Invited paper

A Study of Fog Characteristics using Free-Space Optical Wireless Links

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Abstract. *A technique for modeling the fog droplet size distributions using modified gamma distribution has been demonstrated by considering two separate radiation fog events recorded in Graz (Austria) and Prague (Czech Republic). The measurement of liquid water content (LWC) and the optical attenuations at visible wavelength are used to form equations to obtain the three parameters of the modified gamma distribution i.e., the slope (Λ), the intercept (N_0) and the shape parameter (m). Calculated attenuation or LWC from the retrieved parameters are in excellent agreement with attenuation or LWC obtained from the measurement. Hence this method is useful in the study of fog microphysics and in modeling the fog attenuations for terrestrial FSO links in situations when our measurement data contains values of attenuations only, or liquid water content only or both at a particular location. For the two case studies, Graz and Prague, we obtained the DSD parameters Λ=3.547 $\pm$ 1.935, N_0=3.834 $\pm$ 2.239, m=6.135 $\pm$ 2.692 and Λ=5.882 $\pm$ 2.889, N_0=13.41 $\pm$ 3.875, m=5.288 $\pm$ 3.113, respectively. It is evident that the observed behavior of computed modified gamma distribution parameters for Graz and Prague is closely the same and is consistent with the previous literature for the radiation (continental) fog. Moreover, we observed the variation of the computed DSD parameters at the different stages of fog (formation, maturity and dissipation phases) indicating different microphysical processes at each stage.*

Keywords

Free-space optics, optical attenuations, continental fog, droplet size distribution, modeling.

1. FSO Introduction

Free-space optical communication (hereafter FSO) is an emerging transmission technique to transmit high data rates without any cabling. This technology is expected to revolutionize the present communication system architectures especially the ground-space and in-space communication system architecture. One can witness a linear increase in the trends of data rate for earth observation satellites with the every coming year. In addition, this technology can complement the existing conventional RF transmission links for different terrestrial and ground-space communication scenarios.

A tremendous progress has been made in electro-optics and opto-electronics components, that has already been incorporated and disseminated into today's optical communication systems [1],[2]. But the biggest challenge to terrestrial free-space optical communication links is still different fog conditions, that has variable microphysical characteristics that change with the location and time [3]. Various modeling approaches were adopted in the past, but none yet resulted in a significant breakthrough in overcoming the attenuating effects caused by different fog conditions, which raises the need of proper modeling of the microphysical parameters of fog like drop size distribution (hereafter DSD), particle concentrations and the amount of liquid water content. Especially the optimum scaling of the fog DSD would result in the fine tuning of the methods used for the modeling of optical attenuations over terrestrial free-space links [4].

In this contribution we focus on the size distribution of fog droplets that has been a subject of continued interest as more attenuation data from different locations around the globe are made available. We model the optical attenuations experienced over the terrestrial FSO link installed in Graz, Austria, caused by the continental fog conditions, by a three parameter modified gamma drop size distribution (hereafter MGDSD). Upon performance analysis of the newly computed MGDSD parameters, it is found out that the modeling of optical attenuations can be very accurately modeled by the three parameter MGDSD function.

This paper is organized as follows: after the introduction part in Section 1, in Section 2 theoretical background of the modeling optical attenuations in radiation fog conditions by MGDSD will be mentioned followed by MGDSD parameters retrieval algorithm against two representative fog events. In Section 3, the analysis of the attenuations measured against the two representative fog events is presented. In Section 4, the analysis of the computed MGDSD parameters is presented by mentioning the performance evaluation of the mentioned technique and the explanation of the computed results. Finally, the conclusions are drawn in Section 5.

2. Theoretical Background

The size distribution (DSD) of fog droplets is not an easy function to quantify. It has been customary to express DSD in the form of a distribution function. In the literature, different distribution functions have been considered in the past to model the fog droplet size distributions [5]. However, the modified gamma distribution function is considered the best fit in case of fog and clouds. Actually, the Exponential DSD (hereafter EDSD) has been used in many studies to describe the DSD such as those described by Marshal and Palmer [6]. Then in 1983, Ulbrich introduced the three parameter Gamma DSD (MGDSD) model that seems a very suitable candidate to model fog distributions [5, 7]. The distribution specified by MGDSD can be written as

$$C(r) = N_0 r^m exp(-\Lambda r^{\sigma})\Delta r, \qquad 0 \le r \le \infty \tag{1}$$

where $C(r)$ is the number concentration of fog droplets of radius r (μm) per unit size interval, while Λ (the slope), N_0 (the intercept) and m (the shape parameter) are the three parameters of MGDSD. The value of parameter σ in the above equation is considered as equal to 1 for the fog case [5]. EDSD is a special case of the MGDSD when its shape parameter m=0. While computing optimal distribution parameters of the MGDSD for the selected continental fog event, there are few assumptions that are made. Firstly, the fog droplet radius r is considered as the independent variable of a size distribution as it prevails in the literature probably because it is favored in electromagnetic and Mie theory. Secondly, the size of fog particles considered is in the range between 0.1 μm – 50 μm with an interval size of 0.1 μm. And Mie scattering efficiency is calculated by employing the complex refractive index of a particular wavelength considered using Ray method [8].

In 1964, Deirmendjian analyzed a large number of empirical size spectra of fog droplets by a computational method using three parameters MGDSD having a general form as given in (1) [9]. These three parameters completely determine the shape of a distribution curve. In 1971-73, Garland measured the parameter sets of the MGDSD of small water droplets corresponding to 1 km visibility range by giving the best-fit curves relative to eight size spectra, which are representative size distribution models of haze droplets [9], [10]. The corresponding mode radius (r_c) range of these small water droplets studied was between 0.038 μm and 0.734 μm. Later on in 1976, Tomasi and Tampieri listed parameters sets of MGDSD for fog droplets against 0.2 km horizontal visibility for different fog types [9]. The r_c for radiation fog water droplets case studied, was in the range between 2.13 μm and 12.22 μm. Shettle (in 1989) and Harris (in 1995) listed the MGDSD parameters corresponding to different fog conditions [5], [11]. The reference values of MGDSD parameters (m, N_0 and Λ) and the total actual concentration N_d for the radiation fog case from the above mentioned references are listed in Tab. 1, below.

