

# Evolution Strategies in the Multipoint Connections Routing

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**Abstract.** *Routing of multipoint connections plays an important role in final cost and quality of a found connection. New algorithms with better results are still searched. In this paper, a possibility of using the evolution strategies (ES) for routing is presented. Quality of found connection is evaluated from the view of final cost and time spent on a searching procedure.*

*First, parametrical analysis of results of the ES are discussed and compared with the Prim's algorithm, which was chosen as a representative of the deterministic routing algorithms.*

*Second, ways for improving the ES are suggested and implemented. The obtained results are reviewed. The main improvements are specified and discussed in conclusion.*

## Keywords

Evolution strategies (ES), routing of multipoint connections, optimization.

## 1. Introduction

Routing of multipoint connections in telecommunication networks means building connections among at least three users or nodes. Requirements for the connection are for example maximum delay, used bandwidth or efficiency of routing in using less loaded parts of network. Modified algorithms for single connection routing are usually used for routing of multipoint connections.

A lot of algorithms were designed for routing. They can be divided into static and dynamic, centralized and distributed or local and global [1].

In case of static routing, a new user cannot be added into a connection and an existing user cannot be removed after a connection was established. Centralized algorithms are run by one central node, which means that the whole network can fail in case of a central node malfunction. Global algorithms need information about every node and connection in a network.

Each of the mentioned deterministic routing algorithms has some weaknesses and other algorithms for routing are searched. The paper presents routing of a multi-

point connection by the ES as a possible new method and analyses its results.

## 2. Prim's Algorithm

The Prim's algorithm [2] was chosen as a representative of deterministic algorithms. It is well known from the graph theory. The algorithm how was discovered by Czech mathematician Jarník in 1930, later independently by computer scientist Robert C. Prim in 1957 and rediscovered by Dijkstra in 1959.

The Prim's algorithm finds a minimum-cost spanning tree of an edge-weighted, connected, undirected graph. The algorithm constructs the minimum-cost spanning tree of a graph by selecting edges from the graph one-by-one and adding those edges to the spanning tree. This algorithm is usually used for single routing, but it can be used for routing of multipoint connections with few modifications.

A network has to be presented as a graph, where nodes are vertices and links between the nodes are represented as edges. Cost of the edges evaluates the current load of a link in a network.

For the purposes of multipoint connections, the graphic model of the network has to be transformed into a sub-graph of the shortest paths among target users by Dijkstra algorithm [3]. Then the Prim's algorithm can be applied to receive the minimum-cost spanning tree to interconnect target nodes. The last step is a reverse transformation to the origin network.

## 3. Evolution Strategies

The evolution strategies [4] (ES) were developed at the Berlin Technical University by Ingo Rechenberg and Hans Peter Schwefel in early 1960's. The main idea of this iterative method is to simulate rules of natural selection – only the best individuals will survive to become parents in the next generation.

Approaches based on evolution strategies can be used for simulation of different processes in many fields, which can be expressed by optimizing function, like for example [5].

The process of creating offspring is based on mutation, which can be calculated as follows:

$$\mathbf{x}' = \mathbf{x} + \mathbf{N}(0, \sigma) \quad (1)$$

where  $\mathbf{x}'$  is a vector of a new solution,  $\mathbf{x}$  is a vector of the actual solution and  $\mathbf{N}(0, \sigma)$  is a vector of independent random variables with zero mean value and standard deviation  $\sigma$ . The density distribution function of a random variable for normal Gaussian distribution is:

$$f(x) = \sqrt{\frac{1}{2\pi\sigma^2}} e^{-0,5(x-\mu)^2 / \sigma^2} \quad (2)$$

where  $\mu$  is the mean value of the variable  $x$  and  $\sigma$  is standard deviation.

Another method of creating a new individual from parents is a process of recombination (crossover). In general, recombination means exchange of partial information between parents. Recombination produces only one new solution. Different methods of recombination, between two or more parents, are used in ES.

The ES can be divided into PLUS and COMMA strategies. The main difference between them is in the logic of selection of individuals for the following generation. In the PLUS ES a new generation consists of the best solutions from offspring and parents, in the COMMA ES a new generation is chosen only from offspring.

The PLUS ES are usually less time-consuming than the COMMA ES, but the PLUS ES can bog in a local extreme and never reach the global one.

A model of a telecommunication network represented by an edge-weighted, connected, undirected graph, which is shown in Fig. 1, was used for simulation purposes.

