Ternary Chaotic Pulse Compression Sequences

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Abstract. *In this paper method available for generating ternary sequences is discussed. These sequences are useful in many applications but specifically in synchronization of block codes and pulse compression in radar. The ternary sequences are derived from chaotic maps. It is feasible to achieve simultaneously superior performances in detection range and range resolution using the proposed ternary sequences. The properties of these sequences like autocorrelation function, Peak Side Lobe Ratio (PSLR), ambiguity diagram and performance under AWGN noise background has been studied. The generation of these sequences is much simpler, and the available number of sequences is virtually infinite and not limited by the length of the sequence.*

Keywords

Pulse compression, autocorrelation, peak side lobe ratio.

1. Introduction

Pulse compression is a technique that plays an important role in various fields like radar, sonar and spread spectrum communication to achieve the high transmit energy of a long pulse while preserving the range resolution of a short pulse. It has been extensively used in operational radar systems since 1950s to overcome the practical problem of extending the operating radar range while maintaining the required range accuracy and resolution. Various waveforms have been used for this purpose, including the linear FM signal, the nonlinear FM signal, and biphase codes [1], [2], [3]. The code or sequence is used to describe the phases of the individual sub pulses of a phase coded waveform [1]. Each has certain desirable properties and the choice often depends on the application.

The performance of range resolution radar depends on the autocorrelation pattern of the coded waveform which is nothing but the matched filter compressed output. The binary sequences having ±1 as elements find more importance in pulse compression as they have good aperiodic autocorrelation function and ideal energy efficiency [4]. The Energy efficiency is defined as the ratio of energy in the actual energy in the sequence to the energy if every element in the sequence had the maximum amplitude. The Binary sequences can be easily generated, processed and stored in digital circuitry. But the limitation comes when longer length sequences with lower Peak Side Lobe Ratio (PSLR) are needed. The Peak Side Lobe Ratio is defined from the autocorrelation pattern as the ratio of the peak side lobe amplitude to the main lobe peak amplitude and is expressed in decibels. It was proved that the binary barker sequences exists only up to a length of 13 with a PSLR of -22.3 dB [5].But in some applications sequences with lower PSLR are required. To achieve this, Boehmer [6], Linder [7], Rao and Reddy [8] have obtained longer length binary codes violating the Barker conditions. But as the length increases, the PSLR did not reduce much. Therefore it became necessary to switch over from biphase to either polyphase or multilevel sequences.

The multilevel or non binary sequences have elements of unequal magnitude. Hence their energy efficiency is less than unity. The sequences having 0, ±1, ±2 as alphabets are known as quinquenary sequences and those with elements 0, ±1 are known as ternary sequences. Moharir has shown that the ternary Barker sequences exist for lengths greater than 13 [9], [10].These ternary Barker sequences are obtained with the help of the sieve known as terminal admissibility. It has been proved that the PSLR obtained reduce faster than that of binary sequences as the length increases. There are also many other sieves which have been already developed for the optimum code generation [11], [12].

In this paper a unique class of radar signals called as chaotic ternary phase sequences with good correlation properties is described. The use of chaotic sequences in spread spectrum communications is well documented [16]. A binary phase code generated using chaotic logistic map used to achieve a low PSLR was reported earlier [13]. Similar concept has been extended to the generation of good ternary sequences using different chaotic maps and their performances are compared. The method discussed here has an advantage over the previous methods already available for ternary sequence generation. Here, it is possible to generate sequences at larger lengths easily as it is not an exhaustive search method.

2. Sequence Generation

2.1 Chaotic Maps

The use of chaotic signals in radar has received a lot of attention in the recent years because they behave like noise, have a wide band and are easy to generate [14]. A Chaotic dynamical system is a bounded, deterministic system that exhibits random-like behavior through its sensitive dependence on its initial conditions. Since, in practice initial conditions can never be specified with infinite precision, the behavior of a chaotic system is unpredictable and therefore noise like. Chaotic Sequences can be generated using different types of chaotic maps. Some of the maps are Henon Map, Lorenz Map, Logistic Map, Improved logistic map, tent map and cubic map. In this work an attempt is made to generate ternary sequences using these maps and their properties at different lengths are studied [15], [16], [17].

