Analysis of Simulation Methods for Far-end Crosstalk Cancellation

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Abstract. *The information capacity of current digital subscriber lines is limited mainly by a crosstalk in metallic cables. The influence of near-end crosstalk (NEXT) can be well cancelled by frequency duplex method, but the elimination of far-end crosstalk (FEXT) is not so easy. Therefore FEXT is the dominant source of disturbance in current digital subscriber lines (xDSL). One of the most promising solutions for far-end crosstalk cancellation is Vectored Discrete Multi-tone modulation (VDMT). For the testing of VDMT modulation efficiency it will be necessary to implement advanced methods for modeling of far-end crosstalk to obtain required predictions of the crosstalk behavior in a cable. The actual simple FEXT model is not very accurate and does not provide realistic results. That is why the new method for modeling of far-end crosstalk was developed and is presented in this paper. This advanced model is based on the capacitive and inductive unbalances between pairs in a cable and it also respects the cable's internal structure. The results of the model are subsequently used for the simulation of VDMT modulation and its impact on the FEXT cancellation. These simulations are based on the estimations of transmission speed of VDSL2 lines with VDMT modulation.*

Keywords

Digital subscriber line, Crosstalk, FEXT, VDMT modulation, VDSL2.

1. Introduction

The actual access telecommunication networks still consist mostly of metallic pairs and cables and are often used for high-speed digital transmission systems, such as digital subscriber lines (xDSL). These subscriber lines use existing metallic symmetrical pairs of local cables and thanks to that provide affordable and cheap connections mainly for households and small business companies. However, the requirements for the amount of transmitted data and transmission speed are still continually increasing, and this trend is supposed to continue further. The next generation of xDSL digital subscriber lines, e.g. VDSL2, could provide higher transmission speeds, but there are several problems related with the usage of metallic lines and cables, which need to be solved first. The major problem, which appears in large metallic infrastructure, is crosstalk between the digital systems operating within the same metallic cable [1]. One of the most promising is Vectored DMT modulation [2]. This modulation is an upgrade of previous Discrete Multi-tone modulation (DMT) and it offers the elimination of FEXT crosstalk by coordinating the transmitted DMT symbols according to the transmission functions of all pairs and crosstalk in the metallic cable [3]. However, the practical implementation of VDMT modulation into the present DSLAMs multiplexors is not possible due to the overall complexity and computational demands of VDMT modulation for the full coordination of transmissions in large metallic cable systems. One of the possibilities how to simplify this process would be performing the VDMT modulation only for a limited number of pairs or even only for several xDSL sub-channels [4].

The elementary unit of a standard telecommunication cable is generally two insulated wires twisted uniformly to form a balanced pair. By twisting four insulated wires together a quad is formed. Several quads are typically twisted together to form a subgroup of pairs (or quads), these subgroups can be further twisted and gathered according to a cable internal structure. Quads and subgroups can be also covered with screening, sheeting or taping to form grounded shielding and to separate each subgroup of pairs. Interstices between pairs, quads and subgroups are usually filled with a gel or air. The resulting transmission parameters of final cable are determined according to a method of construction, types of used materials and their processing. During the process of cable's manufacturing, several parameters have to be measured and checked, and must meet specified tolerances. Based on these tolerances, pairs, quads and subgroups in a cable demonstrate towards themselves small irregularities and unbalances. Capacitive and inductive unbalances and couplings are the main source of crosstalk between them. These capacitive and inductive couplings in a quad of four wires form an unbalanced bridge. Using the star-polygon transformation it is possible to express resulting capacitive unbalance C_{ub} and inductive unbalance M_{ub}. The calculation of these unbalances based on the geometrical structure of the quad and other parameters, such as permittivity and permeability of the materials, was presented e.g. in [5] based on mathematical equations in [6].

New derivation procedure for FEXT crosstalk modeling is presented in section 2. The results of the modeling are used for the simulation of VDMT modulation and its impact on the FEXT cancellation. These simulations are based on the estimations of transmission speed of VDSL2 lines and are presented in section 3.

2. Advanced Far-end Crosstalk Modeling

Several models of FEXT crosstalk using capacitive and inductive unbalances and their impedance matrices have been already presented, e.g. [7], or models using pseudo-randomly generated components [8], but these models are mathematically quite complex and require many parameters. The proposed innovative method of FEXT modeling, which will be presented here, and which is based on the description of sub-elements and sections by using cascade matrices, can offer less complexity and computational demands while maintaining a sufficiently accurate method of modeling. The main idea of this innovative FEXT model is dividing the whole cable into several subsections, which consist of the transmission lines, the crosstalk coupling and the bridge taps from the unused ends of both symmetrical pairs. Each section is described by its cascade matrix and the final crosstalk current is calculated by their multiplication. First, several assumptions are necessary.