2.1 Methodology to Retrieve MGDSD Parameters

The optimum scaling of the fog DSD leads to the fine tuning of the methods used for the prediction of attenuation over terrestrial free space optical links, and for the reliable behavior of these transmission links under harsh environments. In the following sections, two continental fog events will be discussed and analyzed, while drop size distribution parameters will be estimated by considering MGDSD. The DSD parameters computation method is based on iterative procedure that selects those optimal parameters of a certain distribution for which the residual errors between the actual measured value and the computed value are minimal. The two fog events are selected from two continental locations Graz (Austria) and Prague (Czech Republic). The algorithm adopted to determine individual MGDSD parameters is explained below.

2.1.1 Graz Fog Event of 18-19 Nov. 2009

In order to compute MGDSD parameters N_0, m and Λ against the fog events recorded in Graz, Maitra and Gibbins procedure is adopted [12]. This procedure considers three nonlinear equations to determine three parameters of the log-normal distribution in case of raindrops, but we consider here two equations to compute three parameters of MGDSD. The two Eqs. (2) and (3) are as given below,

$$\gamma(\lambda) \cong \beta_a(\lambda) = 10^5 \int_{0.1\mu m}^{50\mu m} Q_d\left(\frac{2(\pi)r}{\lambda}, n'\right)\pi r^2 C(r)dr\,, \tag{2}$$

and

$$LWC = \rho_w N_d \frac{4}{3}\int_{0.1\mu m}^{50\mu m} \pi r^3 C(r)dr \tag{3}$$

where $\gamma(\lambda)$ is the specific attenuation measured in dB/km, n' is the real part of the complex refractive index of the fog particles, Q_d is the normalized Mie scattering cross-section. The factor πr^2 in (2) is introduced for denormalizing with respect to the geometrical cross-sectional area of the fog droplets and r is the fog particle radius. Eq. (2) computes fog attenuation coefficient (dB/km) from Mie scattering efficiency and the respective DSD i.e., MGDSD here, and the second equation (3) computes the LWC (g/m^3) from the respective DSD i.e., MGDSD here. It is important to mention that the optical attenuations measured correspond to 950 nm wavelength and the measured attenuation data considered is taken averaged over a minute scale for the computation of MGDSD parameters. The three MGDSD parameters, namely, m, N_0 and Λ can be computed theoretically by taking the ratio of the above mentioned two equations as,

$$\frac{\gamma_{exp}(950nm)}{LWC_{exp}} = \frac{10^5 \int_{0.1\mu m}^{50\mu m} Q_d(\frac{2(\pi)r}{\lambda}, n')\pi r^2 C(r)dr}{\rho_w \frac{4}{3}\int_{0.1\mu m}^{50\mu m} \pi r^3 C(r)dr}. \tag{4}$$

where "exp" denotes the experimentally measured values. Assuming the three parameters MGDSD distribution as given by (1), then the ratio of two equations become,

$$\frac{\gamma_{exp}(950nm)}{LWC_{exp}} = \frac{10^5 \int_{0.1\mu m}^{50\mu m} C_{Ext} N_0 r^m exp(-\Lambda r)dr}{\frac{4}{3}\pi\rho_w \int_{0.1\mu m}^{50\mu m} r^3 N_0 r^m exp(-\Lambda r)dr}\,, \tag{5}$$

Size distributions	m	N_0 $\mathbf{cm^{-3} \cdot \mu m^{-1}}$	Λ $\mu\mathbf{m}^{-1}$	N_d $\mathbf{cm^{-3}}$	Fog Type
Garland [10]	1 - 7	0.54 - 1.28	1.362 - 184.2105	718.61 - 9230.25	Small water droplets
Tomasi and Tampieri [13]	4 - 5	7.54×10^{-6} - 4.32×10^{0}	0.3273 - 2.3474	15.90 - 249.93	Radiation fog
Shettle, Harris [5, 11]	6	2.37 - 607.5	1.5 - 3.0	20 - 200	Radiation fog

Tab. 1. Ranges of MGDSD parameters from literature survey.

or

$$\frac{\gamma_{exp}(950nm)}{LWC_{exp}} = \frac{10^5 \int_{0.1\mu m}^{50\mu m} C_{Ext} r^m exp(-\Lambda r)dr}{\frac{4}{3}\pi\rho_w \int_{0.1\mu m}^{50\mu m} r^{3+m} exp(-\Lambda r)dr} . \tag{6}$$

By this procedure, the parameter N_0 can be eliminated, and we are left with the only two parameters m and Λ. Now the iterative procedure is adopted to compute the two remaining MGDSD parameters. For the computation of these two parameters the values of parameters m and Λ are taken as m = 0.1, 0.2, ..., 10 (100 values), and Λ = 0.1, 0.2, ..., 10 (100 values). It seems appropriate not to restrict the MGDSD parameters, while computing them against the measured attenuation values against the radiation fog events at the two mentioned locations, in the reference ranges as existing in the previous literature since the fog DSD parameters characteristics vary from location to location. Therefore, the ranges mentioned for MGDSD parameters m and Λ between 0.1 to 10, for our computation here, seem reasonable. These ranges of the respective parameters are taken from Tomasi and Tampieri [9], [13]. Then for each combination of m and Λ, residuals of the computation are calculated out of 100 x 100 matrix combination. Finally, the residuals are compared and that optimal combination of m and Λ is taken for which the residual of ratio of attenuations at 950 nm and LWC (as given by (6)) is minimal. After finding the optimal set of m and Λ, the third parameter N_0 can be computed from the ratio of the measured attenuation and the computed attenuation at a particular wavelength given by

$$N_0 = \frac{\gamma_{exp}(950nm)}{\gamma_{thr}(950nm)} , \tag{7}$$

$$N_0 = \frac{LWC_{exp}}{LWC_{thr}} \tag{8}$$

where "thr" denotes the theoretically computed values using the MGDSD parameters and the LWC_{exp} as mentioned in (8) corresponds to LWC values computed from fog density values (measured by a fog sensor device) [14], [15].