2.1.1 The Logistic-map

A great deal of chaotic behavior can be described by one, simple, fairly innocuous looking equation, the logistic map. This is written as

$$x_{n+1} = \mu x_n (1 - x_n) \tag{1}$$

where μ is some constant and is called the bifurcation parameter $[\mu \in (0, 4)]$ and $[x \in (0, 1)]$. The logistic equation is a key example that shows behavior that is stable, periodic, and chaotic. The behavior depends on the value of μ in the equation. When $\mu = 4$ the logistic map exhibits chaotic behavior.

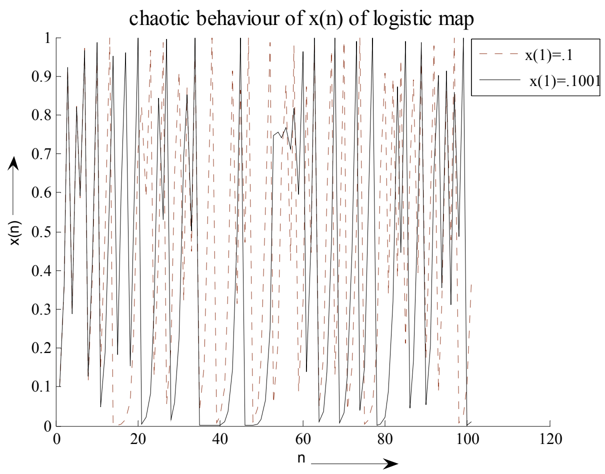

Fig. 1. Chaotic behavior of logistic map for $x_0 = 0.1$ and $x_0 = 0.1001$.

The logistic-map sequences are inherently deterministic however a tiny disturbance in the initial condition x_0 gives rise to a tremendous difference in the outcome [13]. Fig. 1 shows two such sequences for $\mu = 4$ with $x_0 = 0.1$ and 0.1001 respectively. The small difference leads to different waveforms after a few iterates.

2.1.2 The Cubic Map

This map is described by the following equation

$$x_{n+1} = 4(x_n)^3 - 3x_n \text{ where } x \in (-1,1). \tag{2}$$

This equation shows chaotic behavior for an initial value of x_0 varying from 0 to 1. When $x_0 > 1$ as n tends to infinity, x_n also tends to infinity.

2.1.3 The Improved Logistic Map

This map is governed by the equation

$$x_{n+1} = 1 - 2(x_n)^2 \text{ where } x \in (-1,1). \tag{3}$$

Like cubic map, this equation shows chaotic behavior for an initial value of x_0 varying from 0 to 1. When $x_0 > 1$ as n tends to infinity, x_n also tends to infinity.

2.1.4 The Tent Map

Tent map is a discrete-time piecewise-affine input/output characteristic curve, which is used for chaos-based applications, such as true random number generation. This map is given by the equation

$$x_{n+1} = r x_n (1 - 2|x_n - 1/2|) \text{ where } x \in (0,1). \tag{4}$$

r is the control parameter and when $½ < r > 1$ the map exhibits chaotic behavior [17]. When $r > 1$ the map or x_n tends to infinity as n tends to infinity for any initial value of x. In this work, for generation of ternary sequences r was selected as 0.9.

2.2 Method of Generation of Ternary Sequences

Ternary sequences have alphabets 0, ±1 where as binary sequence alphabets are ±1. The algorithm or method used for generating ternary sequences of length n using a chaotic map is given below

Step 1. Ensure that initially the chaotic map is in the chaotic region. In logistic map the bifurcation parameter μ is selected to be 4 so that the map is in chaotic region.

Step 2. Select an initial value for x_n.

Step 3. Generate a raw sequence x_n using the chaotic map equations .

Step 4. The sequence is then quantized into three defined levels based on the threshold levels a and b as per equation (5) to obtain the ternary sequence y_n. The threshold levels were chosen randomly between x_{min} and x_{max} so that a low value of PSLR defined by equation (8) is obtained. For example the values of x_{min} and x_{max} for logistic map are 0 and 1 respectively as per equation (1). The threshold levels are also chosen considering approximately the mean of the raw sequence obtained which is 0.5 in the case of logistic map. The level a is chosen above the mean of the sequence and level b is chosen below the mean of the sequence

$$y_n = \begin{cases} -1 & \text{if } x_n < b, \\ 0 & \text{if } a < x_n < b, \\ 1 & \text{if } x_n > a. \end{cases} \quad (5)$$

The various maps are extremely sensitive to the initial condition. By varying the initial condition x_0, a totally uncorrelated ternary sequence can be obtained. Therefore, the number of available sequences at a given length is virtually infinite. So the above procedure was repeated by varying the initial values of x_n with very small increments and search for good sequences or sequences with low PSLR was done. The computation time for searching good sequences depends on the length of the sequence.