The model does not include impact of the crosstalk through the third circuits, or an indirect effect of the crosstalk arising from reflections from the ends of the unused lines. The total crosstalk coupling is summarily expressed by its inductive and capacitive components, but the inductive part is approximated by the capacitive unbalance. This assumption is based on previous theoretical considerations, according to which the impact of inductive coupling can be modeled by an additional capacitance unbalance and these two parts are included in the summary capacitive unbalance C'. Another assumption comes in a simplification of the current conditions in both pairs. The proposed model does not include the loss of current effect due to crosstalk. The last simplification of the model concerns the question of simulation and determination of the capacitive unbalance. It could be very complicated to express its values mathematically. Moreover, these values are usually pseudo-random and are influenced by many internal and/or external effects. That is why a simple method by generating pseudo-random values is used and formulas of normal distribution with the proper statistical values are used in the model.

Based on the previous assumptions it is possible to provide a schematic model of the whole situation. Standard models for crosstalk between two pairs are usually based on the description of 4-port network, or two coupled 2-port networks, but for the basic crosstalk modeling, the simple 2-port model is sufficient.

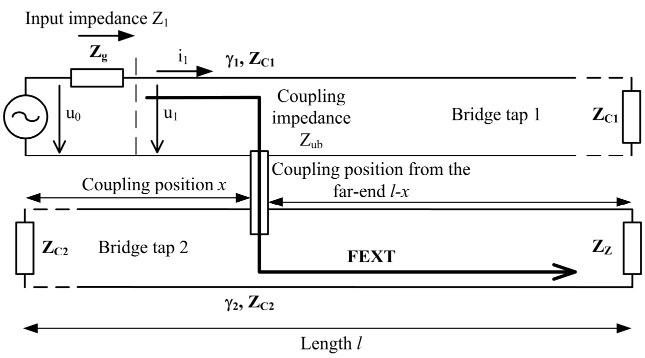

Fig. 1. The cascade elements of the proposed FEXT model.

The signal generator with output voltage u_0 and internal impedance Z_g is located at the input of a disturbing pair. The input impedance of the whole system Z_I provides the total current i_I and voltage u_I. The summary capacitive coupling C', as the impedance Z_{ub}, is situated in the position x from the beginning of a cable and l-x from the far-end of the cable, while l is the length of the cable. This unbalance is situated in series with the generator from the perspective of the FEXT crosstalk. The first bridge tap consists of the unused part of the disturbing pair and has the length l-x. It is connected to the unbalance in parallel. Also the unused section of the disturbed pair, which forms the second bridge tap of the length l, is connected in parallel. The rest of the disturbed pair with the length l-x is connected in series from the prospective of FEXT crosstalk. The far-end of the disturbed pair is terminated by the load impedance Z_Z. The propagation constant of the disturbing pair is γ_1, while the propagation constant of the disturbed pair is γ_2. The ends of both bridge taps are opened, but the model could be further modified by terminating the taps by impedances Z_{C1} and Z_{C2}.

The optimal solution for calculating transmission lines consisting of many sections are cascade matrices. The final result can be obtained by their multiplication. The essential expression of the cascade matrix for the 2-port configuration is:

$$\begin{pmatrix} u_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} u_2 \\ i_2 \end{pmatrix} \tag{1}$$

where a, b, c, d represent the elements of the cascade matrix.

The expression of cascade matrix for standard telecommunication line can be obtained from telegraph equations [6]. For the symmetrical pair with characteristic impedance Z_C, propagation constant γ and length l the cascade matrix can be defined:

$$\begin{pmatrix} u_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} \cosh\left(\gamma(f)\cdot l\right) & Z_C(f)\cdot\sinh\left(\gamma(f)\cdot l\right) \\ \dfrac{\sinh\left(\gamma(f)\cdot l\right)}{Z_C(f)} & \cosh\left(\gamma(f)\cdot l\right) \end{pmatrix} \cdot \begin{pmatrix} u_2 \\ i_2 \end{pmatrix}. \tag{2}$$

The cascade matrix for bridge tap comes from the derivation of cascade matrix for parallel impedance:

$$\begin{pmatrix} u_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \dfrac{1}{Z_C(f)\cdot\coth\left(\gamma(f)\cdot l\right)} & 1 \end{pmatrix} \cdot \begin{pmatrix} u_2 \\ i_2 \end{pmatrix} \tag{3}$$

and the general cascade matrix for the impedance Z connected in series:

$$\begin{pmatrix} u_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} 1 & Z \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} u_2 \\ i_2 \end{pmatrix}. \tag{4}$$

Now, it is possible to express the cascade matrices for the situation described in Fig. 1 using previous formulas.

