Design of Discrete Frequency Coded Sequences using PSOCM for Target Detection with CAF.

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Abstract. *This paper presents the design of Discrete Frequency Coded (DFC) sequences with good auto correlation properties using Particle Swarm Optimization with Cauchy Mutation (PSOCM) technique. The cross-ambiguity function method is used to detect the stationary and moving targets in various target detection scenarios. The contour plots of the cross-ambiguity function are computed as a function of delay and Doppler frequency shift. The results are compared with DFC sequences of the sequence lengths N = 32 and N = 100.*

Keywords

Discrete Frequency Coded Sequences, Cross Ambiguity Function, Particle Swarm Optimisation.

1. Introduction

Traditionally, pulse compression sequences are being used to solve the problem of detection and separation of two closely spaced targets in multi-target environment. However, this separation is accomplished at the expense of introducing sidelobes in the matched-filter response, which may mask weak targets and possibly prevent their detection altogether. In the process of designing pulse compression sequences, Ambiguity Function (AF) is an important tool to understand the performance of designed waveform in terms of the measurement accuracy, target resolution, ambiguities in range and radial velocity, and its response to the clutters.

J. P. Costas [1] suggested the design procedure for the design of frequency-coded waveforms, which ensure high delay-Doppler resolution. In this context, many researchers have been working with different ways of using Costas sequences effectively in the radar signal design [2–7]. An important property of Costas sequences is that a sequence of length N when used in the radar signal design would yield an AF with sidelobes of maximum height 1/N times of its mainlobe height. Hence, use of longer Costas sequences would result in better approximation of the ideal thumbtack like AF, which is actually desired for high-resolution radar applications.

Alternatively, H. Deng [8] suggested an innovative way of designing the DFC sequences for netted radar systems using global optimization algorithm. He showed that the auto and cross correlation sidelobes of level L/N times of the mainlobe can be achieved using such algorithm, where L is the number of sequences optimized simultaneously to achieve the good auto and cross correlation properties, and N is the length of the sequences. However this paper demonstrates the design of DFC sequences for the sequence lengths of N = 32, N = 100, N = 128 and N = 200, using PSOCM algorithm. In the present work the value of L = 1 (i.e. one sequence at a time is optimized). Further, application of such optimized sequences for the detection of targets in various detection scenarios using cross-ambiguity function technique is also proposed. In Section 2, a brief description of DFC sequence is presented. Section 3 explains the design procedure of DFC sequences using PSOCM. Cross-ambiguity function is discussed in Section 4 whereas various target detection scenarios and results are discussed in Section 5.

2. Discrete Frequency Coded Sequences

Consider a discrete frequency coded sequence with N adjacent sub-pulses of time duration 'T' and each is modulated with a distinct carrier frequency. The coding waveform can be represented as

$$s(t) = \sum_{n=1}^{N} A_n(t) e^{j2\pi f_n t} \tag{1}$$

where f_n is the coding frequency of n^{th} pulse and

$$A_n(t) = \begin{cases} \frac{1}{T} & (n-1)T \le t \ge nT, \\ 0 & \text{otherwise.} \end{cases}$$

The resulting coding sequence can be written as $\{f_1, f_2, f_3, \dots, f_N\}$ for waveform of a DFC sequence, which is a permutation of $\{0, \Delta_f, 2\Delta_f, 3\,\Delta_f, \dots, (N-1)\Delta_f\}$. The value of Δ_f is chosen as $\Delta_f = 1/T$ for convenience, when T is selected as 1, given $\Delta_f = 1$ and a DFC sequence is simply represented as {0, 1, 2, 3… N-1} and this sequence is termed as Discrete Frequency Coded sequence (DFC).

The process of optimization involves the selection of the order of the sequence that gives nearly ideal noise like autocorrelation properties. The auto correlation function is given as

$$A(s,\tau)=\frac{1}{N}\int_t s(t)s^*(t-\tau)dt=\begin{cases}1, & \tau=0,\\ 0, & \text{otherwise.}\end{cases} \tag{2}$$

Where $A(s,\tau)$ is the aperiodic auto correlation function of the signal 's(t)'.Therefore, to design DFC sequences for detection purposes, the optimized sequence must satisfy the conditions given in (2).