Using the above mentioned procedure good ternary sequences were generated for lengths from 20 to 4000 using the four chaotic maps. For logistic map the threshold levels a and b were selected as 0.7 and 0.3 respectively. For tent map a, b were 0.65 and 0.4 and for improved logistic map values of a, b were 0.7 and -0.7 respectively. The values of a, b were chosen as 0.6 and -0.4 for cubic map. Thus the generation of sequences is simple, fast and reproducible.

3. Properties of the Ternary Sequences

3.1 Correlation Function

The aperiodic autocorrelation $r(k)$ of a sequence of length N which is nothing but the output of the matched filter or the matched filter compressed output is given as

$$r(k) = \sum_{i=0}^{N-1-k} y_i y_{i+k}, \; k = 0, 1, 2, \ldots, N\text{-}1. \quad (6)$$

For best performance, the autocorrelation pattern of the optimum coded waveform must have a large peak value for zero shift (main lobe) and zero value for non-zero shifts. By varying the initial conditions a search for sequences with good autocorrelation pattern was done at all lengths for all the maps. The autocorrelation pattern of the good sequence generated using cubic map at length 4000 is shown in Fig. 2.The figure has the shape of an impulse which is desired.

The cross correlation $r(k)$ of two ternary sequences y_n and z_n of length N is given

$$r(k) = \sum_{i=0}^{N-1-k} z_i y_{i+k}, \; k = 0, 1, 2, \ldots, N\text{-}1. \quad (7)$$

By varying the initial conditions a search for two sequences with negligible cross correlation was done and the cross correlation pattern of codes of length 100 generated using cubic map is shown in Fig. 3. From the figure it is observed that the sequences have very less cross correlation which allows the code to be used for multiple access communications as well as in mulistatic radar.

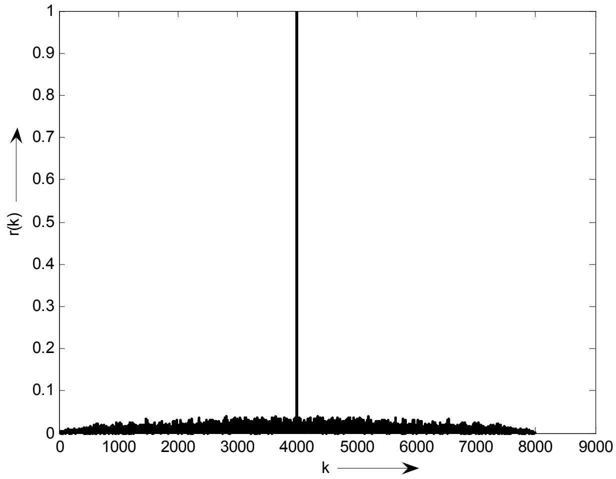

Fig. 2. Normalized Autocorrelation pattern of sequence of length 4000.

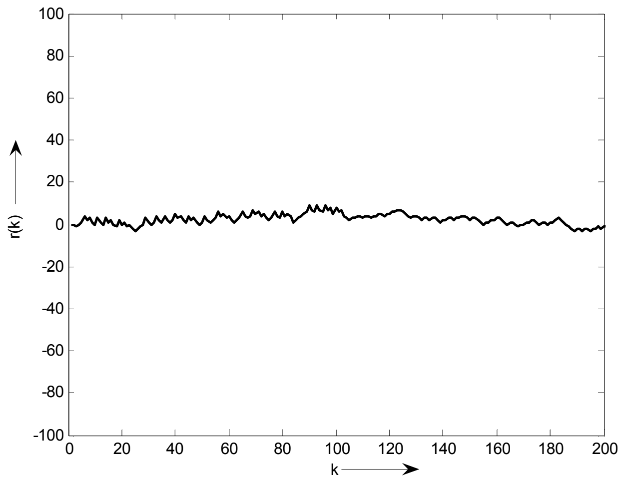

Fig. 3. Cross correlation pattern of sequence of length100.

