Other Computational Technique for Estimation of Lower Bound on Capacity of Two-Dimensional Diamond-1 Constrained Channel

Juraj MAČICA[1], *Peter FARKAŠ*[2], *Srećko KRILE*[3]

[1] MicroStep-MIS, Monitoring and Information Systems, Čavojského 1, 841 04 Bratislava 4, Slovakia,
[2] Dept. of Telecommunications, Slovak University of Technology, Ilkovičova 3, 812 19 Bratislava, Slovakia,
[3] Dept. of Electrical Engineering and Computing, University of Dubrovnik, Dubrovnik, Croatia

juraj.macica@microstep-mis.com, p.farkas@ieee.org, srecko.krile@unidu.hr

Abstract. *Computational technique for lower bound on Two-dimensional Diamond-1 constrained channel capacity is presented in this paper. It is basically an amalgamation of Matrix Fractal Grow Method (MFGM) and a new Matrix Fractal Reduction Method both applicable to state transition matrices of the corresponding constrained channels and Rayleigh Quotient Iteration method. Also other programming tricks are presented which improve its implementation. Estimation of lower bound values on the mentioned capacity is made using it. The results are in good alignment with known exact results, which is a verification of the new method. The method could be generalized for other constraints which are restricted only on one neighbor symbol in 2D constrained channel such as for example Square-1 and Hexagonal-1.*

Keywords

Two-dimensional, constrained channel, run-length limited (RLL) channel, hard square, Diamond-1, capacity, estimation, lower bound, Rayleigh Quotient Iteration method, state transition matrix, fractal, computation.

1. Introduction

Coding for two-dimensional (2-D) constrained channels become interesting topics of research thanks to page oriented memories e. g. holographic and other optical memories [1], [2]. However it is not a straightforward process to apply coding methods developed for 1-D in 2-D.

2-D Run-length limited coding problem was treated in [3], [4] in connection with multi-track magnetic recording. In [5] some related theoretical problems were studied in connection with cascading arrays. The main goal of code design in practice is to get as efficient code as possible. The efficiency of the solution could be evaluated via comparison with theoretical achievable rate given by capacity.

However, it is also difficult to compute or estimate the theoretical capacity of such 2-D constrained channels (C_{2D}) [6]. There were not many publications on capacity of 2-D constrained channels [7-10]. Most related to the approach presented in this paper is the one presented in [10]. The capacity estimation for some 2-D constrained channels was treated there as a checker code on check board problem. The lower and upper bounds on capacity of some selected 2-D constrained channels were calculated there. The precision of the corresponding values there did not exceed 14 decimal places.

Probably the most interesting 2-D constraint is the *hard square* constrained because there is model of gases used in statistical physics called hard square gas model. It in its simplifications assumes that all of the gas molecules are positioned at grid points in plane and only interact with their four grid-neighbors. The grid is taken to be square and fixed. From this assumption the name "hard square" comes [7]. Other name prevalently used in information theory community for this constraint is Diamond-1. It is presented in Fig. 2.

This paper will use this constraint as an example on which the new lower bound capacity estimation method will be explained, tested and verified. It was selected namely because Baxter in [11] calculated the hard square entropy constant to forty three decimal places using Calkin and Wilf's bound [8].

In this paper another method is presented, which allows making estimation of lower bound on the capacity of Diamond -1 constrained 2 -D channel.

The paper is organized as follows. In Section 2, the basic nomenclature for 1-D and 2-D constrained channel capacity is given and some of the known 2-D transition matrix properties are stressed for convenience of the reader. In Section 3, overview of known techniques which form the basis for the method presented in this paper as well as presentation of the new ones for lower bound on 2-D constrained channel capacity calculation are presented. In Section 4, calculation results of the lower bound value on 2-D capacity for Diamond-1 constrained channel are presented, which were obtained using the new technique on ordinary personal computer. The comparison with known exact values verifies the validity of the method. In Section 5 some concluding remarks on are given.