3. Design of DFC Sequences using PSOCM

Kennedy and Eberhart [9] developed an algorithm known as Particle Swarm Optimization (PSO), which is motivated from the organisms such as bird flocking and fish schooling. The major limitation of PSO is its tendency of trapping into local minimum. Changhe Li et.al [10] proposed an algorithm which is a combination of PSO and Cauchy Mutation and they showed that the trapping at local optima points can be minimized with higher probability by introducing Cauchy mutation into the position and velocity equations of PSO. In the present work, PSOCM algorithm is used to optimize the order of the DFC sequence to achieve good auto correlation properties. The basic step to apply the PSOCM algorithm to the design of DFC waveforms / sequences is as follows.

Step 1. (Initialization): Generate initial particles by randomly generating the position and velocity for each particle (in this case, these are discrete frequency sequences).

Step 2. Evaluate each particle's fitness, which is sum of the squares of all the auto correlation sidelobes energy of a DFC sequence and can be calculated as

$$E=\int_\tau \left| \underset{\tau\ge T}{A}(s,\tau)\right|^2 d\tau. \tag{3}$$

Step 3. For each particle, if the fitness (E) is less than its previous best (P_{id}) fitness, update P_{id}.

Step 4. For each particle, if the fitness (E) is less than the best one (P_{gd}) of all the particles, update P_{gd}.

Step 5. For each particle, do

a) Generate a new particle p according to the following formulae

$$X_{id}^{'}=X_{id}+V_{id}^{'}, \tag{4}$$

$$V_{id}^{'}=\omega V_{id}+\eta_1 rand()(P_{id}-X_{id})+ \\ +\eta_2 rand()(P_{gd}-X_{id}) \tag{5}$$

where $X_{id}^{'}$ and X_{id} represent the current and the previous position of the id^{th} particle, whereas V_{id} and $V_{id}^{'}$ are the previous and the current velocity of the id^{th} particle. Here X_{id} represents the position of the binary equivalent values of a frequency coded sequence.P_{id} and P_{gd} are the individual's best positions and the best position found so far, $0 \le \omega \le 1$ is an inertia weight which determines how much the previous velocity is preserved (chosen $\omega = 99$). This signifies that the previous velocity is almost preserved but not preserved completely to avoid escaping from the optimum value, η_1 and η_2 are the accelerating constants assigned a random value picked between 0 to1 from a uniform distribution, *rand*() picks a value from a uniform probability distribution.

b) Generate a new particle p' according to the formulae

$$V_{id}^{'}=V_{id}\exp(\delta), \tag{6}$$

$$X_{id}^{'}=X_{id}+V_{id}^{'}\delta_{id} \tag{7}$$

where δ and δ_{id} denote Cauchy random numbers.

c) Compare p with p', choose the one with lesser fitness function.

Step 6. (Stopping criterion): If the stop criterion is satisfied, then stop, else go to Step 3.

Based on the above described algorithm DFC sequences for the sequence lengths $N = 32$, $N = 100$, $N = 128$ and $N = 200$ are optimized. The Autocorrelation Sidelobe Peak (ASP) values for all the four sequences are shown in Tab. 1. For comparison purpose, the ASP values for the sequence lengths $N = 32$ and $N = 128$ reported in literature [8], using Simulated Annealing (SA) algorithm are also shown in Tab. 1. The results show that the ASP values of synthesized sequences are better than that of the results shown in the literature [8]. The Max (ASP) of designed sequences is calculated using Eqn. (8).

$$\text{Max(ASP)}=\max\{\underset{\tau\ge T}{A}(s,\tau)\}. \tag{8}$$

Figs. 1-4 show the Autocorrelation Functions (ACF) of the optimized/synthesized sequences. The sidelobe levels achieved are better than -26.1 dB, -34.5 dB, -39.3 dB and -42.8 dB for the sequence lengths $N = 32$, 100, 128 and 200, respectively.