2.1.2 Prague Fog Event of 07 Feb. 2009

To find out, first, the two parameters m and Λ of the MGDSD for the representative fog event of Prague as well, the same procedure as we adopted in case of Graz fog event is used here, too. In case of Prague fog event we have an additional possibility of computing the MGDSD parameters from the ratio between two attenuations (at 1550 nm and 850 nm), besides taking the ratio of attenuation at a particular wavelength (either 850 nm or 1550 nm) and the LWC. It is important to mention here that we mainly considered attenuations at 1550 nm wavelength, because of the preference of this wavelength for future FSO link designs, to compute the three MGDSD parameters against the Prague fog event. Therefore, the final equation in case of taking the ratio between optical attenuations at 1550 nm and LWC thus becomes

$$\frac{\gamma_{exp}(1550nm)}{LWC_{exp}} = \frac{10^5 \int_{0.1\mu m}^{50\mu m} C_{Ext} r^m exp(-\Lambda r)dr}{\frac{4}{3}\pi\rho_w \int_{0}^{\infty} r^{3+m} exp(-\Lambda r)dr} . \tag{9}$$

Finally, the third parameter of MGDSD (N_0), for the representative fog event of Prague can be computed by taking the ratio of measured and computed attenuations at 1550 nm wavelength as

$$N_0 = \frac{LWC_{exp}}{LWC_{thr}} . \tag{10}$$

The other possibility of computing the two MGDSD parameters (m and Λ) is to take the ratio between measured attenuations at 1550 nm and at 850 nm as given by the following equation

$$\frac{\gamma_{exp}(1550nm)}{\gamma_{exp}(850nm)} = \frac{\int_{0.1\mu m}^{50\mu m} C_{Ext} r^m exp(-\Lambda r)dr}{\int_{0.1\mu m}^{50\mu m} C_{Ext} r^m exp(-\Lambda r)dr} . \tag{11}$$

Finally, the third parameter N_0 can be computed either by using (10) or by taking the ratio between the measured and the computed LWC values as given by the following equation

$$N_0 = \frac{\gamma_{exp}(1550nm)}{\gamma_{thr}(1550nm)} . \tag{12}$$

3. Analysis of Measured Fog Events

In this section, the two fog events are discussed and evaluated statistically.

3.1 Graz Fog Event of 18-19 Nov. 2009

The first fog event analyzed for the determination of optimum distribution parameters is recorded in Graz over an 80 m FSO link operating at 950 nm. This fog event started at 3:15 pm on 18 Nov. 2009 and lasted to 7:50 am on 19 Nov. 2009. The total duration of this fog event was about 990 minutes. Fig. 1 shows the time series of measured optical attenuations and LWC against this recorded fog event.The maximum value of attenuation reached up to 141 dB/km, with an average attenuation of about 93 dB/km and median attenuation of about 111 dB/km when averaged over a minute scale. The corresponding analysis of changes in attenuation shows that a maximum change of $\pm$ 37 dB/km was

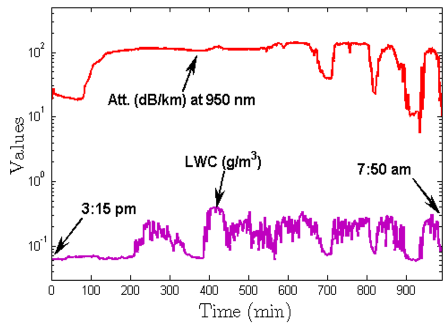

Fig. 1. Time series of specific attenuations (dB/km) and LWC (g/m^3) measured on a minute scale against the fog event recorded on 18-19 Nov. 2009 in Graz.

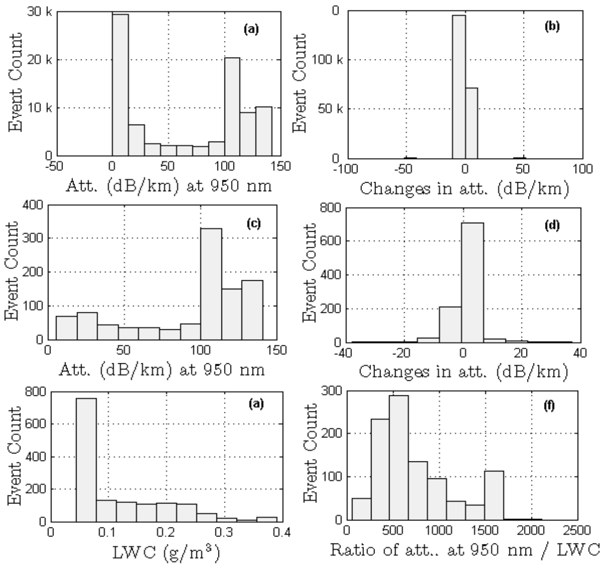

Fig. 2. Histograms of measured optical attenuations (dB/km) (a), corresponding changes in attenuations (dB/km) (b) on a seconds scale scale, the measured optical attenuations (dB/km) (c), and corresponding changes in attenuations (dB/km) (d), and histogram of measured LWC (g/m^3) (e), and the ratio of optical attenuations (dB/km) and LWC (g/m^3) (f), over a minute scale scale against the fog event of 18-19 Nov. 2009 in Graz.

recorded with an average variation of about 0.003 dB/km. Fig. 2(a)-(d) shows the histograms of optical attenuations and the corresponding changes in attenuations when measured over a second scale, and then averaged on a minute scale, respectively.

In Fig. 2(e) the histogram shows the variation behavior of LWC (g/m^3) measured, whereas the histogram shown in Fig. 2(f) depicts the behavior of ratio between measured optical attenuation and the LWC for the mentioned fog event. From the analysis of the measured LWC data, it was observed that the maximum value of LWC recorded against this fog event was about 0.394 (g/m^3), while the average and median values of LWC were about 0.1466 and 0.127 (g/m^3). Tab. 2 summarizes the relevant statistics of different measured parameters related to this fog event.