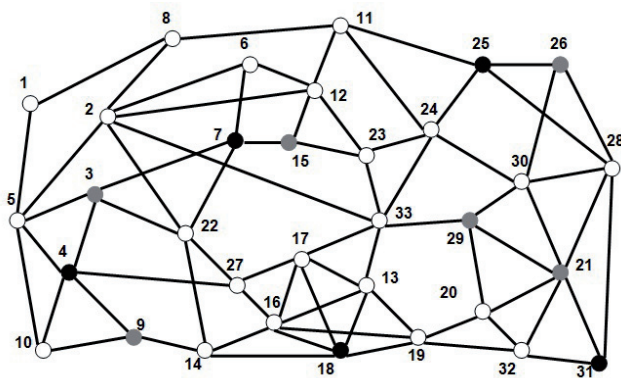


Fig. 1. Model of the telecommunication network.

This network consists of 70 links interconnecting 33 nodes. Every link, represented by an edge of the graph, has defined a cost, which determines the actual state of the link. The cost of the link [6] can be influenced by its capacity, delays, jitter or throughput considering error rate [7]. A higher value of the cost means the link is less lucrative for a final connection.

Two simulation models were used for every method of routing. In the first model, the target group consists of

nodes 3, 9, 15, 21, 26, 29 (grey colored nodes in Fig. 1) and for the second scenario of nodes 4, 7, 18, 25, 31 (black colored nodes in Fig. 1). These sets were chosen with the aim of creating a target group with many ways of interconnecting individual nodes.

In the implemented ES algorithm, the parent's and the offspring's trees are represented by Steiner vertices [8] to simplify computation. The initial generation for computation consists of random trees. These trees are searched only with one condition – they have to interconnect all the selected nodes and create a connected tree.

The parametrical and time dependences were simulated for the ES. For all parametrical dependences, the count of generations is set to 30, the number of parents moves in range from 2 to 10 with step 2, and offspring obtain values from 10 to 50 with step 4. Each measurement was performed 10 times and the average values are displayed in the following graphs. For time dependences, the number of generation is 30, the count of offspring is set to 50 and parent's value is set to 4 to ensure higher parallelism. For time dependences, only one measurement is made, which gives only one example of evolution among generations, because the process of the ES is very stochastic and progress of convergence to the best solution may vary.

The Prim's algorithm was simulated for both scenarios to obtain values of the currently used routing algorithm with a purpose of gaining values for evaluation of the ES. For the first simulation scenario, the final cost is 192 and for the second it is 227. Computing time for both scenarios is less than 1 ms.

### 3.1 ES Parametrical Dependences

The parametrical dependences for the first scenario can be seen in Fig. 2 for the COMMA ES and in Fig. 3 for the PLUS ES.

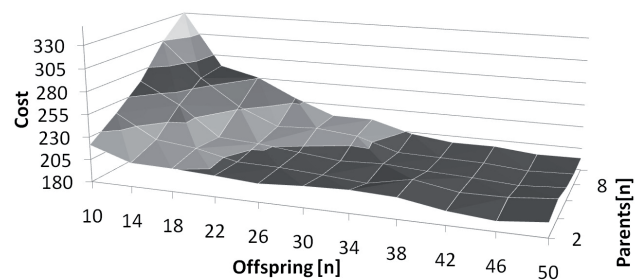


Fig. 2. ES COMMA parametrical dependence for the first simulation scenario.

The cost of the best tree in generations reaches values from 190 to 331 for the COMMA ES and from 190 to 210 for the PLUS ES. The ES PLUS converge to the final tree with cost 190 faster, the range of the costs for this method is narrower, as a result of choosing individuals for a next generation from parents and offspring. The surface of the ES COMMA graph is smoother, because the change of the tree's cost through a generation was slow. In the COMMA

ES the parents for a next generation are chosen only from the offspring, and better solution is sometimes discarded due to fact that strong parents may have weak offspring.

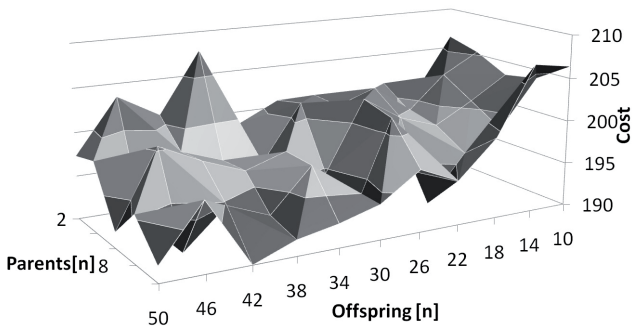


Fig. 3. ES PLUS parametrical dependence for the first simulation scenario.