Sequence length (N)	Max(ASP)	
	synthesized	Literature[8]
32	0.0495	0.0764
100	0.0188	-
128	0.0108	0.0235
200	0.0072	-

Tab. 1. Comparison of Autocorrelation Sidelobe Peaks.

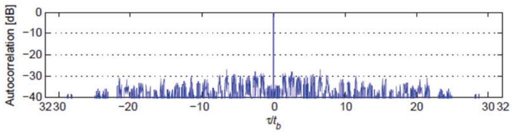


Fig. 1. ACF of DFC sequence of N=32.

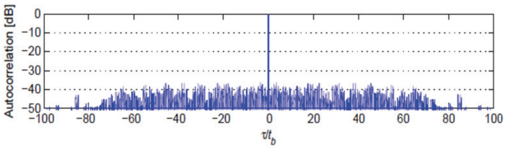


Fig. 2. ACF of DFC sequence of N=100.

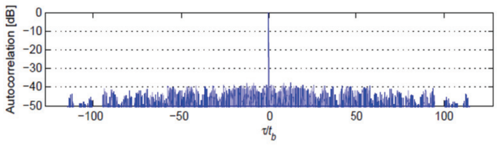


Fig. 3. ACF of DFC sequence of N=128.

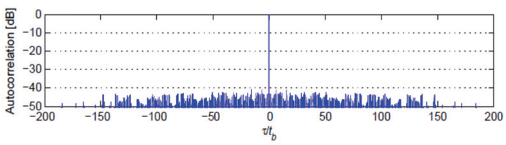


Fig. 4. ACF of DFC sequence of N=200.

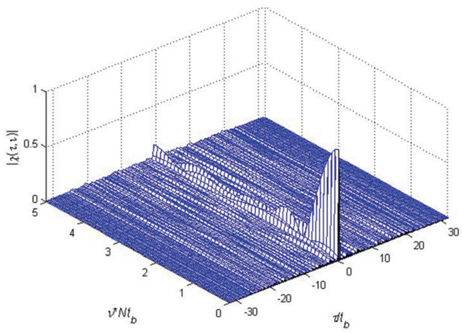


Fig. 5. AF of DFC sequence of N=32.

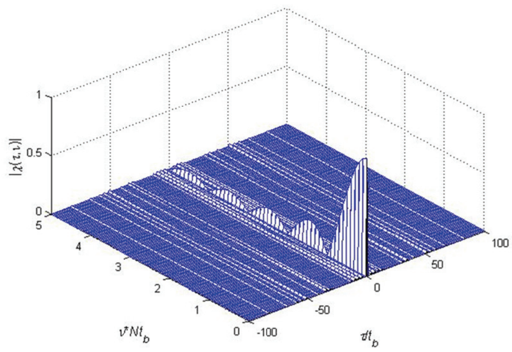


Fig. 6. AF of DFC sequence of N=100.

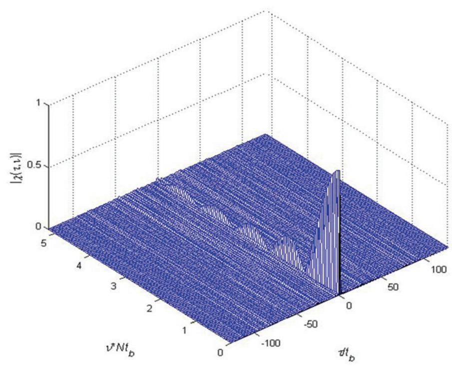


Fig. 7. AF of DFC sequence of N=128.

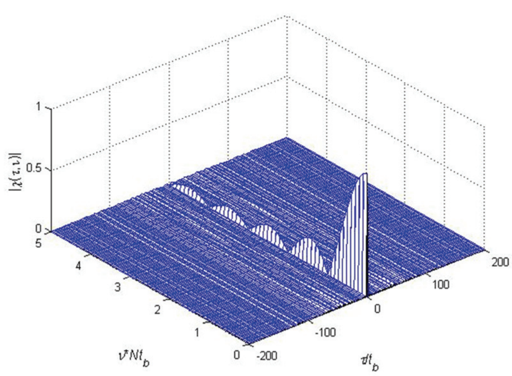


Fig. 8. AF of DFC sequence of N=200.