The cascade matrix of the transmission section of the disturbing pair with the length x:

$$\mathbf{P_1} = \begin{pmatrix} \cosh\left(\gamma_1(f)\cdot x\right) & Z_{C1}(f)\cdot\sinh\left(\gamma_1(f)\cdot x\right) \\ \dfrac{\sinh\left(\gamma_1(f)\cdot x\right)}{Z_{C1}(f)} & \cosh\left(\gamma_1(f)\cdot x\right) \end{pmatrix}. \tag{5}$$

The cascade matrix of the first bridge tap, which consists of the unused section of disturbing pair with the length l-x:

$$\mathbf{O_1} = \begin{pmatrix} 1 & 0 \\ \dfrac{1}{Z_{C1}(f)\cdot\coth\left(\gamma_1(f)\cdot(l-x)\right)} & 1 \end{pmatrix}. \tag{6}$$

The cascade matrix of the coupling impedance Z_{ub}:

$$\mathbf{V} = \begin{pmatrix} 1 & Z_{ub} \\ 0 & 1 \end{pmatrix} \tag{7}$$

in which the impedance Z_{ub} according to the previous assumptions can be calculated:

$$Z_{ub} = \frac{1}{j\omega C'}. \tag{8}$$

The cascade matrix of the second bridge tap, which represents the unused near-end of the disturbed pair with the length x:

$$\mathbf{O_2} = \begin{pmatrix} 1 & 0 \\ \dfrac{1}{Z_{C2}(f)\cdot\coth\left(\gamma_2(f)\cdot x\right)} & 1 \end{pmatrix}. \tag{9}$$

And finally, the cascade matrix of the rest transmission part of the disturbed pair, which is terminated by the impedance Z_Z at its far-end:

$$\mathbf{P_2} = \begin{pmatrix} \cosh\left(\gamma_2(f)\cdot(l-x)\right) & Z_{C2}(f)\cdot\sinh\left(\gamma_2(f)\cdot(l-x)\right) \\ \dfrac{\sinh\left(\gamma_2(f)\cdot(l-x)\right)}{Z_{C2}(f)} & \cosh\left(\gamma_2(f)\cdot(l-x)\right) \end{pmatrix}. \tag{10}$$

The resulting cascade matrix $\mathbf{W}$ can be expressed by the multiplication of the previous cascade matrices for all sections:

$$\mathbf{W} = \mathbf{P_1}\cdot\mathbf{O_1}\cdot\mathbf{V}\cdot\mathbf{O_2}\cdot\mathbf{P_2}$$
$$\mathbf{W} = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix}. \tag{11}$$

The input impedance Z_1 of the whole situation is defined:

$$Z_1(f) = \frac{Z_Z(f)\cdot w_{11}(f) + w_{12}(f)}{Z_Z(f)\cdot w_{21}(f) + w_{22}(f)}. \tag{12}$$

The far-end crosstalk current, which comes from one unbalance situated in the position x, can be calculated:

$$\begin{pmatrix} u_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix} \cdot \begin{pmatrix} u_{Fx} \\ i_{Fx} \end{pmatrix}, \tag{13}$$

$$\frac{i_{Fx}(f)}{u_0(f)} = \\ = \frac{Z_g(f)+Z_Z(f)}{Z_Z(f)\left[Z_Z(f)w_{11}(f)+w_{12}(f)+Z_g(f)Z_Z(f)w_{21}(f)+Z_g(f)w_{22}(f)\right]}$$

To calculate FEXT attenuation, it is necessary to summarize all contributions of crosstalk currents for the whole length of the cable l:

$$\frac{i_F(f)}{u_0(f)} = \sum_l \frac{i_{Fx}(f)}{u_0(f)}. \tag{14}$$

Therefore, the FEXT attenuation in dB can be expressed:

$$A_{FEXT}(f) = 20\cdot\log\left|\frac{1}{Z_Z(f)\cdot\sum_l \dfrac{i_{Fx}(f)}{u_0(f)}}\right|. \tag{15}$$

The results obtained by the presented method for FEXT crosstalk modeling are presented for the metallic cable described in the previous chapter, with the specification TCEPKPFLE 75x4x0.4 and length 400 m. The primary parameters can be calculated using British Telecom model and by using appropriate formulas, it is possible to obtain the characteristic impedances Z_{C1}, Z_{C2} and the propagation constants γ_1 and γ_2 [9]. It is also necessary to divide the cable into several sub-sections with different crosstalk couplings. For that reason, the whole cable was divided into the section with the length of 1 m each, which means 399 capacitive unbalances (400-1) for the whole cable with the length 400 m. The crosstalk currents from all sections are then summarized.

According to the previous conclusions, it is possible to calculate summary capacitive unbalance from the crosstalk parameter K_{FEXT}. Based on the previous conclusions about the influence of the internal structure of the cable on the resulting FEXT crosstalk, the K_{FEXT} parameter can be calculated for three main categories - the pairs within the same subgroup, pairs from two surrounding subgroups and pairs from two distant subgroups. The value of capacitive unbalance C' for each category can be therefore calculated using measured K_{FEXT} parameter and equation (16).

$$K_{FEXT} = \left|Z_C\right|^2 \cdot 4\pi^2 \cdot C'^2 . \tag{16}$$

The K_{FEXT} parameter is usually derived for a cable with the length of 1000 m that is why it is necessary to provide recalculation for the situation of capacitive unbalance for shorter sections - 1 m in this case. Equation (16) could be therefore modified to get the capacitive unbalance for the length of 1 m:

$$C'\left[F/\sqrt{m}\right] = \frac{C'\left[F/\sqrt{km}\right]}{\sqrt{1000}} = \sqrt{\frac{K_{FEXT}}{\left|Z_C\right|^2 \cdot 4\pi^2 \cdot 1000}} . \tag{17}$$

The values of the summary unbalance, calculated according to (17) for cable type TCEPKPKFLE, are presented in Tab. 1.