3.2 Prague Fog Event of 07 Feb. 2009

The second continental fog event selected was recorded in Prague on 07 Feb. 2009 over a 100 m FSO link established with two parallel links using 1550 nm and 850 nm wavelengths. For this particular fog event besides the measurement of optical attenuations and visibility range (m), the LWC and PSA are also measured (by the PVM-100 instrument) in parallel in order to demonstrate the relation between microphysical parameters of fog and the optical attenuations. The total duration of this fog event was 480 minutes and Fig. 3(a) shows the time series of the measured parameters including the optical attenuation in dB/km at 1550 nm, liquid water content LWC (g/m^3) and particle surface area PSA (cm^2/m^3) over a minute scale.

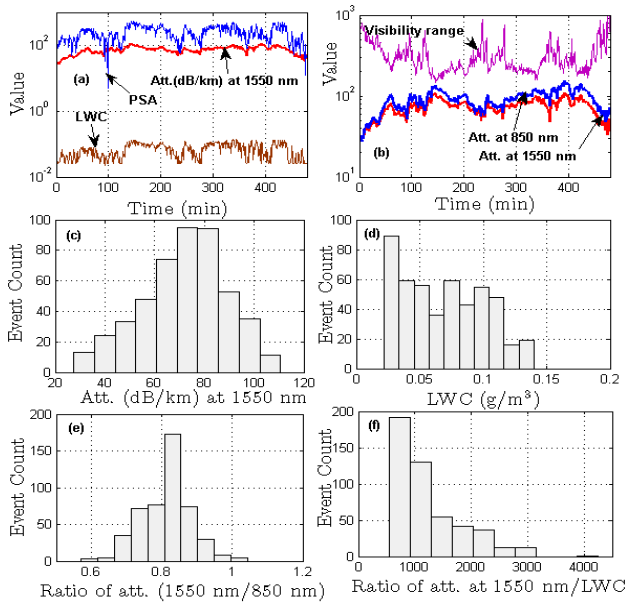

Fig. 3. Time series of measured parameters against the fog event of 07 Feb. 2009 in Prague on a minute scale; measured attenuations (dB/km) at 1550 nm, LWC (g/m^3) and PSA (cm^2/m^3) (a), time series of visibility range, attenuations measured at 1550 nm and 850 nm (b), histogram of attenuations (dB/km) at 1550 nm (c), histogram of LWC (g/m^3)(d), histogram of ratio between measured attenuations at 1550 nm to 850 nm (e), histogram of ratio between attenuations at 1550 nm to LWC (g/m^3) (f).

Fig. 3(b) shows the time series of measured attenuations in dB/km at 1550 nm and 850 nm and the visibility range. Figs. 3(c)-(f) show the histograms of measured attenuations at 1550 nm, the LWC, the ratio between attenuations at 1550 nm and 850 nm, and the ratio between attenuations at 1550 nm to the LWC, respectively. From Fig. 3(b) it is clearly visible that measured optical attenuations are higher in case of 850 nm as compared to 1550 nm. Tab. 3 summarizes the relevant statistics of the mentioned measured parameters related to this fog event. The maximum value of attenuation over 1550 nm link reached up to 111 dB/km, while at the parallel 850 nm FSO link the maximum value was about 155 dB/km. The average and median values

Parameters	Attenuations (dB/km) seconds	Changes in Attenuation (dB/km) seconds	Attenuations (dB/km) minutes	Changes in Attenuation (dB/km) minutes	LWC (g/m^3) minutes
Min.	-0.03477	5.884	5.7369	-37.923	0.058612
Max.	142.6	-5.894	140.99	37.117	0.39044
Mean	65.15	-0.001096	93.51	0.002724	0.1466
Median	77.08	0	110.935	-0.009	0.12718
Std. dev.	53.33	0.9575	39.47	4.594	0.08144
Range	142.6	11.78	135.3	75.04	0.3318

Tab. 2. Statistics of measured optical attenuation (dB/km) and LWC (g/m^3) against the fog event of 18-19 Nov. 2009 in Graz.

Parameters	Visibility Range meters minutes	Attenuations (1550 nm) (dB/km) minutes	Attenuations(850 nm) (dB/km) minutes	LWC (g/m^3) minutes	PSA (cm^2/m^3) minutes
Min.	148	27.41	27.62	0.022	0
Max.	1521	110.7	154.6	0.14	551.3
Mean	317.5	71.92	89.96	0.06942	248.8
Median	265.5	74.56	90.59	0.0695	237.5
Std. dev.	160.1	17.2	25.19	0.03229	118.5
Range	1373	83.27	127	0.118	551.3

Tab. 3. Statistics of measured optical attenuation (dB/km) and other parameters against the fog event of 07 Feb. 2009 in Prague.

of optical attenuations were about 72 dB/km and 75 dB/km over 1550 nm, and 90 dB/km and 91 dB/km. For the determination of MGDSD parameters the attenuation data measured with 1550 nm only is considered, and later on, using the same computed MGDSD parameters the attenuations at 850 nm are computed for our further analysis and comparison.

4. Analysis of Computed MGDSD Parameters

This section deals with the computation of three parameters m, Λ, and N_0 of the MGDSD by employing the iterative procedure to compute against the two fog events; Graz fog event recorded on 18-19 Nov. 2009 and the Prague fog event recorded on 07 Feb. 2009. The iterations of different combinations of two MGDSD parameters m and Λ are repeated unless the residuals of the respective parameters to model are minimized.

4.1 MGDSD Parameters for Graz Fog Event of 18-19 Nov. 2009

In this section, first the performance analysis of the newly computed three MGDSD parameters m, Λ, and N_0 will be presented by comparing the measured and the computed optical attenuations (dB/km) at 950 nm, measured and computed LWC (g/m^3) and the ratio of measured and computed attenuations (dB/km) at 950 nm to the LWC (g/m^3). This will be followed by an analysis of the computed quantities and the respective three parameters of the MGDSD.