4. Cross Ambiguity Function

Time-frequency signal representations (TFRs) are widely used mathematical tools for efficient coding of signals and as a statistics for signal detection and parameter estimation [11]. Ambiguity Function (AF), which is a quadratic TFR, has been used extensively for investigating the ambiguity properties of the modulated waveforms used in various fields such as radar, sonar, radio astronomy, communications etc. The ambiguity function represents the response of a filter matched to a given finite energy signal when the signal is received with a delay τ and a Doppler shift ν relative to the nominal values (zeros) expected by the filter [11]. The Ambiguity Function is defined as

$$|\chi(\tau,\nu)| = \left| \int_{-\infty}^{\infty} s(t)s^*(t+\tau)e^{j2\pi\nu t}dt \right| \tag{9}$$

where s(t) is the complex envelope of the signal. A nonzero ν implies a target moving at a certain radial velocity with respect to radar. Positive τ refers to round trip delay time when the target is away from the radar by a certain distance.

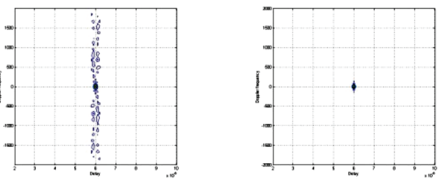
In (9), if $s(t)$ and $s^*(t+\tau)$ are the complex envelopes of the transmitted signal then the resulting equation is the ambiguity function (AF) or auto-ambiguity function. Ambiguity function is used for determining the characteristics of the designed waveform in respect of measurement accuracy, ambiguities in range and velocity, and target resolution. [11].

Figs. 5 – 8 show the ambiguity function of the optimized DFC sequences of lengths N = 32, 100, 128 and 200, respectively. The AF of each sequence exhibits a thumbtack like ambiguity function that has a sharply peaked mainlobe at the centre and well-controlled sidelobes in remaining part of the delay and Doppler plane, which is desired for better delay and Doppler resolution measurement capability of a radar system.

On the other hand, in equation (9), when $s(t)$ is the complex envelope of transmitted signal and $s^*(t+\tau)$ is the complex envelope of the received signal (with or without noise) then $\chi(\tau, \nu)$ is called cross-ambiguity function (CAF). Therefore cross-ambiguity function is useful for determining the waveform effects in response to the clutters [11]. Hence, this paper presents the use of cross-ambiguity function for the detection of targets in various target scenarios.

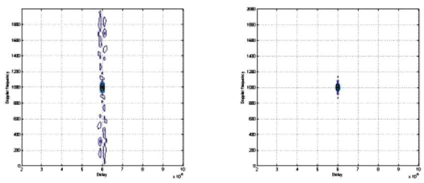
5. Target Detection Scenarios

Figs. 9 – 13 show the contour plots of various target scenarios. In each figure the delay (proportional to range) and Doppler (proportional to radial velocity) are plotted on x-axis and y-axis respectively. In Figs. 9 – 10 contour plots are showing single stationary and moving target scenarios respectively. In each plot because of the symmetry, the target appears in two quadrants, for convenience here we consider only right quadrant. In Fig. 9 the target is stationary (zero Doppler shift) whereas range is corresponding to τ = 60 µs. However, in moving target scenario the target Doppler shift is taken as ν = 1000 m/s and τ = 60 µs, which is evident in Fig. 10. Fig. 11 shows the detection capability in presence of two stationary targets (Target.1: τ = 80 µs, Target: 2 τ = 60 µs).



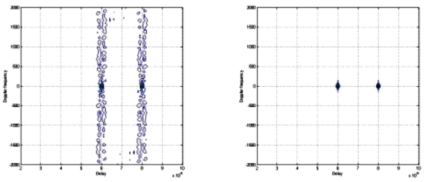
(a) (b)

Fig. 9. Stationary single target scenario with τ = 60 µs. sequence length N=32, (b) sequence length N=100.