The recalculation	K_{FEXT}	C' [F/√m]
Pairs within the same subgroup	$9.9462\cdot10^{-17}$	$5.0194\cdot10^{-13}$
Pairs from two surrounding subgroups	$1.292\cdot10^{-17}$	$1.8090\cdot10^{-13}$
Pairs from two distant subgroups	$3.2040\cdot10^{-18}$	$9.0087\cdot10^{-14}$

Tab. 1. The calculation of capacitive unbalances.

As it was described before, the behavior of capacitive unbalance is varying along the cable in the interval of values with pseudo-random characteristic which can be predicted using the formulas for normal distribution. Therefore, the values of capacitive unbalance C' in Tab. 1 were subsequently used as a standard deviation for generating the character of capacitive unbalance $C'(x)$ with the zero mean value.

Using the previous equations of the proposed FEXT cascade model (11), (13), (14) and (15) together with the pseudo-randomly generated $C'(x)$ characteristic according to the values in Tab. 1, several examples of results were obtained. These results were compared with the measured characteristic of a cable and also with the standard FEXT model. The comparisons are presented in the following graphs.

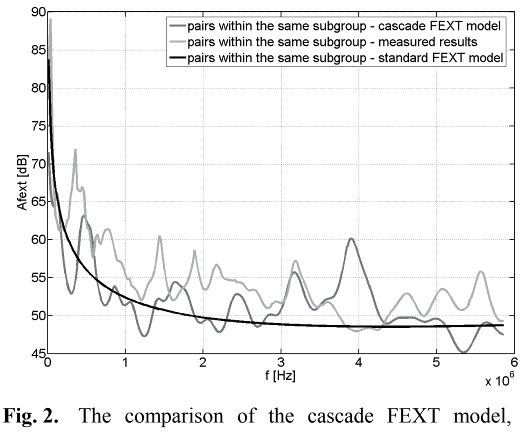

Fig. 2. The comparison of the cascade FEXT model, the standard FEXT model and measured results for the pairs within the same subgroup.

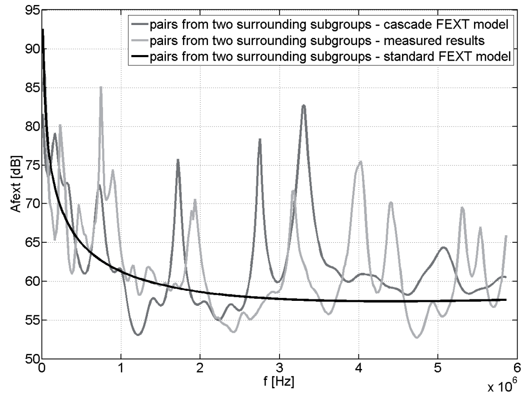

Fig. 3. The comparison of the cascade FEXT model, the standard FEXT model and measured results for the pairs from two surrounding subgroups.

The characteristics in Fig. 2 and 3 give an example of the presented cascade FEXT method of modeling, the standard FEXT model and measured results for the frequency band to approx. 6 MHz. It is obvious that unlike the standard FEXT model (presented in the graphs as a black line), the proposed cascade modeling method provides more precise and realistic results. The standard model comes from only average values for the whole cable, the cascade method based on the varying function $C'(x)$ of capacitive unbalance together with the influence of internal structure of the cable provides final results very close to the characteristics in real applications.

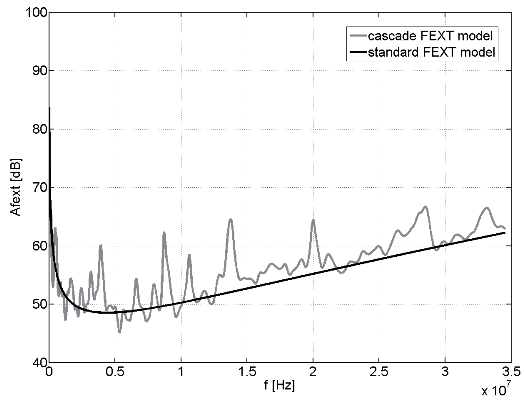

Fig. 4. The comparison of the cascade FEXT model and the standard FEXT model in the frequency band to approx. 35 MHz.

The situation for the frequency band to approximately 35 MHz is very similar. The standard model provides characteristics that comes from only average values for the whole cable and thus offers only very approximate estimations. The proposed cascade model brings more accurate results and reaches more realistic shapes of the transmission and crosstalk characteristics in the cable.

