The PAPR and Simple PAPR Reduction of the 2D Spreading Based Communication Systems

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Abstract. *This paper deals with Peak to Average Power Ratio (PAPR) characteristics and the theory of variable spreading factor orthogonal frequency and code division multiplexing (VSF-OFCDM) systems. Comparison and evaluation of PAPR of VSF-OFCDM systems with various 2D spreading factors (SF) are done in time and in frequency domain. The simple PAPR reduction approach based on random chip interleaving is also evaluated.*

Keywords

VSF-OFCDM, OFDM-CDMA, 2D Spreading, PAPR.

1. Introduction

OFDM with its orthogonal subcarriers has been known since the 70's, but due to high computational requirements of the Fourier transform, is used in the relatively new standards only (e. q. ADSL, LTE or DVB-T).

CDMA was developed by the military as a communication system resistant to jamming, but CDMA can also be used as a channel access method. This feature is nowadays the main advantage of CDMA that is used, for example, in the Qualcomm standard IS-95 and in GPS.

The combination of OFDM and CDMA techniques can be called variously – OFDM-CDMA, Multi Carrier (MC) CDMA or Direct Sequence (DS) CDMA. All these systems are different, but only OFDM-CDMA offers two-dimensional (2D) spreading. OFDM-CDMA is the other name for VSF-OFCDM [1].

Variable spreading is a very attractive feature. We can set the spreading factor in two dimensions to get transmitted data more resistant to fading in the transmission channel, but we can not leave behind the big disadvantage of a system based on OFDM and CDMA - namely high envelope fluctuations of the transmitted signal and thus the PAPR (Peak to Average Power Ratio) parameter which is, as we consider, changing with changing spreading factor.

2. VSF-OFCDM as Combination of CDMA and OFDM

Data spreading in the VSF-OFCDM system can be done in two dimensions – in the frequency domain and in the time domain. This is the main difference between the OFDM and the CDMA approach. Two-dimensional spreading factor (SF) is expressed as:

$$\mathrm{SF} = \mathrm{SF}_{time} \times \mathrm{SF}_{freq} \tag{1}$$

where SF_{time} is the spreading factor in time domain and SF_{freq} is the spreading factor in frequency domain.

Variable spreading means that we can change the spreading factor according to the actual transmission channel conditions to achieve lower bit error rate (BER) [2].

It was shown in [7], that proper setting of the spreading factor is able to reduce the BER and. Moreover, to reduce BER, proper setting of separate SF_{time} parameter as well as SF_{freq} is of greater importance than the whole spreading factor SF.

We can suppose that PAPR will change with changing SF, SF_{time} or SF_{freq}.

2.1 Simulation of VSF-OFCDM

Fig. 1 shows the scheme of 2D spreading in the VSF-OFCDM system. At first, a certain number of data symbols is spread by 1D spreading code of length SF. For this, the Hadamard sequences are used. After that, *S/P* block is used to divide the data stream from 1D spreader into SF secondary 2D spread data streams according to the desired SF pattern. These streams form the input of the IFFT block.

The IFFT block creates the orthogonality of the subcarriers according to the equation:

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{m=0}^{M-1} a_n^m Rect_T(t-nT) e^{j2\pi m \frac{t}{T}} \tag{2}$$

where m is the subcarrier number, n is the symbol order, a_n^m denotes the n-th symbol transmitted in the m-th subcarrier

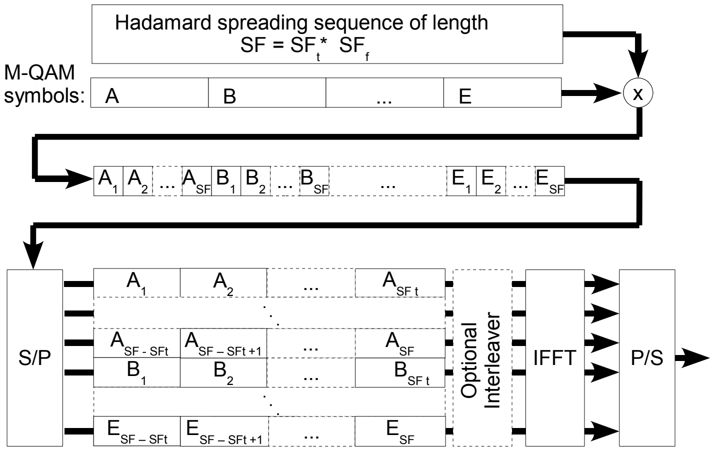

Fig. 1. Scheme of 2D spreading in VSF-OFCDM.

and finally $Rect_T$ is the rectangular window function with durability of T, which defines the OFDM symbol durability.

