Electrical Analogy to an Atomic Force Microscope

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Abstract. *Several applications of the atomic force microscopy (AFM), such as measurement of soft samples, manipulation with molecules, etc., require mechanical analysis of the AFM probe behavior. In this article we suggest the electrical circuit analogy to AFM cantilever tip motion. Well developed circuit theories in connection with fairly accessible software for circuit analysis make this alternative method easy to use for a wide community of AFM users.*

Keywords

Atomic Force Microscopy, electrical mechanical analogs, circuit theory.

1. Introduction

Atomic force microscopy was invented by Binnig *et al.* [1] in 1986. Atomic force microscope (AFM), the principle of which is given in Fig. 1, has become widely used as a tool making the nanoscale world accessible in various branches of contemporary science, including the material engineering, biosciences, electronics, etc. An alternative model describing the probe motion or cantilever behavior, which are to be analyzed in the application of the AFM, is formulated in this article. The method is based on electrical circuit analysis of the electrical analogy to the AFM.

The modeling of dynamic properties of the AFM, and cantilever based sensors generally, was performed by several authors using various methods. Simple, nevertheless functional, approach uses Newton's equation of motion of the damped oscillator [2], [3]. Other techniques are based on advanced methods of mechanical engineering [3], [4], [5], [6]. Yaralioglu and Atalar [7] employed dissipative linear passive electrical network to simulate the noise analysis of the cantilever. Salapaka [8] published control engineering approach to almost all modes of the AFM. Electrical analogy to the AFM presented here is based on electrical–mechanical analogs developed by Firestone [9] in 1930s.

2. The Model

To start with, we use the most simple configuration of the model[1]. Let us suppose, that motion of all components is limited to the direction of the z coordinate. Consider the cantilever with effective mass m as a spring with quasi–elastic restoring force linearly proportional to the load. The spring (force) constant of the cantilever is thus k. Resisting force representing the losses in the cantilever and its surroundings may be assumed to be proportional to the speed of the cantilever's tip with dumping constant β.

The cantilever is driven by the sample–tip interaction force $\mathbf{F}$, which is given by so called force–distance (F–d) curve. The interaction distance d depends on the movement of the cantilever, z_1, and the vertical movement of the sample, such as $d = z_1 - z_2$. z_2 includes the topography of the sample, z_3, transformed by the stiffness of the sample[2]. Since the measurement is provided in contact mode

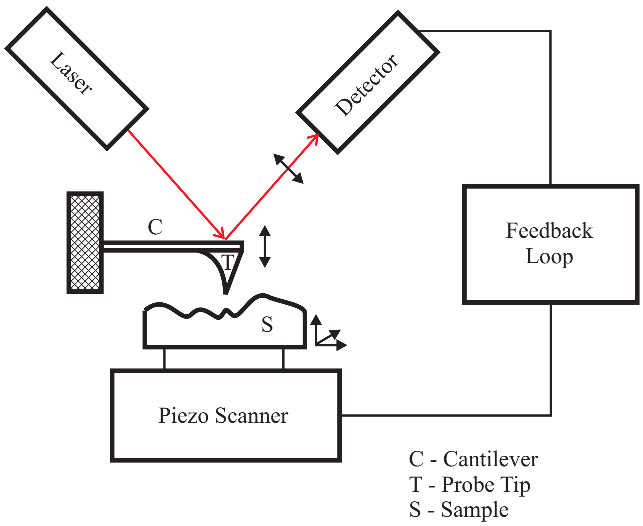

Fig. 1. The principle of AFM. A sharp probing tip is attached to a cantilever type spring which deflects according to the sample-tip interaction force while being scanned across the surface of the sample. The topography is reconstructed on the basis of cantilever deflection signal which is observed using laser beam optical lever.

[1]One of suggested “full version” models is depicted in Fig. 6.
[2]The stiffness of the sample is responsible for indentation of the tip into the surface of the sample.

and the deflection amplitudes are supposed to be small, we may consider the eligible part of the F–d curve to be linear. The stiffness of the sample is considered linear as well. Both properties are, however, inseparable within the AFM measurements, because the stiffness is determined from the F–d curve, which thus contains the "atomic" interaction and elasticity as well. Since we considered both to be linear (see Fig. 2) we can represent them by one force constant κ. With the exclusion of other sources of the vertical motion (feedback, noise, *etc.*), the z_3 is directly given as an integral of the velocity v_3 with which the topography is changing below the probe.