3. Simulation of FEXT Cancellation

Crosstalk FEXT is a dominant type of self-disturbance at VDSL2 lines, which are expected to be a subscriber line up to 1 km long. To counteract the negative effects of FEXT, VDMT modulation is proposed as an extension of standard Discrete Multi-Tone modulation.

Modulation VDMT adjusts particular transmitted data symbols, so that even after the passage through a transmission environment with negative influence of crosstalk FEXT they could be detected on a reception side without any errors. In this way, modem at a subscriber side does not have to be equipped with advanced functions enabling correction of a received signal [10].

It would be a very demanding computational process to carry out a coordination of each DMT symbol (which represents transmitted user data) for all user lines and for all tones [2]. Those complicated calculations can be found in [3]. Gaining a real-time control of the transmission environment parameters requires additional mathematical operations. Mentioned demanding calculations make impossible performing simple simulation of VDMT benefits when developing and designing access networks.

The classic method for calculating a transmission performance is based on the summarization of the total disturbance in the line, determination of a signal to noise ratio and estimation of the transmission rate. Interference is expected only from the system in the same class (in terms of spectral compatibility). The dominant source of a noise in digital subscriber lines is the FEXT crosstalk. To simulate a crosstalk usually a consortium FSAN (Full Service Access Network) model is used:

$$K_n = K_1 \cdot n^{0.6} \tag{18}$$

where K_n is the crosstalk parameter (crosstalk coupling) of disturbance from n sources of the same class (within the meaning of spectral compatibility), K_1 is the crosstalk parameter (crosstalk coupling) of disturbance from one source (so-called worst case), n is the number of disturbing lines (interval 1 to 49).

To reduce demanding crosstalk computing, it is possible to use the findings of spatial selection. A spatial selection divides lines into groups, according to a geometrical construction of a metallic cable [11], [12].

Then (18) can be expressed by:

$$K'_n = \left(\sum_{i=1}^{S} \left(K_i \cdot n_i^{0,6} \right)^{1/0.6} \right)^{0.6} \tag{19}$$

where K_i is the crosstalk parameter of disturbance from one source (so-called worst case) in group i, K'_n is the crosstalk parameter with use of the spatial selection method, n is the number of disturbing lines in group i, interval 1 to 49, S is the number of disturbing groups.

To reduce the demanding computations even more it is possible to carry out only so-called partial coordination. In that case only some of disturbing lines (so-called worst case) are taken under consideration. When applying partial coordination, the process starts with lines so-called worst case that have the highest crosstalk parameter (or crosstalk coupling) in the given group, and ends with lines that have the lowest crosstalk parameters. Equation (20) shows how to determine the crosstalk parameter with use of spatial selection method for a partial coordination (within the meaning of partial crosstalk cancellation):

$$K_n^* = \left(\sum_{i=1}^{S} \left(K_i \left(n_i - n_{i,k} \right)^{\varphi} \right)^{1/\varphi} \right)^{\varphi} \tag{20}$$

where $n_{i,k}$ is the number of coordinated lines in group i, n_i is the total number of lines in group i, K^*_n is the crosstalk parameter with use of spatial selection method for a partial coordination, φ is a coefficient in interval <0.6;1>, so between values for an empiric worst case model and a linear model.

The new method of modeling the transmission performance for the purposes of theoretical simulation, determines the reduction of crosstalk disturbance with repeating calculation of the crosstalk parameter K_i (from one group of lines or inside of the one group).

Equation (20) represents the sum of crosstalk couplings from surrounding groups or from all symmetrical pairs to the tested pair. When using the modulation VDMT value of the K^*_n will decline along with the growing number of coordinated systems. Reduction of the crosstalk couplings can theoretically be simulated by changing the value of crosstalk coupling K_i. The advantage of such a crosstalk calculation is the ability to use verified FSAN model (18). This procedure also does not require the empirically determining a new coefficient φ in (20). The new procedure for calculating crosstalk coupling in the FSAN model must respect two factors. The first is to respect the changing location of the so-called worst case, therefore a symmetrical pair with the greatest crosstalk coupling when compared with the tested pair. The second factor is the need to respect the change in the number of disturbance sources (symmetrical pairs) compared to the tested pair.

3.1 Complying with a Changing Location of a Disturbing Line

When calculating, the following assumptions are taken under consideration:

- Using a standard 1% worst case model to calculate crosstalk PSD (Power Spectral Density) in a group of 50 pairs [13].
- Because 1% of 50 pairs is not an integral value, the closest higher number is used, in this case 2% (it matches 1 pair).
- Using parameters of a real, metallic cable TCEPKPFLE.
- Using a normal distribution of those parameter values with average value, which equals to the crosstalk parameter in the linear crosstalk model [10]. Distribution function of the normal distribution has 0.02 value

for so-called worst case crosstalk parameter $K_i = 1.303 \cdot 10^{-16}$ (obtained by a real measurement), respectively for recalculated crosstalk parameter k_i in dB (k_i = -10·log K_i, for frequency in Hz and length in m). A dispersion of the normal distribution could be calculated by numerical solution of a distribution function with the required parameters.