(a) (b)

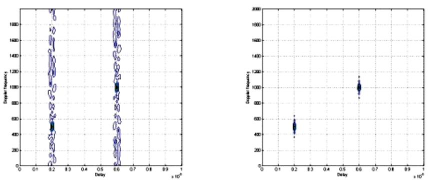
Fig. 10. Moving single target scenario with ν = 1000 m/s, τ = 60 µs. (a) sequence length N = 32, (b) sequence length N = 100.



(a) (b)

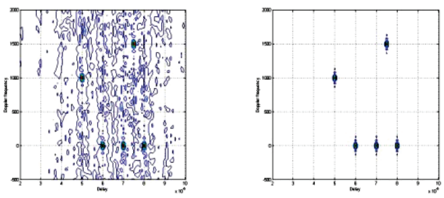
Fig. 11. Stationary multi target scenario with Target.1 τ = 80 µs, Target: 2 τ = 60 µs. (a) sequence length N = 32, (b) sequence length N = 100.

Similarly in multi-target scenarios, both stationary as well as moving targets are also considered in Figs. 12 – 13. Fig. 12 shows the multi stationary targets, Target 1 is taken with τ =80 µs and ν = 500 m/s, target 2 values are τ = 60 µs, ν = 1000 m/s. The positions of the detected targets corresponding to the above values could be seen in Figs 12(a) and 12(b). Fig 13 shows the multi moving targets with different values of τ and ν.



(a) (b)

Fig. 12. Moving multi target scenario with Target: 1(τ = 80 µs, ν = 500 m/s). Target: 2 (τ = 60 µs, ν = 1000 m/s). (a) sequence length N = 32, (b) sequence length N = 100.



(a) (b)

Fig. 13. Stationary and moving multi-target scenario Target: 1 (τ = 80 µs), stationary Target: 2 (τ = 70 µs), stationary Target: 3(τ = 60 µs), stationary Target: 4 (τ = 75 µs, ν = 1500 m/s), moving Target: 5 (τ = 50 µs, ν = 1000 m/s), moving (a) sequence length N = 32, (b) sequence length N = 100.

In Fig. 13, combination of multi stationary and moving targets is considered. In all above figures (Figs. 9 – 13) it is observed that using sequence length N = 32, the sidelobe levels are higher which leads to an ambiguous detection, whereas using sequence length N = 100 all the targets are detected without any ambiguity, which can be evidently seen in Figs. 9 (b) – 13 (b). This clearly shows that the detection capabilities of DFC sequences are increasing with the increase in sequence length. Since the design of longer length sequences using algebraic construction method is difficult and advocate the specific sequence length, however with the available methods, DFC sequences of any lengths can be designed as per the desired accuracy.

The detection performances in all above cases are considered with and without noise (Noise level of 10 dB), but it is observed that that there is no appreciable difference in detection and measurements of the target range and velocity. Therefore, the results are not shown for noise cases.

6. Conclusion

In general auto ambiguity function is used for the design of radar signals to achieve desired delay-Doppler resolution. However, this paper demonstrates the detection of target (targets) using CAF technique, which is shown to be an alternative technique for the detection of targets. The work is mainly focused on – (i) Design of DFC sequences using PSOCM optimization technique. Using this optimization technique DFC sequences for sequence lengths N = 32, 100, 128 and 200 are optimized. The sidelobe levels obtained are better than -26.1 dB, -34.5 dB, -39.3 dB and -42.8 dB for the sequence lengths N = 32, 100, 128 and 200 respectively. Figs. 1 – 8 show that the optimized DFC sequences have impulse like autocorrelation function and thumbtack like ambiguity function. It shows that that PSOCM is also capable of optimizing the DFC sequences. (ii) Detection of targets using CAF technique in various target scenarios. Figs. 9 – 13 demonstrate detection of target(s) in different target scenarios such as single and multi-targets for stationary scenario, moving scenario, and both stationary and moving scenario for sequence lengths $N = 32$ and $N = 100$. Figs. 9 – 13 are showing that in each case the sidelobe levels in delay-Doppler plane are higher when sequence length is $N = 32$, whereas these sidelobes are eliminated when sequence length is considered $N = 100$. This clearly shows that the resolution and detection capabilities of a radar system can be improved when one can go for higher sequence lengths.

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