Fig. 4 and Fig. 5 show the parametrical dependences for the second simulation model.

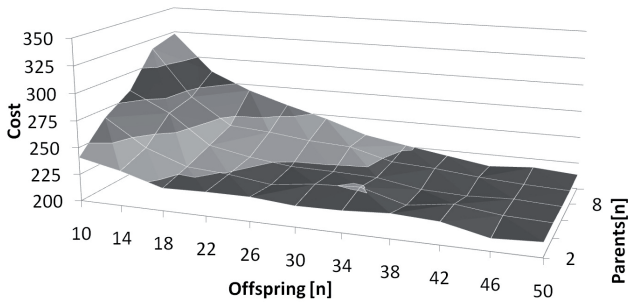


Fig. 4. ES COMMA parametrical dependence for the second simulation scenario.

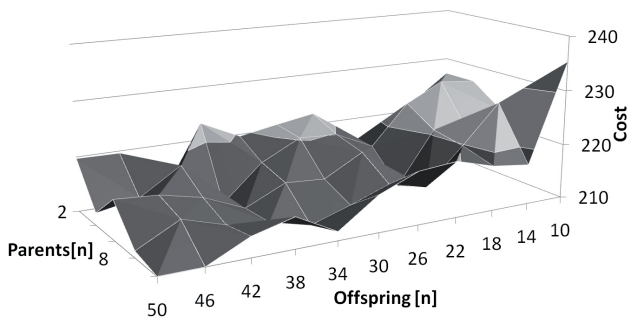


Fig. 5. ES PLUS parametrical dependence for the second simulation scenario.

The range of the costs for the second simulation scenario for the COMMA ES is from 202 to 330 and for the PLUS ES it is from 210 to 235. The range of the tree's costs is narrower again for the PLUS ES, but the best final cost 202 was achieved by the COMMA ES. The PLUS ES did not reach this value within 30 generations. This case shows bogging in a local extreme. When a strong parent is located near a local extreme, the parent is chosen for every next generation and forces the PLUS ES algorithm to search a narrow set of solutions only.

The cost of the final tree is better for both models and methods in comparison to the Prim's results. ES were able to find a tree with better cost, because Prim's algorithm selects the edges with the minimum cost to create spanning tree. There exist situations when ES select the edge with a higher cost to interconnect two nodes, while Prim's algorithm uses two edges with minimum cost. The sum of edges selected by Prim's algorithm is higher than the one chosen by ES to interconnect nodes directly.

### 3.2 ES Time Dependences

Dependences for both ES methods are shown in Fig. 6 and Fig. 7.

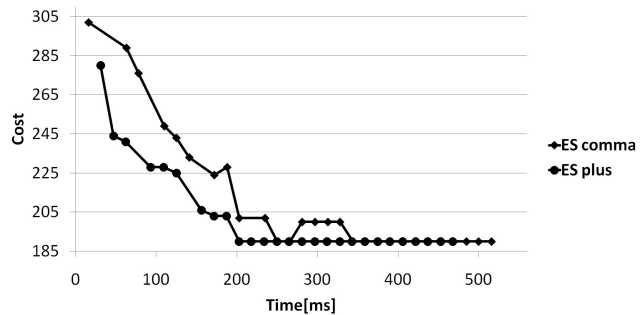


Fig. 6. Time dependences for the first simulation scenario.

For the first simulation scenario, the PLUS ES needed about 200 ms to reach better final cost than the Prim's algorithm. The COMMA ES achieved this cost of 190 in 250 ms.

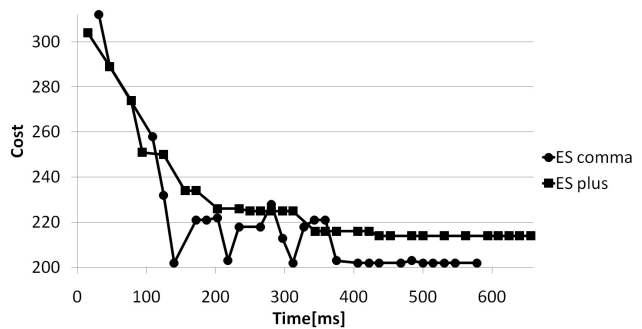


Fig. 7. Time dependences for the second simulation scenario.