When modeling the partial coordination, it all begins from pairs with the highest crosstalk. The value of a residual crosstalk parameter $K_{i,z}$ (respectively $k_{i,z}$ in dB) depends on the decreasing number of the strongest sources of crosstalk and the changing location of line with the highest crosstalk (e.g. new so-called worst case). When a full coordination of all 49 lines is gained the $K_{i,z}$ parameter equals to 0. In this case the crosstalk disturbances are suppressed under a level of N_{AWGN} (Additive White Gaussian Noise).

Values $K_{i,z}$ calculated for real metallic cable TCEPKPFLE with worst case crosstalk parameter $K_i = 1.303 \cdot 10^{-16}$ (value obtained from real measurements) are shown in Tab. 2.

Number of coordinated lines	$k_{i,z}$ [dB]	$K_{i,z}$
0	158.85	$1.303 \cdot 10^{-16}$
1	159.84	$9.470 \cdot 10^{-17}$
2	160.49	$7.810 \cdot 10^{-17}$
5	161.74	$5.325 \cdot 10^{-17}$
10	163.06	$3.477 \cdot 10^{-17}$
25	165.77	$2.480 \cdot 10^{-17}$
40	168.62	$4.350 \cdot 10^{-18}$
48	172.37	$7.357 \cdot 10^{-19}$
49	0	0

Tab. 2. $K_{i,z}$ parameter in dependence on the number of coordinated lines.

3.2 Complying with a Changing Number of the Disturbing Lines

To be able to use a crosstalk worst case model, it is necessary for the residual crosstalk $K_{i,z}$ to respect decreased number of disturbing lines. In other case it would be necessary to determine again the value of the coefficient φ in (20).

For the total FEXT crosstalk without using VDMT coordination applies:

$$PSD_{FEXT,C}(f) = K_{i,z} \cdot n_i^{0.6} \cdot f^2 \cdot l \cdot |H(f)|^2 \cdot PSD_{TX}(f) \quad (21)$$

where n_i is the total number of disturbing lines in group i, $K_{i,z}$ is a parameter of the residual crosstalk in group i, $PSD_{FEXT,C}(f)$ is a total crosstalk FEXT without using VDMT, $PSD_{TX}(f)$ is a transmission PSD from one xDSL system, $|H(f)|$ is module of line transmission function.

The same method will be used, when the partial coordination is applied successively to lines with the highest crosstalk. Thanks to application of the partial coordination total FEXT will decrease by:

$$PSD_{FEXT,k}(f) = K_{i,z} \cdot n_{i,k}{}^{0.6} \cdot f^2 \cdot l \cdot |H(f)|^2 \cdot PSD_{TX}(f) \quad (22)$$

where $n_{i,k}$ is the number of coordinated disturbing lines in group i, $K_{i,z}$ is a parameter of the residual crosstalk in group i, $PSD_{FEXT,k}(f)$ is reduced FEXT crosstalk from so-called worst, $PSD_{TX}(f)$ is the transmission PSD from one xDSL system, $|H(f)|$ is module of line transmission function.

In (22) the same parameter of the residual crosstalk $K_{i,z}$ as in (21) can be used, because crosstalk from so-called worst case line was eliminated. The crosstalk parameter of this line characterizes also the crosstalk parameter of the whole group i (or cable).

A difference between crosstalk level $PSD_{FEXT,C}(f)$ (21) and eliminated crosstalk $PSD_{FEXT,k}(f)$ (22) is a $PSD_{FEXT,z}(f)$ of the residual crosstalk:

$$\begin{aligned} PSD_{FEXT,z}(f) &= K^*_{i,z} \cdot n_{i,z}{}^{0.6} \cdot f^2 \cdot l \cdot |H(f)|^2 \cdot PSD_{TX}(f) = \\ &= PSD_{FEXT,C}(f) - PSD_{FEXT,k}(f) \end{aligned} \quad (23)$$

where $PSD_{FEXT,C}(f)$ is the total crosstalk FEXT without using VDMT, $PSD_{FEXT,k}(f)$ is the reduced FEXT crosstalk from so-called worst, $PSD_{FEXT,z}(f)$ is the residual FEXT crosstalk.

This residual crosstalk will have influence on the tested line. The FSAN model (18) can still be used to calculate crosstalk, because the group of lines with existing so-called worst case is still being used. Only its location and total number of disturbing lines have changed. The value of this forced parameter of the residual crosstalk $K^*_{i,z}$ is:

$$\begin{aligned} K^*_{i,z} &= \frac{K_{i,z} \cdot f^2 \cdot l \cdot |H(f)|^2 \cdot PSD_{TX}(f)\left(n_i^{0.6} - n_{i,k}{}^{0.6}\right)}{n_z{}^{0.6} \cdot f^2 \cdot l \cdot |H(f)|^2 \cdot PSD_{TX}(f)} = \\ &= K_{i,z} \frac{\left(n_i^{0.6} - n_{i,k}{}^{0.6}\right)}{\left(n_i - n_{i,k}\right)^{0.6}} \end{aligned} \quad (24)$$

where n_i is the total number of disturbing lines in group i, $n_{i,k}$ is the number of coordinated disturbing lines in group i, $K_{i,z}$ is the parameter of the residual crosstalk in group i, $K^*_{i,z}$ is a forced parameter of the residual crosstalk when transmission from lines $n_{i,k}$ is coordinated with n_i in group i. Equation (24) considers changing number of disturbing lines, if the coordination (within the meaning of crosstalk cancelation) is executed.