4.1.1 Performance Analysis of the Method

The DSD parameters corresponding to representative fog event of Graz are obtained by (6) and (7). Fig. 4 shows the performance analysis of the newly computed parameters of the MGDSD in terms of their computation of optical attenuations (dB/km), LWC (g/m^3) and the ratio between optical attenuations (dB/km) to the LWC (g/m^3). In the plot shown in Fig. 4(a), the optical attenuations computed are compared with the actual measured attenuations at 950 nm. A strong correlation between measured and computed optical attenuations exists as visible through R^2 test and the corresponding linear fitting applied. The equations related to linear fit along with the respective value of R^2 in case of measured and computed optical attenuations at 950 nm are

$$Y = 1.0007X - 0.0813\ , \qquad R^2 = 0.9999\ . \qquad (13)$$

Here Y represents the quantity computed, and X the quantity measured experimentally. Fig. 4(b) shows a comparison between the measured LWC (g/m^3) (from fog density) and computed values LWC (g/m^3) (using MGDSD parameters). Here, again it is evident that a very strong correlation exists between the measured and the computed values of the LWC as visible through the R^2 test and the linear fitting applied. The resultant equation in case of linear fitting with same R^2 value is

$$Y = 1.00X + 1.1284 \times 10^{-05}\ , \qquad R^2 = 0.9999\ . \qquad (14)$$

The same performance test was conducted for ratio between the measured and the computed attenuations at 950 nm and the respective LWC as shown in Fig. 4-(c). It is clearly evident that here again the behavior of newly computed MGDSD parameters is sufficiently acceptable as seen

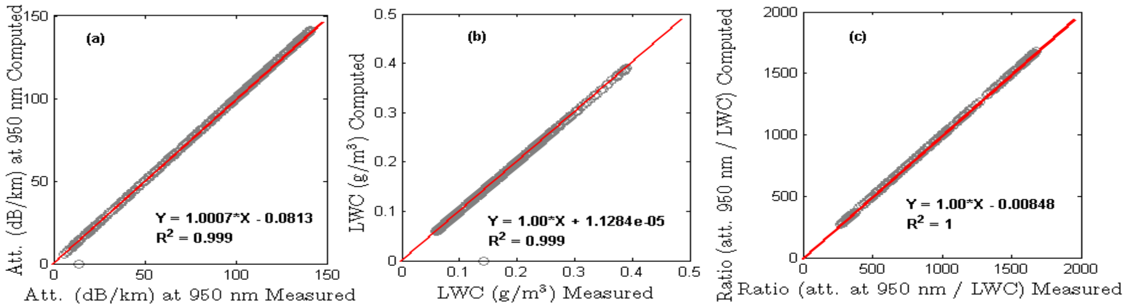

Fig. 4. Performance analysis based on newly computed MGDSD parameters while comparing measured and computed optical attenuations (dB/km) at 950 nm (a), LWC (g/m^3) (b), and ratio between optical attenuations (dB/km) at 950 nm and LWC (g/m^3) (c) against the fog event of 18-19 Nov. 2009 in Graz.

on R^2 values obtained and the linear fit equation is given below

$$Y = 1.00X - 0.0084807\,, \qquad R^2 = 1\,. \tag{15}$$

On the basis of the performance analysis presented for the radiation fog event in Graz, it is clearly evident that the proposed method performs exceptionally well in order to retrieve MGDSD parameters corresponding to attenuations (dB/km), LWC (g/m^3) and the ratio between attenuations (dB/km) and LWC (g/m^3). Since the retrieved parameters are in excellent agreement with the attenuations and LWC obtained from measurements, this method can be quite useful in the study of fog microphysics and in modeling of optical attenuations for terrestrial FSO links with radiation in fog environments.

4.1.2 Analysis of the Computed Quantities and MGDSD Parameters

In Fig. 5 the histograms of residuals of the computed quantities (attenuations at 950 nm, LWC and the ratio between attenuations at 950 nm and the corresponding LWC values computed from newly retrieved MGDSD parameters), and MGDSD parameters are presented that show the behavior of these computed quantities and the MGDSD parameters during the whole fog event of Graz. As visible through histograms of the residuals of computed optical attenuations (dB/km) at 950 nm as shown in Fig. 5(a), computed LWC (g/m^3) as shown in Fig. 5(b), and the ratio between the computed attenuations at 950 nm and the LWC values as shown in Fig. 5(c), the retrieved MGDSD parameters were quite excellent in modeling the optical attenuations, and the corresponding value of LWC for a radiation fog event. The mean and median values of the computed optical attenuations at 950 nm are 93.49 dB/km and 110.9 dB/km with a standard deviation of about 39.51 for the whole fog event. The analysis of the computed LWC values shows that the mean and median values are about 0.1465 and 0.1268 (g/m^3), respectively with a standard deviation of about 0.08157 for the whole fog event.

Similarly, by analyzing the ratio of computed attenuations at 950 nm and the computed LWC, we observe a value of 737.0 and 598.4 for the mean and median values with

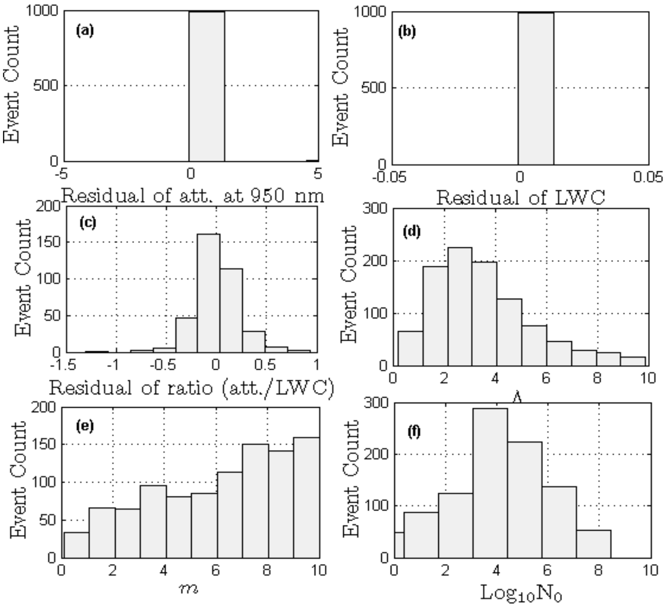

Fig. 5. Histograms of: Residuals of computed attenuations (dB/km) (a), LWC (g/m^3) (b), the ratio of optical attenuation (dB/km) at 950 nm to the LWC (g/m^3) (c), and the computed MGDSD parameters Λ (d), m (e), and $\mathrm{Log}_{10}(N_0)$ (f) against the fog event of 18-19 Nov. 2009 in Graz.

