentially exceeds $J^{aer}(\pi,\lambda)$, for example about 2.5 times.

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