4. Result of Simulation

The method described in section 2 was implemented in a simulation program, which is modeling situation in a real access network. The program is able to estimate the performance parameters of transmission for different types of xDSL lines.

The tested line is VDSL2 with a frequency plan VDSL2 B7-10 (the code 997E30-M2x-NUS0 by ITU-T G.993.2). The transmission environment is a metallic cable TCEPKPFLE 75x4x0.4 mm. The total number of disturbance systems of the same spectral class is 49. The subscriber loop length starts at 0.1 km, ends at 2 km, with step 0.1 km. The transmission rates are calculated based on the partial cancellation of disturbing sources, from no coordinated systems to full coordination of all 49 disturbing sources of the same class.

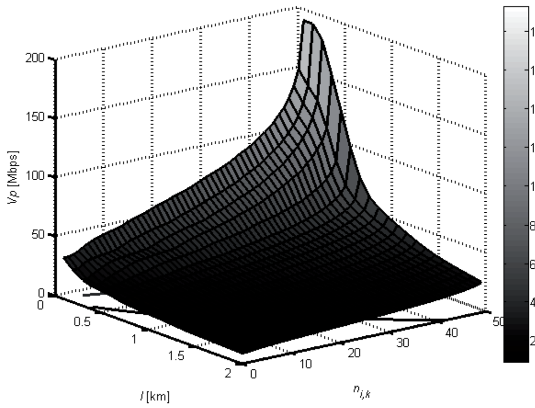

Fig. 5. Transmission rate VDSL2, frequency band plan B7-10 for downstream direction.

For example, for the line 0.4 km long without applying any coordination simulation it shows that it is possible to reach transmission rate of 19.472 Mbps and 18.744 Mbps for simulation with the advanced FEXT model. When applying the transmission coordination for 1 line (so-called worst case), it is possible to reach the value of 20.776 Mbps (resp. 19.600 Mbps). If coordination spreads to 5 lines, the value of 26.464 Mbps (resp. 23.744 Mbps) could be reached. For the line 0.4 km long with full coordination it is possible to gain the transmission rate of 154.720 Mbps (resp. 117.808 Mbps).

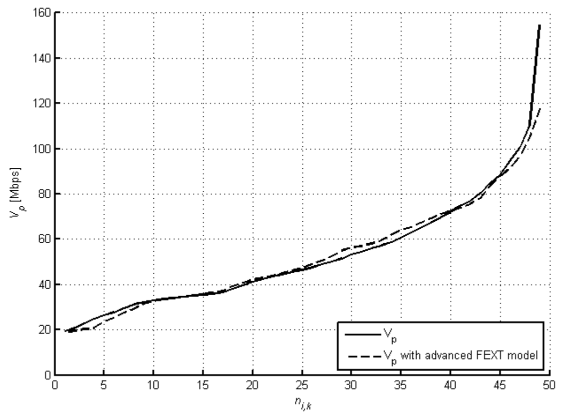

Fig. 6. Dependence of the transmission rate on the number of coordinated lines in downstream direction.

With this simulation program it is also possible to verify the theoretical results of the advanced FEXT model.

Fig. 6 shows the dependence of the maximum transmission rate in the downstream direction on the number of coordinated systems. The solid line shows the maximum transmission rate in downstream direction calculated based on the procedure presented in section 3. The dashed line shows the value of the maximum transmission rate in the downstream direction with values of the crosstalk couplings, which are calculated using the procedure from section 2.

Fig. 7 shows the comparison of crosstalk coupling (crosstalk parameters) that are used in the simulation program to calculate the performance of VDSL2 transmission with VDMT modulation using the method described in section 3 and crosstalk coupling gained with use of the theoretical advanced FEXT model. The full curve shows the probable behavior. Applying coordination in about four or five adjacent lines, which are the largest source of the disturbance, makes sense. An important benefit is an application of full coordination, when the disturbance level of FEXT crosstalk is gradually suppressed below the N_{AWGN} level. The dashed curve (k_{fext} in dB is K_{FEXT} from (16)) takes under consideration a geometric design of the cable and a position within the same subgroup, a surrounding subgroup and a distant subgroup.