Using the mechanical circuits approach we may design the network diagram of such system (see Fig. 3). Applying the D'Alembert's principle

$$\sum_{i=1}^{n}\left(F_i(z) - m_i\frac{\mathrm{d}^2 z_i}{\mathrm{d}t^2}\right) = 0 \qquad (1)$$

in each node[3] of the circuit leads to following system of equations

$$\begin{pmatrix} m\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \beta\frac{\mathrm{d}}{\mathrm{d}t} + k & -k & 0 \\ -k & k+\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} \beta\frac{\mathrm{d}z_3}{\mathrm{d}t} \\ \kappa z_3 \\ \int v_3\,\mathrm{d}t \end{pmatrix} \qquad (2)$$

where z_1 is the actual motion of the probing tip, z_2 and z_3 are the motions of the sample-tip boundary and of the surface of the sample, respectively. After trivial rearrangement we may write one equation of the probe motion

$$m\frac{\mathrm{d}^2 z_1}{\mathrm{d}t^2} + \beta\frac{\mathrm{d}z_1}{\mathrm{d}t} + \left(\frac{k\kappa}{k+\kappa}\right) z_1 = \beta\frac{\mathrm{d}z_3}{\mathrm{d}t} + \left(\frac{k\kappa}{k+\kappa}\right) z_3 . \qquad (3)$$

3. Electrical Analogy

From comparison of mechanical and electrical circuits [9] we may derive the force-voltage electrical analogy (see Fig. 4) to suggested mechanical circuit (Fig. 3). Values of circuit elements are: $C = m$, $L_1 = \frac{1}{k}$, $L_2 = \frac{1}{\kappa}$, $R = \frac{1}{\beta}$, $u_c = z_1$, and $u = z_3$. Using nodal–voltage analysis we may determine equation

$$C\frac{\mathrm{d}u_c}{\mathrm{d}t} + \frac{u_c - u}{R} + \left(\frac{1}{L_1+L_2}\right)\int_0^t (u_c - u)\,\mathrm{d}\tau + i_L(0) = 0 \qquad (4)$$

which can be rearranged to the form identical with Eq. 3. This system has the transfer function

$$H(p) = \frac{\mathcal{L}\left[u_c(t)\right]}{\mathcal{L}\left[u(t)\right]} = \frac{p(L_1+L_2)+R}{p^2(L_1+L_2)RC + p(L_1+L_2)+R} \qquad (5)$$

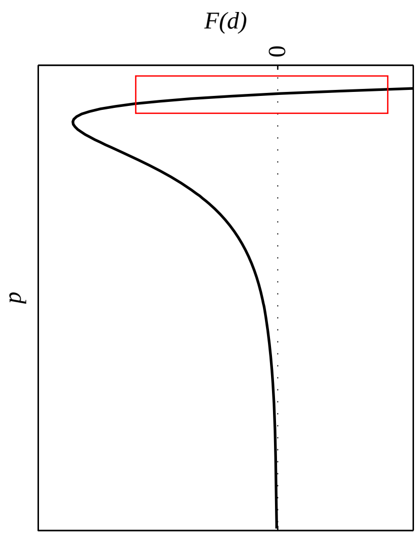

Fig. 2. Typical force-distance (F–d) curve of the sample–tip interaction. A part of the F–d curve which is considered to be linear is inscribed with the rectangle.

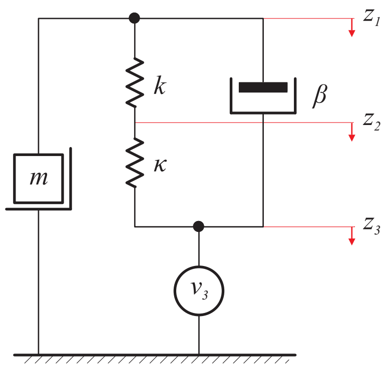

Fig. 3. Mechanical network diagram of the sample-tip-cantilever system.

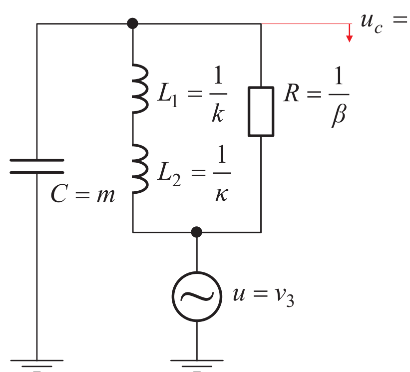

Fig. 4. Force-voltage electrical analogy to the mechanical circuit in Fig. 3.

[3] D'Alembert's principle is a mechanical analogy to Kirchhoff's nodal rule (or current law). The term *node* is used identically as in the electrical circuits theory.

where $\mathcal{L}$ denotes the operator of the Laplace transform. Considering feasible values of L_1, L_2, R, C and applying the relation between Laplace transform and Fourier transform, we may derive the resonant frequency of cantilever in contact with the sample as

$$\omega_o = \left| \frac{-(L_1+L_2) - \sqrt{(L_1+L_2)^2 - 4(L_1+L_2)CR^2}}{2(L_1+L_2)CR} \right| . \quad (6)$$

4. Model Verification

To verify the model proposed, we examined, both experimentally and theoretically, the response of unloaded cantilever to the 1/f-like ambient noise. Veeco MultiMode IV scanning probe microscope with Veeco NP20 D cantilever were used. The cantilever is triangular shaped with experimentally determined resonance frequency $f_o = 19,52$ kHz ($f_o = 18$ kHz nominal) and spring constant $k = 0.0765$ N/m ($k = 0.06$ N/m nominal).