3.2 Peak Side Lobe Ratio (PSLR)

One of the most commonly used performance measures is the Peak Side Lobe Ratio. The Peak side lobe is the largest sidelobe in the correlation of a sequence. The Peak Side Lobe Ratio is defined from the autocorrelation pattern $r(k)$ as the ratio of the peak side lobe amplitude to the main lobe peak amplitude and is expressed in decibels. It is given by

$$PSLR = 20\log_{10} \frac{Max(r(k))}{r(0)} \text{ where } k \neq 0. \quad (8)$$

A sequence is defined as a good sequence if the PSLR of the sequence is low [11].

Although the sequence generation process is quite general not all sequences are good or adequate for pulse compression in radar. So a search for good sequences was done by varying the initial conditions and from the codes y_n obtained good sequences were selected based on the PSLR. The Peak Side Lobe Ratio obtained for good ternary sequences at different lengths for all the maps is tabulated in Tab.1. To clearly understand how the PSLR varies with length a plot of PSLR obtained from the chaotic sequences using the cubic map, improved logistic map and logistic map is shown in Fig. 4.

Length of the sequence	Peak Side Lobe Ratio in dB			
	Tent map	Cubic map	Logistic map	Improved logistic map
20	-20.82	-22.27	-18.06	-22.27
30	-18.58	-20.82	-20.42	-20.8
40	-18.41	-20	-17.79	-19.55
50	-17.38	-20.00	-18.84	-19.78
60	-17.69	-19.55	-19.28	-20
70	-17.41	-19.46	-18.76	-20.21
80	-18.36	-19.85	-18.66	-20
90	-17.14	-19.70	-18.59	-20.17
100	-17.43	-20.24	-19.56	-21.11
200	-17.40	-21.02	-20.12	-21.71
300	-17.52	-21.46	-21.07	-21.79
400	-16.99	-22.03	-22.16	-22.31
500	-17.48	-22.64	-22.02	-23.22
600	-17.142	-22.98	-22.44	-23.19
700	-16.91	-23.74	-22.67	-23.736
800	-17.02	-23.35	-22.96	-24.10
900	-16.95	-24.15	-23.07	-24.17
1000	-17.55	-24.66	-23.50	-24.71
2000	-17.56	-26.13	-26.47	-25.7
3000	-17.67	-27.44	-27.62	-26.76
4000	-17.78	-28.39	-28.67	-29.16

Tab. 1. PSLR of Ternary sequences generated using Chaotic Maps.

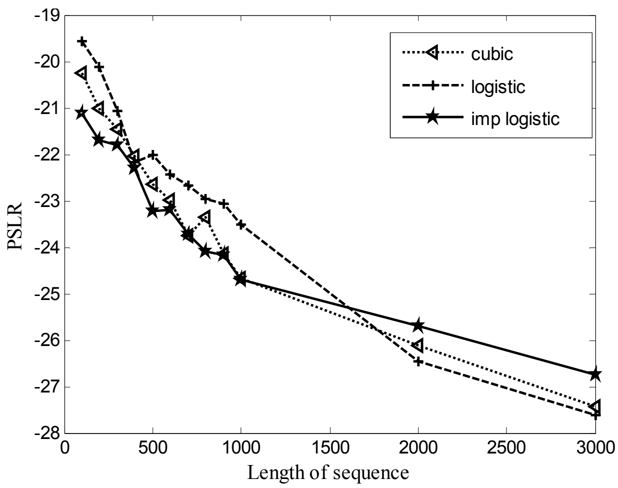

Fig. 4. Plot of PSLR of ternary sequences generated using chaotic maps.

Comparing the four different mapping methods, the performance of codes generated by all the three maps are good except the codes generated by tent map. The codes generated by tent map does not offer a low PSLR and the PSLR also does not decrease with the length of the sequence. For comparison a plot of PSLR of binary sequences generated using improved logistic map is also shown in Fig. 5. The PSLR of ternary sequences is lower than that of binary sequences at every length thus depicting the superior performance of ternary sequences.