For the second simulation model, the COMMA ES achieved better cost according to the Prim's algorithm within the time of about 130 ms. After the time, algorithm of the COMMA ES returned to worse values few times. The PLUS ES needed more than 300 ms to obtain better value than Prim's algorithm and never reached the cost of the COMMA ES.

This simulation showed that the ES can overcome the cost of the Prim's final tree, but the computation time is too high. For purposes of a real use of the ES for routing, time features must be improved.

### 4. Modifications of ES

An idea of improving time features of the ES was based on using trees found by the Prim's algorithm as an initial generation for the ES. Because edges represent Prim's trees, it was necessary to change the whole logic of trees representation in the ES algorithm. The tree representation was changed from Steiner vertices to edges. For these purposes, a tree searching algorithm and mutation and recombination methods were designed. Implementation and few simulations showed that the modification is too computation and time demanding.

These findings led to a change and only the solution tree determining algorithm is used from the implemented procedures. The algorithm based on a recursion seeks possible ways of interconnecting chosen nodes. The maximum possible degree of randomness was brought to the algorithm to ensure diverse found trees.

The presented modifications are based on a different composition of the initial solution set. Several sets were simulated and compared.

#### 4.1 ESH Dependences

A modification, where the initial set of trees is created by a designed trees finding algorithm represented by edges, was named ESH. Trees are found and then transformed to a representation by Steiner vertices. So built initial set should force the ES to search a wide area of solutions.

The parametrical dependences for the first simulation scenario are shown in Fig. 8 for the COMMA and in Fig. 9 for the PLUS ESH.

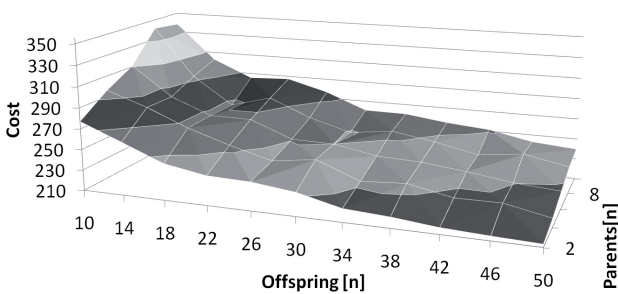


Fig. 8. COMMA ESH parametrical dependence for the first simulation scenario.

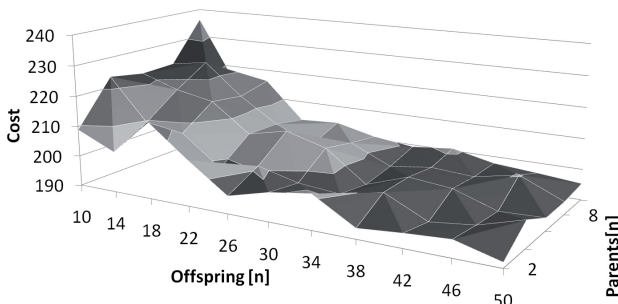


Fig. 9. PLUS ESH parametrical dependence for the first simulation scenario.

The range of costs for the COMMA ESH is from 210 to 345, for the PLUS ESH from 192 to 240. Better result is achieved by the PLUS ESH, but its best cost 192 is worse than the cost 190 reached by the PLUS ESH.

Time dependences for both methods and the first simulation scenario can be seen in Fig. 10.

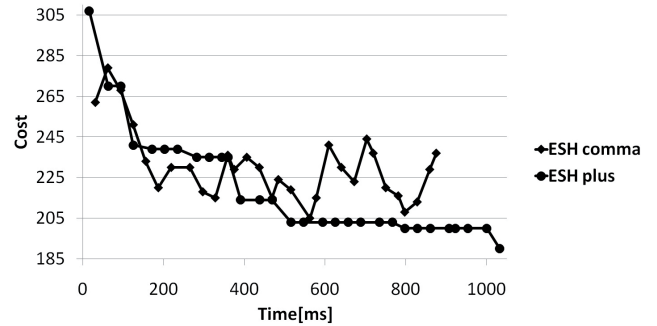


Fig. 10. ESH time dependences for the first simulation scenario.