Another illustration is shown in Fig. 2, where we can observe a matrix of 2D spread and interleaved (marked as *intv y*, noninterleaved data are marked as *intv n*) signal just after IFFT operation. The simulated system was 8×4, $N = 32$. The frequency domain is placed at x-axis, the time domain is at y-axis and finally the power of the 2D signal is at z-axis. It is seen from Fig. 2 that one 16-QAM symbol is spread according to the desired spreading pattern on 4 subcarriers and on 8 timeslots.

The values within the frame of one 16-QAM symbol in direction of axes x and y are not the same due to random interleaving and, in direction of x-asis, due to IFFT operation. The optional random chip interleaving block can be used for reduction of PAPR, as proposed in some papers [9]. The principle of such method is as follows.

The data stream of an OFDM system in the time domain can be considered as a sum of sinusoids. When the peak values of these sinusoids are optimally mutually shifted, the sum of peaks in one timeslot (and also the PAPR parameter) can be reduced. It was shown in [9], that the optimal interleaving pattern gives almost the same results as a random pattern, which is used here and can be generated as a random permutation of number of interleaved chips.

3. PAPR

PAPR in VSF-OFCDM systems can be observed in two dimensions; in time domain and in frequency domain.

Considering that the main drawback of high PAPR is the increase of BER caused by nonlinearity of a power amplifier, the time domain PAPR has more real usage than the PAPR parameter in frequency domain.

The reason for this is obvious from Fig. 3 – we consider the time domain just behind the IFFT block. Behind this block, a power amplifier is inserted as well. The PAPR is critical for this block.

PAPR can be computed as:

$$PAPR(x_\tau,\tau) = \frac{\max_{\tau\in\tau}|x_\tau|^2}{E\left\{|x_\tau|^2\right\}}\ . \tag{3}$$

In (3), τ is the time index used to represent the successive time variable t and also the discrete time index n. The $\max_{\tau\in\tau}|x_\tau|^2$ indicates the maximal value of the power of signal x and finally $E\left\{|x_\tau|^2\right\}$ denotes the mean value of the signal.

Eq. (3) is the general expression of PAPR, [8] shows an alternative equation (4) used for computing PAPR in the frequency domain:

$$PAPR\{x^m(t)\} \leq N\frac{\max|X_k|^2}{E\left\{|X_k|^2\right\}} \tag{4}$$

where N is the number of subcarriers. The marking of signals follows the way of marking in Fig. 3.

A numerical example of PAPR values in the frequency domain for 16-QAM modulation is demonstrated in Tab. 1.

	$PAPR_f$ [dB]	SF patterns
N=80	16,81	10×8
N=160	19,82	20×4, 10×8
N=320	22,83	80×4, 40×8, 20×16
N=640	25,84	640×1, 80×8, 40×16
N=1280	28,85	640×2

Tab. 1. $PAPR_f$ [dB] values for 16-QAM modulation and various SF patterns.

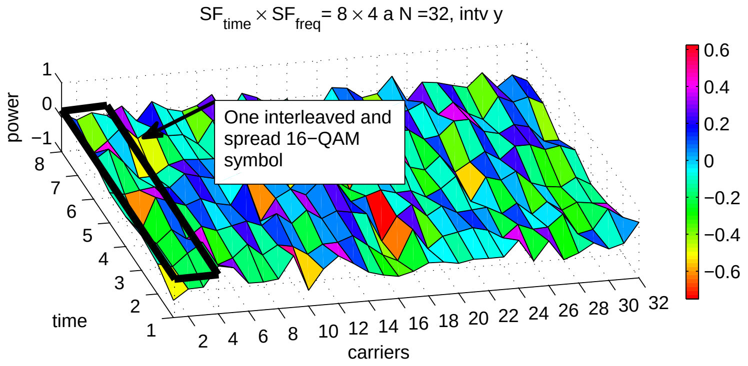

Fig. 2. A 2D spread signal after IFFT operation.