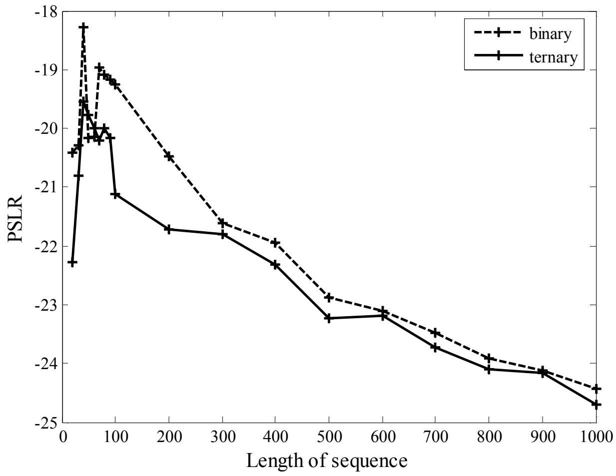

Fig. 5. Plot of PSLR of binary and ternary sequences.

3.3 Ambiguity Diagram

An important tool in the design and analysis of radar signals is the Ambiguity diagram. It represents the time response of a signal filter matched to a specified signal of finite energy when the signal is received with a time delay and Doppler shift relative to the nominal values expected by the filter. The ambiguity function is defined to be [1]

$$\chi(\tau,\nu) = \left| \int_{-\infty}^{\infty} u(t)u^*(t+\tau)\exp(j2\pi\nu t)dt \right| \tag{9}$$

$u(t)$ represents the envelope of the ternary coded waveform. The time delay is indicated by τ and ν is the Doppler frequency. The ambiguity diagram of a chaotic ternary code generated using logistic map is shown in Fig. 6. The code length is 100.

Because of the random nature of the chaotic codes, the ambiguity function approaches a thumbtack shape.

The accuracy with which range and velocity can be measured by a particular waveform depends on the width of the spike centered at $|x(0,0)|$. The resolution of the waveform is also related to the width of the center spike which helps to resolve two closely spaced targets. Since the center spike of the ternary chaotic code is confined in a very narrow region, high resolution in both range and velocity is obtained.

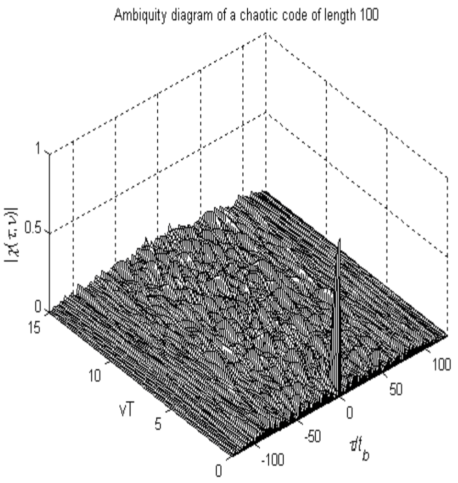

Fig. 6. Partial ambiguity diagram of a chaotic code of length 100.

3.4 Behavior of the Sequence in the Presence of Noise

The behavior of a ternary code in the presence of AWGN noise was studied for different values of signal to noise ratio and the pulse compressed waveform when the signal to noise ratio was considered to be -20 dB is shown in Fig. 7. The peak point of the compressed waveform is clearly distinguishable even in the presence of noise. The length of the code was considered to be 4000.

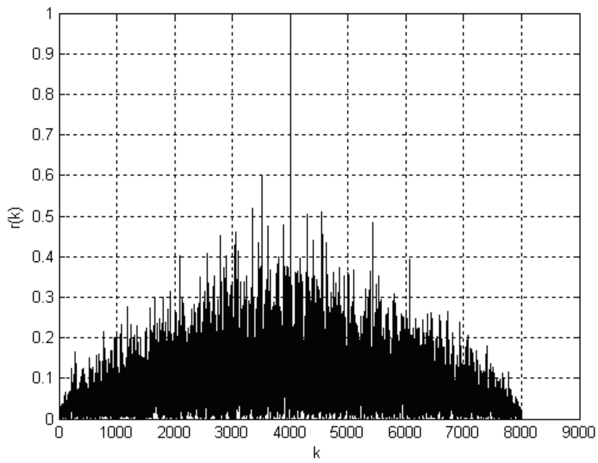

Fig. 7. Pulse compressed waveform in the presence of AWGN noise.