We measured cantilever deflection in false engage mode[4] as a function of time for 8.6486 s with sampling frequency of 60.621 kHz. Noise protection wasn't used. Resulting power spectrum density of noise–induced deflection of the cantilever is depicted in Fig. 5 by crosses.

The spectra calculated using the model proposed is depicted with a dashed line in Fig. 5. Parameters of the model were estimated experimentally. Since the cantilever was kept unloaded in the air, the coil representing spring constant κ (see Fig. 4) hasn't appeared in the model. The 1/f-like noise source was used to feed the circuit.

The calculated response gives only principal insight into the probe behavior. To observe curve, which fits the experimental data well (full line curve in Fig. 5), some expansions have to be introduced into the model.

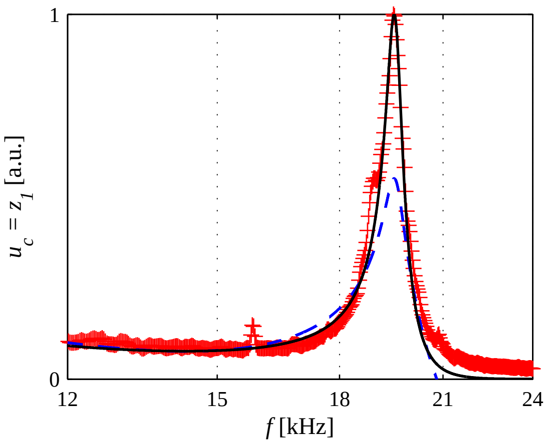

Fig. 5. Response of unloaded cantilever to the ambient 1/f-like noise versus frequency. Calculated curve is depicted with dashed line for linear and with full line for nonlinear model, respectively. Crosses represent experimental data. Both in arbitrary units.

5. Model Expansion

We have done several simplifications at the beginning to make the structure of the model clear and equations "pretty". The suggested analogy should be now expanded to provide more realistic description of behavior of the probe.

Firstly the higher order of the model with more elements is to be used. Secondly elements representing nonlinearities of the sample-tip interaction can be added. Finally the feedback loop and driving loops for various imaging modes have to be taken into account. The model is variable enough to enable analysis of most AFM regimes and apparatus arrangements. It is also possible to replace the lumped parameters of the cantilever (m, b, k) by the finite element network proposed by Yaralioglu [7].

The expansion of the model seems to be easy; however, almost each of parameters added remains undisclosed within the measurement and needs additional experiments to determine its value. Nevertheless, the model can be used to determine unknown parameters in particular conditions. For instance, the damping and the spring constant can be estimated from the oscillative transient phenomenon if the effective mass is known.

6. Conclusion

The variable and easy to use electrical analogy to AFM was presented and verified. The expanded model is suitable for scientific simulations. It enables treatment of several

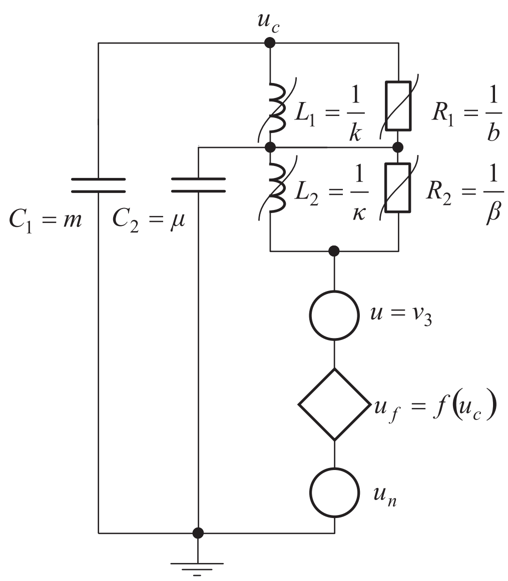

Fig. 6. An example of expanded electrical model of the AFM in the contact mode. The probe is represented by its effective mass m, nonlinear spring constant k, and damping b. The sample has analogous parameters μ, κ, β. u represents topography of the sample, u_f and u_n represent the feedback and the noise, respectively.

[4]False engage mode denotes regime in which the cantilever is kept unloaded in ambient surroundings (air in this case).

AFM probe problems including, but not limited to, response of cantilever to noise and interaction of probe with soft dynamic structures (*e.g.* cells in life sciences). Influence of the feedback loop on the signal measured can be also analyzed. Furthermore, it is a novel application of electrical circuit theory in the field of nanotechnology.

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About Author. . .

Ondřej KUČERA graduated in Biomedical Engineering from the Czech Technical University in Prague in 2008. Currently he is a PhD. student at the same place and he is engaged in the Institute of Photonics and Electronics, AS CR. His research interests include atomic force microscopy, biological electromagnetism, and related techniques.