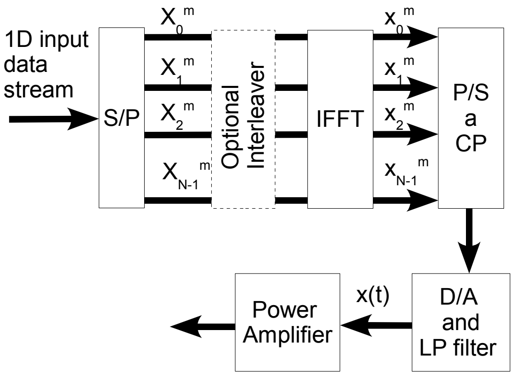

Fig. 3. A OFDM transmitter, signal marking.

4. Results

Results of simulations are shown in Figs. 4, 5 and 6. PAPR was tested in the time domain and the influence of changing SF_{time}, SF_{freq} and N, which represents the number of subcarriers, was observed.

The left part of Fig. 4 shows CCDF (Complementary Cumulative Distribution Function) of PAPR for 16-QAM. SF_{freq} stays const., $\mathrm{SF}_{time} = [8, 16, 32, 64]$ and $N = 120$. Interleaver was not used. PAPR is not significantly rising with the rise of SF_{time}. The reference curve for a pure OFDM system is illustrated in every figure for comparison. OFDM in this figure has significantly lower PAPR than 2D spreading based systems.

The right part of Fig. 4 shows the situation when $\mathrm{SF}_{time} = 10$, $\mathrm{SF}_{freq} = [10, 20, 40, 80]$ and $N = 160$. Interleaver was also not used. PAPR is rising with rising SF_{freq} and is rising with N as well, which is shown in the left part of Fig. 5.

The right part of Fig. 5 illustrates the impact a of random chip interleaver. A 16-QAM, $\mathrm{SF} = 8 \times 16$, $N = 1600$ system is considered. We can observe huge reduction of PAPR up to 9 dB. The system with an interleaver achieves nearly the same PAPR as an OFDM system with the same number of subcarriers N.

In Fig. 6, the amplitude histograms of the transmitted signal are plotted for $\mathrm{SF} = 16 \times 10$ settings and the interleaver is on and off, respectlively. The distribution of the amplitudes of systems with an interleaver is close to the Rayleigh distribution as well as in an OFDM system. If no interleaver is used, the distribution is exponential.

5. Conclusions

This paper describes the VSF-OFCDM system with time-frequency spreading and its Peak to Average Power Ratio (PAPR) performance with different 2D spreading patterns.

An important conclusion is that PAPR is not growing with growing SF_{time}, but with growing SF_{freq} and N, PAPR is growing. This feature can be profitably used in designing various SF patterns in VSF-OFCDM systems. Another conclusion is that PAPR can be simply and effectively reduced nearly to OFDM PAPR level.

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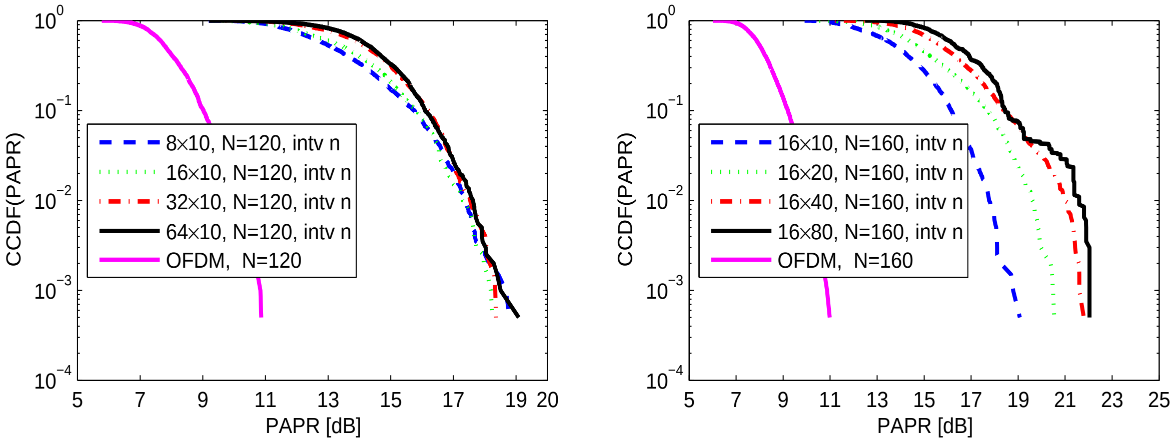

Fig. 4. CCDF of PAPR for 16-QAM. In the left part, SF$_{freq}$ stays constant, SF$_{time} = 8, 16, 32, 64$ and $N = 120$. Interleaver was not used. In the right part, SF$_{time} = 10$, SF$_{freq} = [10, 20, 40, 80]$, $N = 160$. Interleaver was not used.