The time dependence shows the PLUS ESH are better in comparison to the COMMA ESH, but they are slower than the ES. The COMMA ESH algorithm returned back to worse costs in new generations repetitively, what is visible from the graph.

Fig. 11 and Fig. 12 show the parametrical dependence for the COMMA and the PLUS ESH for the second simulation models. The time dependence is in Fig. 13.

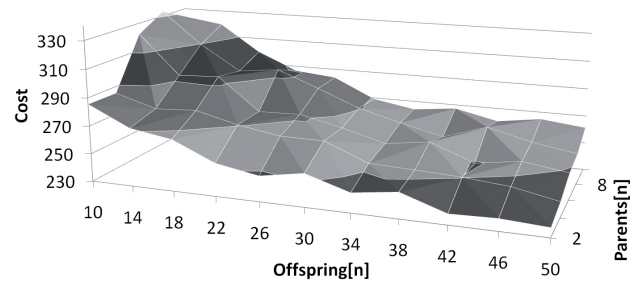


Fig. 11. COMMA ESH parametrical dependence for the second simulation scenario.

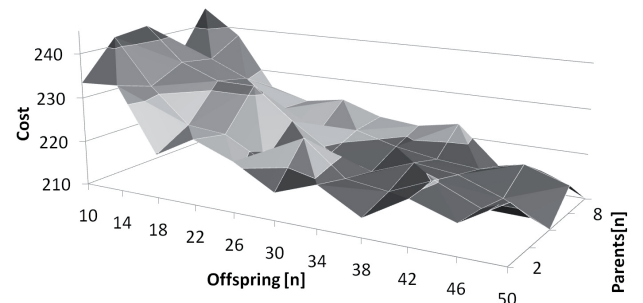


Fig. 12. PLUS ESH parametrical dependence for the second simulation scenario.

The achieved values of the final cost are worse than the ES results, similar to the first simulation model. For the COMMA ESH the range of values is from 230 to 335 and

for the PLUS ESH algorithm from 210 to 245. The PLUS ESH reached a better cost in a better time as it can be seen from the time dependence in Fig. 13.

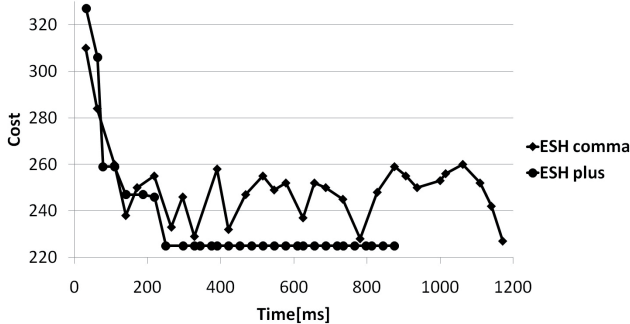


Fig. 13. ESH time dependences for the second simulation scenario.

For both simulation scenarios, the ESH achieved worse final cost than the ES. It is caused by diverse initial generations. The ESH were forced to search wide area of solutions and 30 simulated generations were not enough to reach the optimum tree. But this initial generation helps to overcome bogging of the PLUS ES in a local extreme, as we can see from dependences for the second simulation model, where the PLUS ESH were better than the COMMA ESH.

### 4.2 Prim – ESH Dependences

In the previous modification the diverse initial set led to a deterioration of the ES algorithm’s result, an initial generation built of strong solution should improve the original ES according to theoretical assumptions.

A Prim-ESH modification uses the initial generation formed by a Prim’s tree. The first step is to compute Prim’s algorithm and transform the found tree into a representation by Steiner vertices. The initial set of solution consists of the copies of Prim’s tree with cost 192 for the first simulation scenario and cost 227 for the second simulation scenario.

Parametrical dependences for the first simulation are displayed in Fig. 14 and Fig. 15, while Fig. 16 shows time dependences.

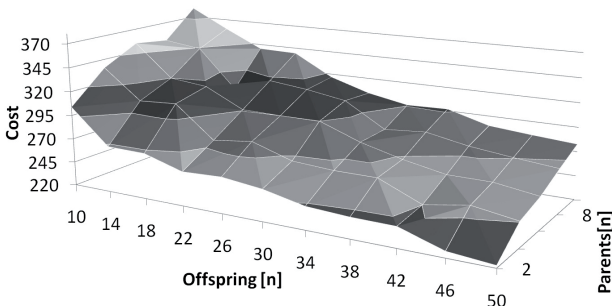


Fig. 14. The COMMA Prim-ESH parametrical dependence for the first simulation scenario.