4. Conclusions

Good ternary sequences were generated using different chaotic maps. At different lengths, good sequences were obtained and it was found that the PSLR decreases with the length of the sequence. The chaotic maps like logistic, improved logistic and cubic map were found to perform well than the tent map. For length equal to 4000 PSLR less than -28 dB was obtained. Thus ternary sequences with low PSLR can be generated using any one of these maps. The ternary sequences can achieve desired detection range and range resolution simultaneously. Compared to the other ternary sequences available it has the advantage of easy generation and vast availability. Because the sequences are reproducible, the matched filter in the radar receiver can be easily designed based on the incoming signal. By using this method, the matched filter can be designed based on the initial condition and mapping method used and need not depend on the noise corrupted phase coded signal. Because of the chaotic nature of the sequences it can be used also for secure communications. Still better sequences can be searched by varying different parameters like the more sensitive initial conditions and threshold levels.

References

[1] LEVANON, N., MOZESON, E. *Radar Signals*. Hoboken, NJ: John Wiley& Sons, Inc., 2004.

[2] SKOLNIK, M. I. *Introduction to Radar Systems. 2nd Ed.* McGraw-Hill Book Company, 1980.

[3] COOK, C. E., BERNFIELD, M. *Radar Signals - An Introduction to Theory and Application.* New York: Academic press, 1967.

[4] ACKROYD, M. H. Amplitude and phase modulated pulse trains for radar. *The Radio and Elect. Eng.,* December 1971, vol. 41, pp. 541-552.

[5] RAO, B. V., DESHPANDE, A. A. Why the barker sequence bit length does not exceed thirteen. *Journal of IETE,* Nov-Dec 1998, vol. 34, no. 6, pp. 461-462.

[6] BOEHMER, A. M. Binary pulse compression codes. *IEEE Trans. IT-13,* April 1967, no. 2, pp. 156-157.

[7] LINDER, J. Binary sequences upto length 40 with best possible autocorrelation function. *Electronic Letters,* Oct 1975, vol. 11, no. 21, pp. 507-508.

[8] VEERABHADRA RAO, K., UMAPATHY REDDY Biphase sequences with low sidelobe autocorrelation function. *IEEE Transactions on Aerospace and Electronic Systems,* March 1986, vol. 22, no. 2, pp. 128-133.

[9] MOHARIR, P. S. Signal design. *Journal of IETE*, Oct. 1976, vol. 41, pp. 381-398.

[10] MOHARIR, P. S. Ternary Barker codes. *Electronics Letters,* Oct. 1974, vol. 10, pp. 460-461.

[11] MOHARIR, P. S., VARMA, S. K., VENKATARAO, K. Ternary pulse compression sequences. *J. Inst. Electron. Telecomm. Engrs.,* 1985, vol. 31, pp. 1-8.

[12] MOHARIR, P. S., RAJARAJESWARI, K., VENKATARAO, K. Barker progressions. In *Proceedings of Workshop on Signal Processing Communications and Networking.* Tata McGraw Hill, New Delhi, 1990, pp. 126-133.

[13] XIN WU, WEIXIAN LIU, LEI ZHAO, JEFFREY S. FU Chaotic phase code for radar pulse compression. In *Proceedings of IEEE National Radar Conference.* Atlanta (USA), 2001, pp. 279-283.

[14] ASHTARI, A., THOMAS, G., KINSNER, W., FLORES, B. C. Sufficient condition for chaotic maps to yield chaotic behavior after frequency modulation. *IEEE Transactions on Aerospace and Electronic Systems,* July 2008, vol. 44, no. 3, pp. 1240-1248.

[15] ASHTARI, A., THOMAS, G., FLORES, B. C. Radar signal design using chaotic signals. In *Proceedings of International Waveform Design and Diversity Conference.* Pisa (Italy), June 2007, p. 353-357.

[16] HEIDARI-BATENI, G., MCGILLEN, C. D. Chaotic sequences for spread spectrum: An alternative to PN- sequences. *IEEE, ICSTWC*, 1992, pp. 437-440.

[17] HILBORN, R. C. *Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers.* Oxford University Press, 1994.

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