2. Basic Nomenclature

For convenience of the reader, the explanation will start with a short overview of basic nomenclature on one-dimensional (1-D) and 2-D constrained channels. More details on 1-D case could be found in [12]. 1-D constrained channel model is a discrete error free channel in which some constraints on the sequences of symbols apply. The constrained channel could be described in many ways. The simplest is by words. For example for binary channel the following constraint can be imposed by practical considerations: "After each transmitted 1 into a channel at least one zero has to be transmitted." Description by words is however most prone to misinterpretation, therefore other tools are used in communication theory for this purpose as well. One is the state diagram. For the simple example it is depicted in Fig. 1. State S_0 denotes "history" that 1 was transmitted and state S_1 means that at least one 0 was transmitted.

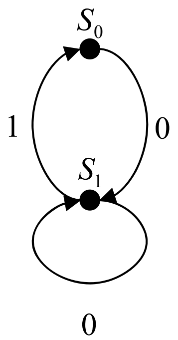

Fig.1. Constrained channel example: "After each 1 at least one 0 has to be transmitted into the channel."

Transition matrix **B** contains the same information as the state diagram in another form. Its *ij*-th entry is a number of possible transitions from state S_i to state S_j. The transition matrix corresponding to the state diagram depicted in Fig. 1 is:

$$\mathbf{B} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}. \tag{1}$$

As already mentioned different constraints are important in practical applications namely in magnetic recording using a one track (spiral or circle) because the magnetic material does not allow flux reversals, which are too close together in the direction of magnetization. Such channel models are named Run-length limited (RLL) channels or more specifically zero–run (ZR) constrained channels. In ZR channels at least r zeros must follow each 1, so the ones are not spaced too closely to each other, in order to avoid the occurrence of interference effects.

However the track structure of the memory medium causes unnecessary capacity losses because the area between tracks can not be used for information storage. More recent optical memories are therefore page oriented. In page oriented memories the mentioned RLL constraints are generalized into 2-D constraints. Similarly like 1-D constrained channel also 2-D constrained channel model is a discrete error free channel in which some constraints on the sequences of symbols apply in two directions (dimensions). One typical 2-D constrained is a Diamond-1 (Fig. 2). This constrain will be used in this paper as a representative for other 2-D constraint.

$$\begin{matrix} & 0 & \\ 0 & 1 & 0 \\ & 0 & \end{matrix}$$

Fig.2. Diamond-1 constraint.

An important theoretical question with practical consequences is how to estimate the capacity C of the constrained channel. For 1-D constrained channel it can be expressed in bits per channel symbol:

$$C_{1D} = \lim_{n\to\infty} \frac{\log_2 N(n)}{n} \tag{2}$$

where $N(n)$ is the number of allowed sequences with n symbols. Lower and upper bounds on C_{1D} could be expressed using a state transition matrix **B** or better to say the *ii*-th or *ij*-th entries $(\mathbf{B}^n)_{ij}$ of the n-step transition matrix $\mathbf{B}^n$ as follows:

$$\frac{1}{n}\log_2(\mathbf{B}^n)_{ii} \le C_{1D} \le \frac{1}{n}\sum(\mathbf{B}^n)_{ij} \; . \tag{3}$$

In [12] there is detailed explanation of the state transition matrix role in 1-D constrained channels and also a proof of the following theorem:

„A noiseless channel has capacity given by $C_{1D}=log_2\,\lambda$, where λ is the eigenvalue of **B** with the largest magnitude“ (real and positive).

The 2-D capacity is defined as:

$$C_{2D} = \lim_{n\to\infty, m\to\infty} \frac{\log_2 N(m,n)}{mn} \tag{4}$$

where $N(m,n)$ is the number of allowed 2-D pattern in an rectangular area with sides m and n. The question how to estimate a capacity C_{2D} of the 2-D constrained channel is even more involved. From [13] the following formula for lower bound on C_{2D} estimation is known

$$C_{2D} \ge \lim_{n\to\infty} \log_2 \frac{\lambda_n}{\lambda_{n-1}} \tag{5}$$

where λ_n is eigenvalue from adjacent matrix ${}_n\mathbf{B}$ for an area $n\times\infty$. This formula will be exploited in the proposed method, which will be described in this paper later.