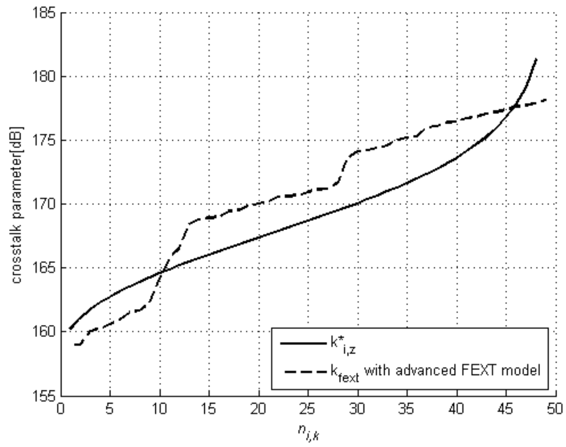

Fig. 7. Dependence of k_{fext} crosstalk coupling values.

5. Conclusion

The proposed advanced FEXT model is based on analysis of the internal configuration of particular cable elements, where the derivation of crosstalk relations between specific pairs is based on the simulation and calculation of capacitive and inductive unbalances (couplings) within the considered pairs. In this way, based on given parameters of the transmission medium, the model allows to generate realistic data for simulation of xDSL lines. Generated curves by the above procedure, by its nature are considerably closer to the actual frequency characteristics of crosstalk.

The modeling program for simulation VDMT modulation benefits allows obtaining the theoretical results of transmission performance in xDSL lines. The program allows performing calculations for given conditions, which represent the typical situation in real access networks. The obtained results allow analyzing, for example, the ability to suppress crosstalk through VDMT modulation without the need to perform time-consuming measurements on real metallic cables, or simulate real operating conditions in metallic cables.

The results obtained on the specific cable with group design confirmed the accuracy of reflection concerning the impact of a structural elements arrangement on the resulting crosstalk coupling. Also reflection on the impact of the relative positions of disturbing and disturbed sources was confirmed.

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References

[1] BRADY, M. H., CIOFFI, J. M. The worst-case interference in DSL systems employing dynamic spectrum management. *Journal on Applied Signal Processing*. Hindawi Publishing Corporation EURASIP. Vol. 2006, Article ID 78524, p. 1-11.

[2] GINIS, G., CIOFFI, J. Vectored transmission for digital subscriber line systems. *IEEE Journal on Selected Areas in Communications*, 2002, vol. 20, no. 5, p. 1085-1104.

[3] CENDRILLON, R., GINIS, G., MOONEN, M., ACKER, K. V. *Partial Crosstalk Precompensation in Downstream VDSL*. UQ Library. The University of Queensland Australia. [online]. [cit. 2008-7-21]. Available at: http://espace.library.uq.edu.au/-eserv/UQ:9944/partial_precode.pdf.

[4] VODRAZKA, J. Multi-carrier modulation and MIMO principle application on subscriber lines. *Radioengineering*, 2007, vol. 16, no. 4, p. 33 – 37.

[5] LAFATA, P. Modeling of far-end crosstalk in multi-quad cables based on capacitive and inductive unbalances between pairs. In *TSP - 32nd International Conference on Telecommunications and Signal Processing* [CD-ROM]. Budapest: Asszisztencia Szervező Kft., 2009, ISBN 978-963-06-7716-5.

[6] HUGHES, H. *Telecommunications Cables: Design, Manufacture and Installation*. Chichester: John Wiley&Sons Ltd., June 1997. ISBN 0-471-97410-2.

[7] CLAYTON, R. P., McKNIGHT, J. W. Prediction of crosstalk involving twisted pairs of wires-Part I: A transmission-line model for twisted-wire pairs. *IEEE Transactions on Electromagnetic Compatibility*, May 1979, vol. EMC-21, no. 2.

[8] CIOFFI, J. M., FANG, J. L. *A Temporary Model for EFM/MIMO Cable Characterization. IEEE 802.3 Standards Contribution*. Los Angeles, CA, October 2001.

[9] CHEN, W. Y. *DSL: Simulation Techniques and Standards Development for Digital Subscriber Line System*. Indianopolis: Macmillan Technology Series, 1998. ISBN 1-57870-017-5.

[10] STARR, T., SORBARA, M., CIOFFI, J. M., SILVERMAN, P. *DSL Advances.* Upper Saddle River, USA: Prentice Hall, 2002. 576 s. ISBN 0-13-093810-6.

[11] LAFATA, P., JARES. P., SYKORA. J. Influence of xDSL spatial selection for DMT modulation. In *Proc. of the 8th International Conference on Research in Telecomunication Technology RTT 2007* [CD-ROM]. Zilina (Slovakia), 2007, p. 227-230. ISBN 978-80-8070-735-4.

[12] LAFATA, P., VODRAZKA, J. Simulations and statistical evaluations of FEXT crosstalk in xDSL systems using metallic cable constructional arrangement. In *TSP - 31st International Conference Telecommunications and Signal Processing* [CD-ROM]. Budapest: Asszisztencia Szervező Kft., 2008, ISBN 978-963-06-5487-6.

[13] UZCATEGUI, R., JAYANT, N. S. A new mathematical interpretation of the FSAN crosstalk-summing method. In *Proceedings IEEE-ICC*, June 2002.

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