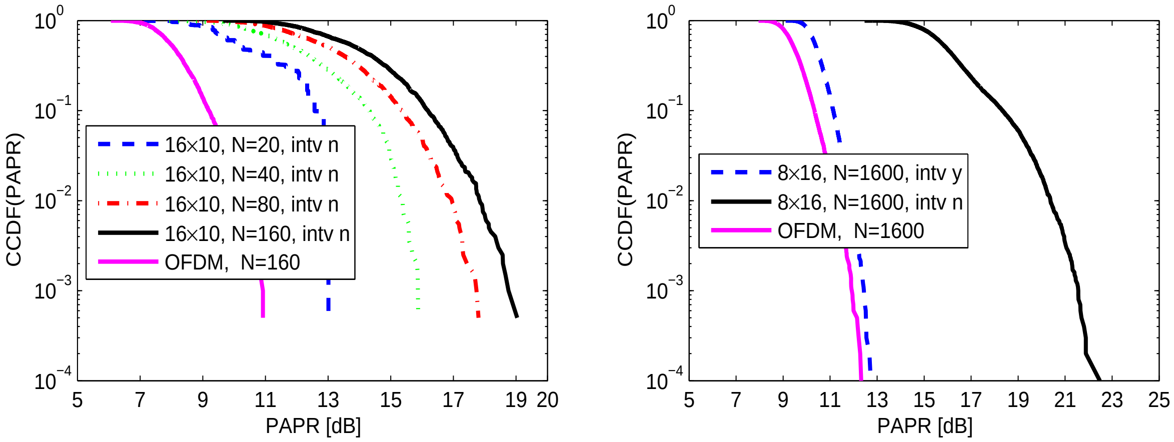

Fig. 5. CCDF of PAPR for 16-QAM. In the left part, $N = [20, 40, 80, 160]$ and SF$= 16 \times 10$. Interleaver was not used. In the right part, the influence of an interleaver on 16-QAM, SF$= 8 \times 16$, $N = 1600$ system is shown.

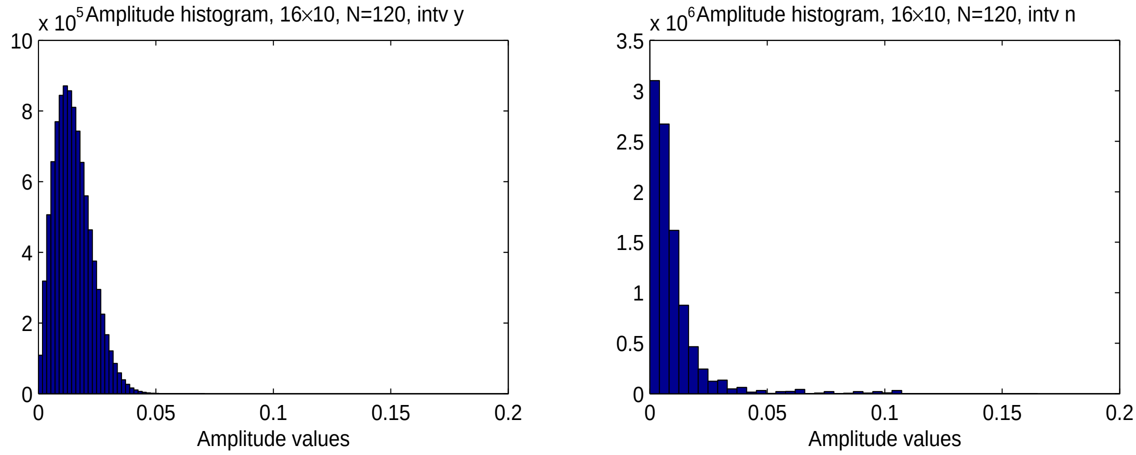

Fig. 6. Amplitude histograms of SF$= 8 \times 16$, $N = 120$ systems. The left part with an interleaver, the right part without interleaver.

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