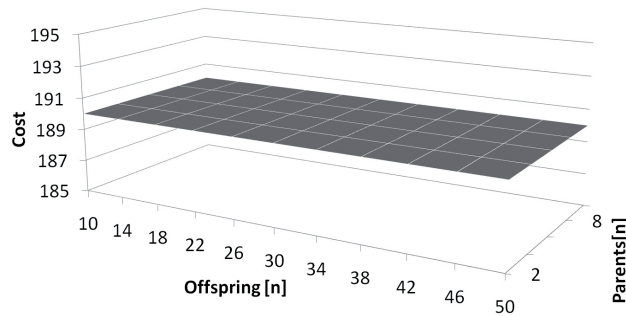


Fig. 15. The PLUS Prim-ESH parametrical dependence for the first simulation scenario.

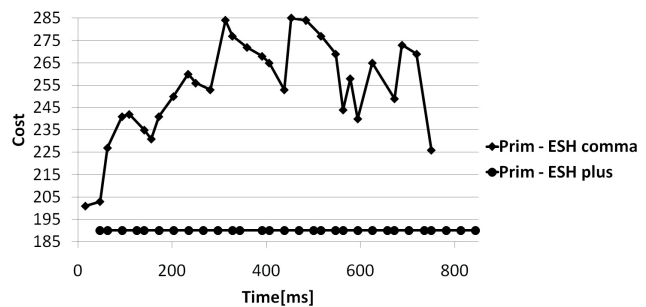


Fig. 16. The Prim-ESH time dependences for the first simulation scenario.

The COMMA Prim-ESH modification deteriorated Prim’s solution as shown in Fig. 14. The cost of final tree in the range contains values from 220 to 380; the cost of the tree in initial generation was 192. This deterioration is caused by choosing solutions to next generation from offspring only. Fig. 15 displays parametrical dependence for PLUS Prim-ESH strategies. As we can see, the final cost of the tree is stable during all generations and the surface of the graph is absolutely smooth. The PLUS Prim-ESH strategies improved Prim’s solution in the first generation and held this value for the whole simulation time, because a next generation is chosen from both parents and offspring.

The time comparison of the PLUS and the COMMA Prim-ESH modification is shown in Fig. 16. Time dependence for the COMMA strategies contains a lot of fluctuation in cost that led to its bad result. The PLUS strategies held the static value of 190 from the first simulation. This value was reached in the first generation in the time of about 30 ms.

The results for the second simulation scenario are very similar to the first one as we can see from the graphs in Fig. 17, Fig. 18 and Fig. 19.

The deterioration of Prim’s result by the COMMA Prim-ESH strategies is visible from Fig. 17. The range of the tree costs takes values from 245 to 390. The PLUS strategies improved Prim’s tree cost from 227 to the final cost of 214. Results from a time point of view are displayed in Fig. 19. The PLUS strategies improved Prim’s solution in the 6<sup>th</sup> generation in time of about 200 ms and held this value until the end of the simulation. The COMMA Prim-

ESH strategies reached better tree cost during the simulation once, but this cost was deteriorated in a later generation.

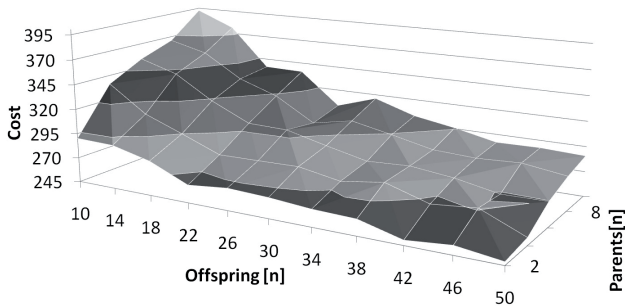


Fig. 17. The COMMA Prim-ESH parametrical dependence for the second simulation scenario.

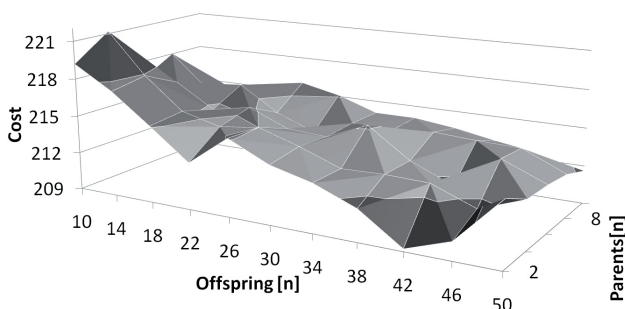


Fig. 18. The PLUS Prim-ESH parametrical dependence for the second simulation scenario.