3. New Computation Method of Lower Bound on Capacity of 2-D Diamond-1 Constrained Channel

In this section we will present the new method for estimation of lower bound of the capacity C_{2D}. Basically it is similar approximation as given in left side of (3) but now for 2-D case. The approximation is based on exploitation of the fractal structure of the corresponding 2-D state transition matrices, which was observed in [14], and also the approximation of dominant eigenvalue using Rayleigh Quotient Iteration together with some other introduced techniques e.g. Matrix Fractal Reduction Method and others.

3.1 Matrix Fractal Grow Method

Let's start with brief review of the known method for construction of higher degree state transition matrices from a known one. It is based on observations in [14]. In this paper we will denote it as Matrix Fractal Grow Method (MFGM).

We will explain it on example of Diamond-1 constraint. However it can be used also for other types of constraints which are restricted on one neighbor symbol such as Square-1 and Hexagonal-1. The method is iterative and therefore it needs some input matrix, which we will denote *starting matrix*. In essence this matrix describes the state transitions in $n \times \infty$ area of the 2-D memory space. The minimal starting matrix is a seed matrix ${}_1\mathbf{B}$ which describes $1 \times \infty$ area. For Diamond-1 constraint the description via a seed matrix contains only two states - zero and one. However the method can start with any other ${}_n\mathbf{B}$ matrix, where n is positive integer. In Fig. 3 the allowed states in state transition matrix for area $3 \times \infty$ are illustrated. Allowed states are such states which do not violate the rules of constraint. State transition matrix describes permitted transitions among the allowed states.

$$n = 3 \qquad \begin{array}{l} S_1 = \ 0\ 0\ 0 \\ S_2 = \ 0\ 0\ 1 \\ S_3 = \ 0\ 1\ 0 \\ S_4 = \ 1\ 0\ 0 \\ S_5 = \ 1\ 0\ 1 \end{array} \qquad {}_3\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

a) b)

Fig. 3. **a)** Allowed states for n=3, **b)** Corresponding adjacent matrix.

The observations presented in [14] allow getting ${}_4\mathbf{B}$ from ${}_3\mathbf{B}$ states simply adding "0" and "1"symbol at the beginning of a state corresponding to ${}_3\mathbf{B}$**.** We get twice as many states as we had before but we have to discard those states which are not allowed. In Fig. 4, which illustrates the procedure, we can see that states S_9 and S_{10} are not allowed. In general this simple procedure will work for any higher step from n to n+1. It can be repeated as many times as needed.

Note: For some constraints different from Diamond-1 however the seed matrix could exist only for integers greater than 1.

$$\begin{array}{llcll}
S_1 & 000 & \xrightarrow{0xxx} & S_1 & 0000 \\
S_2 & 001 & \xrightarrow{0xxx} & S_2 & 0001 \\
S_3 & 010 & \xrightarrow{0xxx} & S_3 & 0010 \\
S_4 & 100 & \xrightarrow{0xxx} & S_4 & 0100 \\
S_5 & 101 & \xrightarrow{0xxx} & S_5 & 0101 \\
S_1 & 000 & \xrightarrow{1xxx} & S_6 & 1000 \\
S_2 & 001 & \xrightarrow{1xxx} & S_7 & 1001 \\
S_3 & 010 & \xrightarrow{1xxx} & S_8 & 1010 \\
S_4 & 100 & \xrightarrow{1xxx} & [S_9] & [1100] \\
S_5 & 101 & \xrightarrow{1xxx} & [S_{10}] & [1101] \\
 & {}_3\mathbf{B} & & & {}_4\mathbf{B}
\end{array}$$

Fig. 4. Getting ${}_4\mathbf{B}$ from ${}_3\mathbf{B}$ states simply adding "0" and "1" symbol at the beginning of a state corresponding to ${}_3\mathbf{B}$ which is denoted as 0xxx resp. and 1xxx over the corresponding arrows respectively. States S_9 and S_{10} are not allowed (must be discarded), which is highlighted by square brackets.