a standard deviation of about 423.3, respectively. The histograms as presented in Figs. 5(d)-(f) describe the behavior of the three MGDSD parameters (Λ, m and N_0) that are retrieved by a standard iterative procedure. The mean and median values are 3.547 and 3.2 for Λ, 6.135 and 6.6 for m, and 3.834 and 4.01 for N_0, respectively. The values of the standard deviation corresponding to this fog event are 1.935, 2.692 and 2.239 for Λ, m and N_0, respectively. An analysis of the computed MGDSD parameter m reveals that about 14.70 % of its values are below 3.0, and about 26.73 % are within the range of 3.0 and 6.0, while major portion of its values – about 58.57 % are above the value 6.0. Analysis of the the computed values, for this whole fog event, in case of second MGDSD parameter lambda (Λ) shows that about 11.62 % are below the value of 1.5, about 32.34 % are within the range between 1.5 and 3.0, while a major portion – about

56.04 % lies above the value of 3.0. Similarly, analyzing the values obtained for the third MGDSD parameter, i.e. N_0, it was observed that about 22.14 % values are below 2.37 and the remaining values – about 77.86 % are all within the range between 2.37 and 607.5.

Fig. 6 presents a comparison of the newly computed MGDSD parameters with the measured attenuation (dB/km) for this fog event through time series analysis. It is evident through this plot that all these parameters of the MGDSD have a strong correlation with the formation stage, maturity and the dissipation phase of this fog event. An increase in attenuation results in an increase in all three MGDSD parameters and vice versa. Tab. 4 summarizes the computed quantities and the three parameters of the MGDSD for this fog event.

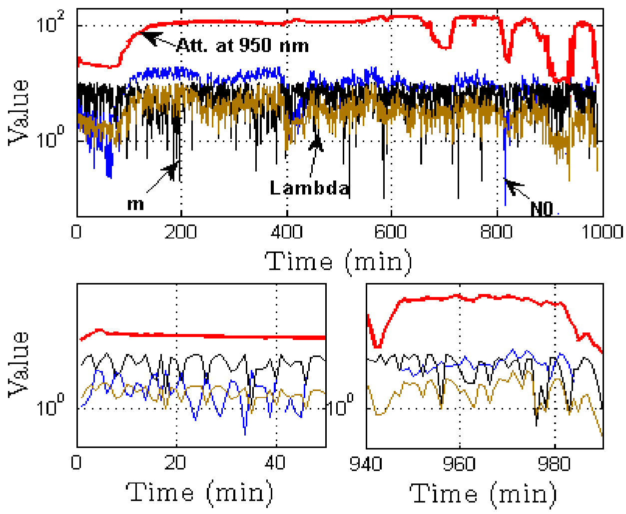

Fig. 6. Time series of measured optical attenuation (dB/km), and the corresponding predicted MGDSD parameters Λ, N_0 and m against the fog event on 18-19 Nov. 2009 in Graz.

4.2 MGDSD Parameters for Prague Fog Event of 07 Feb. 2009

In this section, first the performance analysis of the newly computed three MGDSD parameters m, Λ, and N_0 will be presented by comparing the measured and computed optical attenuations (dB/km) at 1550 nm and 850 nm, measured and computed LWC (g/m^3), the ratio of measured and computed attenuations (dB/km) at 1550 nm to the LWC (g/m^3), and the ratio of measured and computed attenuations (dB/km) at 1550 nm to attenuations (dB/km) at 850 nm. This will be followed by an analysis of the computed quantities and the respective three parameters of the MGDSD corresponding to this fog event.

4.2.1 Performance Analysis of the Method

After the retrieval of DSD parameters using (9) and (10), we analyzed the performance of the newly computed three MGDSD parameters for the case of Prague fog event recorded on 07 Feb. 2009. The linear fit equations obtained along with the respective R^2 value by the two above mentioned equations for the case of measured and computed attenuations at 1550 nm, at 850 nm, measured and computed LWC and the ratio between measured and computed attenuations at 1550 nm to the LWC are given below, respectively.

$$Y = 0.9894X + 0.7428\,, \qquad R^2 = 0.9797\,, \tag{16}$$
$$Y = 0.5636X + 13.265\,, \qquad R^2 = 0.8758\,, \tag{17}$$
$$Y = 0.9X + 0.0083\,, \qquad R^2 = 0.9345\,, \tag{18}$$
$$Y = 0.6X + 4.2 \times 10^{+02}\,, \qquad R^2 = 0.8383\,. \tag{19}$$

We observe that R^2 values, as obtained in the above mentioned four equations, are not as good as they are in case of Graz fog event, even after removing the outliers in the measured attenuations and LWC values. Upon analyzing the data, it seems that there was a slight offset in the values of LWC measured by the measuring device and so requires proper calibration. We noticed that, on average, a difference of about 1 % exists between the actual measured LWC values and the values of LWC computed by MGDSD parameters. Due to this offset the LWC measurement device measured LWC values which were about 1 % lesser than the actual values, on average, when compared with the LWC values computed using MGDSD parameters. That is why a relatively weaker correlation existed between the measured and the computed LWC values and as a consequence it affected the computed attenuation values at 850 nm and the ratio between attenuations at 1550 nm to the LWC, as evident through R^2 test values obtained in the above mentioned equations.