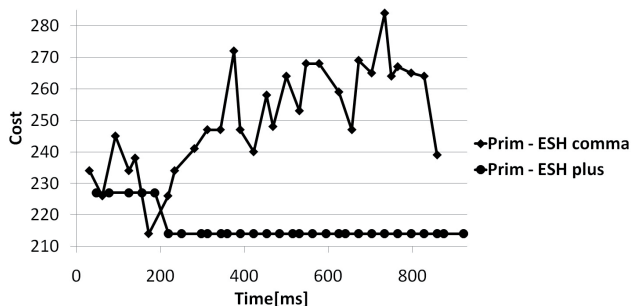


Fig. 19. The Prim-ESH time dependences for the second simulation scenario.

The PLUS strategies showed better results for both simulation scenarios. These values were obtained in a quite pleasant time of 30 ms for the first scenario and 200 ms for the second one. We may say the initial generation built of strong solution improved the result of the ES.

### 4.3 ESH with Combined Inputs Dependences

The next idea of modification was to build an initial set of solutions from Prim's trees and trees found by the ESH in various ratios. This modification was simulated only for the PLUS strategies according to the results of the previous algorithm. We have run simulations for inputs of Prim's tree: tree found by the ESH in ratios 3:1, 2:1, 1:1, 1:2 and 1:3.

For the first simulation scenario, the parametrical dependence is the same as the parametrical dependence for the PLUS Prim-ESH strategies in Fig. 15. The time dependence is very similar to the time dependence for the PLUS Prim-ESH strategies in Fig. 16 for all the simulated ratios.

The same is true for the second simulation scenario. Differences according to the time dependence for the COMMA Prim-ESH strategies are in the number of generations, which holds the cost of Prim's tree.

Ratio 2:1 requires our attention, because the cost of the final tree was improved to the value of 202 as it is shown in Fig. 20.

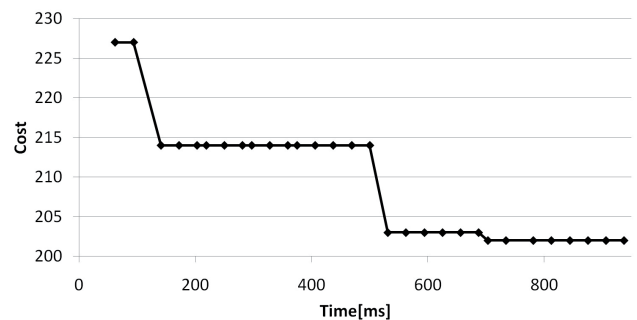


Fig. 20. The combined PLUS ESH 2:1 time dependence for the second simulation scenario.

## 5. Conclusion

In this paper, three modifications of evolution strategies for the multipoint connections routing were described. These modifications were designed and implemented to improve computation time of algorithm. The goal was to design an algorithm that enables the use of the evolution strategies for routing in real time traffic (this approach can be extended for the presence of self-similar traffic [9] using the appropriate model).

During every simulation thirty generations were created and the final ones were compared. The obtained results proved the theoretical assumptions – the PLUS strategies usually give a better result than the COMMA ones, but there is a risk of bogging in a local extreme. The first simulated modification helped to overcome the weakness, but the algorithm is very slow. The second modification introduces a hypothesis that a good initial generation should make an algorithm more efficient. This procedure required a six times shorter period of time for computation than the original one (time needed for finding the final tree for the original algorithm was about 200 ms, for designed modification only 30 ms). The last modification gave very similar results to the second one.

These modifications made routing by the evolution strategies less time-consuming, but the answer if we can use them in real time networks requires more experiments with larger simulation network. But we definitely can say that the PLUS evolution strategies are more suitable for the

mentioned modification, because they respond to changes of the initial solution's set more sensitively.

For future work we would like to design the way how to determine that the solution of the current generation is optimal and stop the algorithm for searching connection among chosen nodes. Next we would like to implement this kind of routing into existing protocols and run simulations to verify results of both mentioned approaches.

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