The number of allowed states raises according to Fibonacci sequence.

$$a_{n+1} = a_n + a_{n-1} \tag{6}$$

As already mentioned for Diamond-1 constraint a_1=2 and the set of states is {0,1}. Now we will describe how using MFGM the state transition matrices for higher n could be obtained from a starting matrix. Observing Fig. 5 one can see that permitted transitions between states S_1-S_5 (upper left corner of ${}_4\mathbf{B}$**)** are the same as given by ${}_3\mathbf{B}$ in Fig. 3. Also the transitions between group S_1-S_5 and group S_6-S_8 are the same. This can be deduced from binary representation of the states in Fig. 4. The binary representations of S_6-S_8 are exact copies of binary representations of S_1-S_3 except the "added" (first) symbols, which are different. This observation has as a consequence that the areas highlighted by rectangles with full and dashed lines could be simply copied as also the corresponding arrows show in Fig. 5. It remains to explain why the square in bottom right corner is filled with zeros only. The transitions inside this set (only among S_6-S_8) are not allowed in Diamond-1 constraint because they have "1" in "added" (first) symbols. This observation causes that adjacent matrix will grow in general in accordance with (7).

$${}_{n+1}\mathbf{B}_{a_{n+1},a_{n+1}} = \begin{pmatrix} {}_n\mathbf{B}_{a_n,a_n} & {}_n\mathbf{B}'_{a_n,a_{n-1}} \\ {}_n\mathbf{B}'_{a_{n-1},a_n} & \mathbf{0}_{a_{n-1},a_{n-1}} \end{pmatrix} \tag{7}$$

$$ {}_4\mathbf{B}_{8,8} = \begin{pmatrix} {}_2\mathbf{B}_{5,5} & {}_2\mathbf{B}'_{5,3} \\ {}_2\mathbf{B}'_{3,5} & \mathbf{0}_{3,3} \end{pmatrix} $$

$$ {}_4\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} $$

Fig. 5. Example of creating ${}_4\mathbf{B}$ from ${}_3\mathbf{B}$.

where ${}_{n+1}\mathbf{B}_{a_{n+1},a_{n+1}}$ is the n+1-th element of matrix sequence with dimensions $a_{n+1} \times a_{n+1}$ and ${}_n\mathbf{B}'_{a_n,a_{n-1}}$ is part from the $n^{\text{-th}}$ matrix with dimensions $a_n \times a_{n-1}$. $\mathbf{0}_{a_{n-1},a_{n-1}}$ is zero matrix with dimensions $a_{n-1} \times a_{n-1}$.

The new method is also based on (5). Because in practice we can not reach $n = \infty$ we can only estimate the lower bound of the C_{2D} capacity. The higher the n, the more precise the estimation will be. Therefore the goal is to reach as high n as possible, which results in necessity to deal with quite huge state transition matrices. The new method presented in this paper overcomes this problem partially thanks to new so called Matrix Fractal Reduction Method (MFRM) amalgamated with Rayleigh Quotient Method.

Let's turn our attention first to the MFRM. The main advantage of it is that we have not to store the actual ("huge") state transition matrix ${}_n\mathbf{B}$. Instead if we need a value ${}_nb_{i,j}$ in the actual matrix we can iteratively (backward) get this value from a seed matrix. In other words we can backward iteratively reduce the indexes of the needed entry in the actual matrix so that the reduced indexes point into the seed matrix with the correct needed value. For example in Fig. 6, the value in position i= 8, j=5 is traced back to position i=1, j=2 in the seed matrix. In essence the back tracing is reverse MFGM, which was explained earlier. Using this method we can save significant amount of memory. For example for Diamond-1 only 2×2 (seed matrix) bits are needed instead of any actual (possibly huge matrix), which is needed on particular iteration step.

$$ \mathrm{B} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} $$

Fig. 6. Tracing back value from larger matrix to seed matrix (Check also Fig. 5.)