Due to the above mentioned problem, we again retrieved DSD parameters for this selected fog event (by the same iterative procedure) but now from the ratio of attenuations at 1550 nm to 850 nm by using (11) and (12) given above. The linear fitting equations, along with the respective R^2 values, obtained now for optical attenuations at 1550 nm, attenuations at 850 nm and the ratio between attenuations at 1550 nm to 850 nm are given below in order.

$$Y = 1.00X - 2.8 \times 10^{-14}\,, \qquad R^2 = 1.0\,, \tag{20}$$
$$Y = 1.00X + 0.00043\,, \qquad R^2 = 1.0\,, \tag{21}$$
$$Y = 1.00X - 0.0001\,, \qquad R^2 = 1.0\,. \tag{22}$$

We observe now a very good correlation between the measured and computed attenuation values and their ratio, duly evident by respective R^2 values obtained and the corresponding linear fit applied over the scatter plots between measured and computed attenuations at 1550 nm, at 850 nm, and the ratio between attenuations at 1550 nm to 850 nm as shown in plots shown in Figs. 7(a)-(c). Thus, in general, it is quite evident that the proposed method based on standard iterative technique performs exceptionally well towards the retrieval of MGDSD parameters from the optical attenuations (dB/km) and the LWC (g/m^3). The proposed method can be quite useful towards the study of fog microphysics at a particular place and in modeling the optical attenuations for terrestrial FSO links in different fog environments.

Parameters	Attenuations (dB/km) minutes	LWC (g/m^3) minutes	Attenuations / LWC (dB/km)/(g/m^3)) minutes	Λ μm^{-1}	N_0 $cm^{-3} \cdot \mu m^{-1}$	m
Min.	0	0	55.39	0.2	-5.0	0.1
Max.	140.99	0.3904	2111	9.9	8.44	10
Mean	93.49	0.1465	737	3.547	3.834	6.135
Median	110.9	0.1268	598.4	3.2	4.01	6.6
Std. dev.	39.51	0.08157	423.3	1.935	2.239	2.692
Range	141	0.3904	2055	9.7	13.44	9.9

Tab. 4. Statistics of computed optical attenuation (dB/km), LWC (g/m^3) and corresponding MGDSD parameters against the fog event of 18-19 Nov. 2009 in Graz.

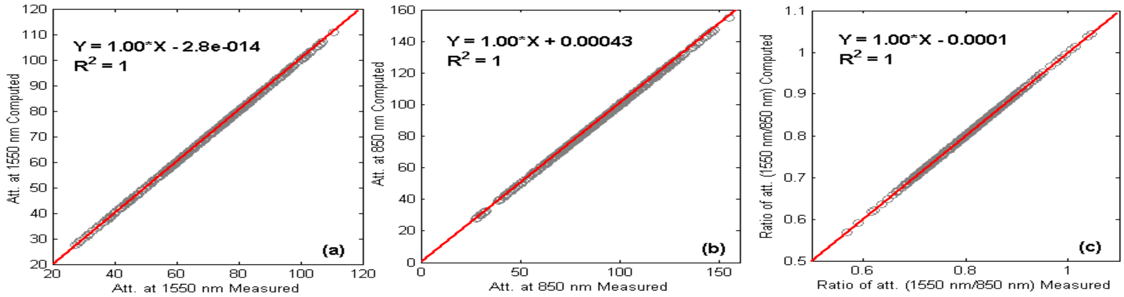

Fig. 7. Performance analysis based on newly computed MGDSD parameters while comparing measured and computed optical attenuations (dB/km) at 1550 nm (a), measured and computed attenuations (dB/km) at 850 nm (b), and the ratio between measured and computed attenuations at 1550 nm and 850 nm (c), against the fog event of 07 Feb. 2009 in Prague.

4.2.2 Analysis of the Computed Quantities and MGDSD Parameters

In order to perform analysis of the computed parameters of MGDSD for the Prague fog event case, histograms of residuals of actual measured and computed quantities (optical attenuations (dB/km) at 1550 nm, at 850 nm, the ratio of measured and computed attenuations at 1550 nm to 850 nm), and the three computed MGDSD parameters m, Λ and N_0 (using standard iterative procedure) are shown in Fig. 8. As evident through the histograms of residuals of measured and computed optical attenuations at 1550 nm as shown in Fig. 8(a), residual of measured and computed attenuations at 850 nm as shown in Fig. 8(b), and the residual of measured and computed ratio of attenuations (1550 nm/850 nm) as shown in Fig. 8(c), the retrieved MGDSD parameters are quite accurate in modeling the optical attenuations for this representative radiation fog event. Excellent results are achieved for the selected fog event in Prague as were achieved in the case of representative fog event of Graz, and thus the proposed method is accurate enough to model the optical attenuations in radiation fog environments.

An analysis of the computed optical attenuations reveals that the maximum attenuation value, for the 1550 nm wavelength, using retrieved MGDSD parameters is about 110.7 dB/km, whereas the mean and median values are about 71.92 and 74.56 dB/km respectively, with a standard deviation of about 17.2. In case of the computed attenuations at 850 nm, the maximum value reached is about 154.5 dB/km and the mean and median values are about

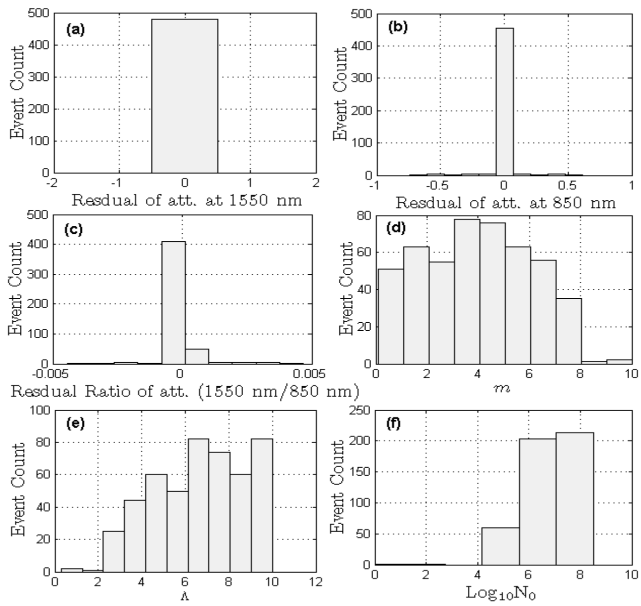

Fig. 8. Histograms of: residuals of computed optical attenuations (dB/km) at 1550 nm (a), residuals of computed optical attenuations (dB/km) at 850 nm (b), ratio between Attenuations (dB/km) at 1550 nm and 850 nm (c), MGDSD parameters m (d), Λ (e), and the Log10N_0 (f) retrieved by taking ratio of attenuations at 1550 nm to 850 nm, against the fog event of 07 Feb. 2009 in Prague.

89.96 and 90.59 dB/km with a standard deviation of about 25.19. The histograms as shown in Figs. 8(d) to 8(f) reveal the behavior of the three computed MGDSD retrieved by iterative procedure against this representative fog event. The mean and median values are 3.933 and 3.9 for m, 6.669 and 6.8 for Λ, and 6.81 and 6.918 in case of Log10(N_0). A further analysis of these three parameters shows that in case of parameter m, about 35.20 % values are below 3.0, about 45.20 % are within the range 3.0 to 6.0 and about 19.58 % are above the value of 6.0. Similarly, analyzing the values of Λ retrieved against the whole fog event shows that about 0.4 % are below the limit of 1.5, about 2.5 % are within the range 1.5 to 3.0, while a major portion of the computed values for Λ, about 97.08 %, is above the value of 3.0. A similar analysis of the 3rd parameter N_0 reveals that about 0.833 % are below the value of 2.37 and the remaining values about 99.16 % lie within the range between 2.37 and 607.5.

In order to study the behavior of these three computed MGDSD parameters during the whole fog event, Fig. 9 shows the time series of plot between the measured attenuation and the three retrieved MGDSD parameters.

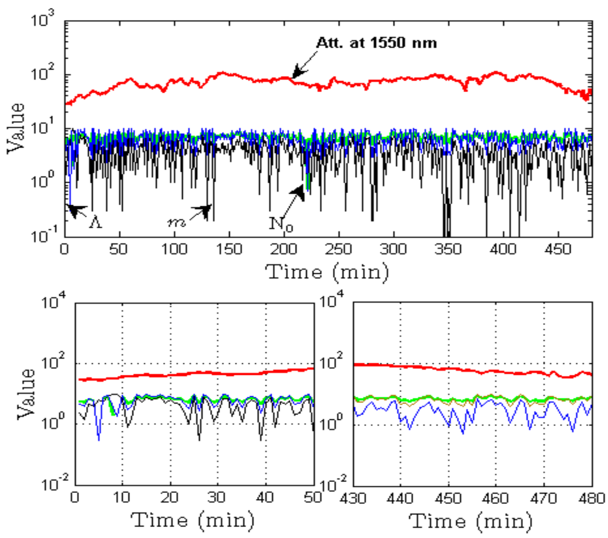

Fig. 9. Time series of measured optical attenuation (dB/km), and the corresponding computed MGDSD parameters Λ, N_0 and m against the fog event of 07 Feb. 2009 in Prague.

During the early phase of the representative fog event (fog formation stage) and the last phase (fog dissipation stage) of the fog event, there exist a strong correlation between all three MGDSD parameters and the measured attenuations at 1550 nm. With the increase in attenuation, a corresponding increase in all these three parameters of MGDSD is observed and vice versa. During the maturity stage of the fog event, fog attenuations are in high correlation with the parameter N_0 as compared to the other two parameters.

This time series behavior of the three parameters of MGDSD is exactly the same as observed in case of their influence for the Graz fog event case. Thus it can be safely concluded that the fog distributions at the two locations Graz and Prague are closely the same and they behave in the similar fashion during different phases of the continental fog event. Moreover, it may also be concluded that the MGDSD parameters for similar kind of fog environments may have the same behavior of DSD parameters. The Table 5 summarizes the computed quantities and the three parameters of MGDSD for the representative fog event of Prague.

5. Conclusions

In order to study the characteristics of fog using the free-space optical wireless links, we adopted Maitra and Gibbins technique, that employs iterative procedure to compute three distribution parameters of a modified gamma distribution. For the mentioned purpose, two individual radiation fog events were analyzed, one each in Graz and Prague, mainly to indicate the effectiveness of the technique adopted to obtain near-instantaneous radiation fog droplet size distribution in terms of modified gamma distribution. Based on the performance analysis of the mentioned techniques, it was observed that this technique is quite useful in terms of simplicity and efficiency and yields excellent results while computing optimal parameters for the MGDSD. Thethree newly computed parameters of the modified gamma distribution, both for Graz and Prague locations, are consistent with the previously computed distribution parameters existing in the literature and are in a very strong correlation with the actual measured quantities like optical attenuations and the LWC. Hence it may safely be inferred that the behavior of computed MGDSD parameters in both mentioned locations is quite similar which implies that the radiation fog behavior at both locations is somewhat similar. Hence, also proved that this techniques is very useful in computing the distribution parameters at a particular location for FSO links in situations where the measurement of most of the microphysical parameters is not feasible besides measurement of optical attenuations.

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Parameters	**Att. at 1550 nm (dB/km) minutes**	**Att. at 850 nm (dB/km) minutes**	**Ratio of Att. (1550 nm / 850 nm) minutes**	**Λ μm^{-1}**	**N_0 cm$^{-3} \cdot \mu$m^{-1}**	**m**
Min.	27.41	27.62	0.5693	0.3	-5.953	0.1
Max.	110.7	154.6	1.045	10.0	8.515	10.0
Mean	71.92	89.96	0.8124	6.669	6.81	3.933
Median	74.56	90.59	0.8194	6.8	6.918	3.9
Std. dev.	17.20	25.19	0.07235	2.071	1.221	2.128
Range	83.27	127.0	0.4753	9.7	14.47	9.9

Tab. 5. Statistics of computed optical attenuation (dB/km), LWC (g/m^3) and corresponding MGDSD parameters against the fog event of 07 Feb. 2009 in Prague.

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