Now we can turn our attention to the new modification, which allows getting estimations of the lower bound on capacity of Diamond-1 and similar constraints by including the Rayleigh quotient method for approximation of eigenvalues into the calculations. Basically the Rayleigh quotient method improves the precision of the values if compared for example with Power method [15]. Note: The complexity of both methods (by the same number of iterations) is in essence the same.

In engineering language the Rayleigh quotient method is in principle based on the following simple statement [16]: "If $\mathbf{v}$ is an approximation to an eigenvector of $\mathbf{B}$ then $\frac{\mathbf{v}^{\mathbf{T}}\mathbf{B}\mathbf{v}}{\mathbf{v}^{\mathbf{T}}\mathbf{v}}$ is a good approximation to the corresponding eigenvalue."

Note: It can be used only if adjacent matrix $\mathbf{B}$ is real and symmetric, which in our application is fulfilled. Please see for example Fig. 5.

More precisely, the Rayleigh Quocient Iteration method (RQI) for calculation of eigenvalue λ is given by the following expression [17]

$$ \lambda = \lim_{n\to\infty} \frac{\mathbf{v}_n^{\mathbf{T}}\mathbf{v}_{n+1}}{\mathbf{v}_n^{\mathbf{T}}\mathbf{v}_n} \tag{9} $$

where $\mathbf{v}_n$ is the n-th iteration of an eigenvector.

In next iteration eigenvector $\mathbf{v}_{n+1}$ is obtained in accordance with

$$ \mathbf{v}_{n+1} = \frac{\mathbf{B}\mathbf{v}_n}{\|\mathbf{B}\mathbf{v}_n\|}. \tag{10} $$

In (10) the new eigenvector is calculated by multiplication of adjacent matrix with an old version of the same eigenvector and dividing it by its own norm $||\mathbf{B}\mathbf{v}_n||$. Stop condition can be based on the difference of the new eigenvalue and the old one. It is obvious, that in order to find the lower bound estimation of the capacity we have to calculate two eigenvalues in sequence (for n and n+1).

3.2 Some Additional Programming Techniques

In practice the precision of estimation of C_{2D} if calculating using computer is limited. The main restrictions are determined by a limited memory space and a limited time we have available for the task. These limiting factors could be partially overcome by the approaches which will be presented in this part.

The first one follows from the nature of RQI in which it is not necessary to change $\mathbf{B}$ during each iteration. Only the vector $\mathbf{v}_n$ has to be changed for each iteration step (10).

The second one is the possibility to use MFRM, which is reciprocal to MFGM.

The main advantage of MFRM is that we have not to store the actual ("huge") state transition matrix ${}_n\mathbf{B}$. Instead if we need a value ${}_nb_{i,j}$ in the actual matrix we can reduce the indexes of the needed entry in the actual matrix so that the reduced indexes point into the seed matrix with the correct needed value. Using this method we can save significant amount of memory. For example for Diamond-1 only 4 bits are needed.

The third one is quite obvious, namely multiplication of binary matrices can be realized via addition of entries in corresponding rows and columns involved in the operation step.

The fourth one is the usage of own data types, because the double precision type in common computers is too low for this application.

The fifth one is based on observation that it is not necessary to make the normalization in (10) in each iteration step. This normalization is needed in order to prevent overflow of values. The normalization can be done approximately only every 30-th iteration. Even some of the other calculations could be omitted in each iteration. It is possible to realize only the multiplication of vector with a matrix in each iteration.

$$\mathbf{v}_{n+1} = \mathbf{B}\mathbf{v}_n \qquad (11)$$

Then after for example after 30 iterations all operations could be done. Then after the result could be tested on precision. If the needed precision is reached, the calculation could be stopped. This leads to a significant reduction of computational complexity.

4. Results

In this section some results of experiments will be presented. For computing a common PC (Celeron 1GHz and 2 GB RAM) was used. The longest computation took only about 5 hours. It was not possible to compute longer, because the memory space became a limiting factor.

With this method and some tricks mentioned above it was possible to calculate eigenvalues of adjacent matrices with dimensions 24 157 817 × 24 157 817 which represents area 35×∞ on an ordinary PC. The computed value of the eigenvalue of this matrix is approximately 1671753.737815890852403827.

The second matrix which represents area 34 × ∞ has dimensions 14 930 352×14 930 352 and the calculated eigenvalue is approx. 1112242.354258369078361453. The resulting lower bound estimation on the capacity of Diamond-1 channel from these values is 0.587891161775340558936l2331. This value was compared with logarithm with base two of the value of the hard square entropy constant from [11]. The agreement was fair enough. Calculation of any bigger matrix could not be done using the PC, which was available for calculation, because of memory limit. Iterating vector and its copy took 24 157 817 × 2 × 24 byte = 1.16 GB of RAM. Next matrix calculation (in the following iteration cycle) would need about 1.88 GB of RAM. Some more details on computations could be found in [15].

5. Conclusions

In this paper an alternative method for estimation of lower bound on capacity of 2-D constrained channels was presented. It was explained on Diamond-1 constraint. The computational method is a combination of different new approaches which partially eliminate the limitations given by real computers e. g. limited memory space and limited time available. The testing and comparison with results in [11] shows that the new method gives correct values. The main advantage is that it could be used even on ordinary PC thanks to it relatively low computational complexity.

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About Authors

Juraj MAČICA received BSc., MSc degrees in Computer systems and networks and PhD. degree in Telecommunications from the Slovak University of Technology (STU) in Bratislava in 1999, 2001 and 2010, respectively. Currently he is with MicroStep-MIS, Monitoring and Information Systems, Bratislava, Slovakia as Hardware developer. His theoretical interests are concentrated on capacity estimation of 2-D constrained channels. In practice he is working on design, prototyping, and maintenance of different sensors and sensor networks, which operate not only in Europe but also in Middle East and Africa.

Peter FARKAŠ received MSc. and DrSc. degrees both in Telecommunications from STU in 1980 and 1997, respectively, and PhD. degree in Computer Science from St. Petersburg State University (former LPI) in Russia in 1987. He is with STU in Bratislava since 1981, from 1993 as Assoc. Prof. and from 1999 as Prof. During the years he visited and worked in: ATRI, Edith Covan University and Curtin University (Western Australia), Lancaster University and Kingston University (where he was Visiting Professor from 2002 till 2008), (UK), RWTH Aachen (Germany), Sandbridge Tech., (USA), TESLA VRUSE and SWH, SIEMENS PSE and SIEMENS MSS (Slovakia). In 2004 he was named SIEMENS VIP for his innovations and received Werner von Siemens Excellence Award for 2-D Complete Complementary Codes, together with his PhD. Student Matúš Turcsány. He was leading teams participating in projects NEXWAY FP 5, and CRUISE, FP 6 on behalf of STU and WINNER Phase II, FP 6 on behalf of SIEMENS MSS (financed by European Commission). He is an author and co-author of 7 patents and the CRC 11 polynomial he proposed together with Pavol Lunter (SIEMENS MSS) was included into 3GPP standard for EDCH channel. His research interests are communications theory, especially Error Control and Translation Coding, Digital Communcations, Spreading sequences and WSN.

Srećko KRILE was born in 1957 in Dubrovnik, Croatia. He received BSc., MSc. and PhD. degree in 1980, 1988, and 2000 respectively, all in the field of Telecommunications from the Faculty of Electrical Engineering and Computing, University of Zagreb. From 1983 to 1991 he worked on planning and maintenance of functional communication networks for government. Since 1991 he is with University of Dubrovnik, former Polytechnic and Maritime Faculty. He was the first chief of new established department of Electrical Engineering and Computing. Except two books: “Electronic Communications in Shipping” and “Mobile Satellite Networks” he also published in scientific journals and presented papers on many international conferences. Also, he is reviewer in high referenced journals, e.g. IEEE Communications Letters. Research areas: communications in maritime, QoS, resource management and optimal capacity allocation, traffic engineering, mobile satellite networks. Since 1993 he is a member of editorial board for scientific journal "Naše more". He was a member of management board for Croatian Academic Research Network (CARNet) from 2004 - 2008. Also, he is a member of Croatian union of musicians and Croatian society of composers.