Arbitrary Shape Electromagnetic Transparent Device Based on Laplace's Equation

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Abstract. *Transparent device is deliberately-designed electromagnetic structure that is transparent to electromagnetic wave. It can be used as a radome structure which is capable of protection antenna inside without sacrificing its performance. In this paper, two-dimensional (2D) arbitrary shape electromagnetic transparent device is designed based on transformation optics. Laplace's equation is adopted to construct the coordinate mapping between the original space and the transformed space. The design method is flexibly extended to three-dimensional (3D) case, which greatly enhances the applicability of transparent device. The protection of a horn antenna is taken as an example to show the effectiveness of the transparent device. Since the performance of the transparent device is independent on the inner antenna, it can be designed separately. Full-wave simulations are made to validate the results.*

Keywords

Transparent device; metamaterials; coordinate transformation; finite element method.

1. Introduction

Transformation optics [1], [2] provides a new design methodology allowing unprecedented manipulation of electromagnetic wave propagation, with the invisible cloak as the most prominent example [3-6]. Besides, a wide variety of applications other than cloaking has been recently presented, such as field concentrators [6], [7], transparent device [8-10], beam shifters [11], field rotator [12], waveguide bends and corners [13], etc. Among these novel applications, transparent device is a special kind of transformation media that protect electric devices inside but do not affect their performances at all. The 2D circular and the elliptical electromagnetic transparent devices were firstly proposed by Yu et al. [8]. Later, Yang and Zhou et al. [9], [10] developed the generalized material parameter equations for 2D transparent devices, of which the contour can be described by a continuous function $R(\theta)$ with period 2π. But the determination of the corresponding material parameters in analytical form for a transparent device with complex shape is a formidable task, and it is rather difficult to obtain the material parameters with analytical form when the contour can not be described by a simple function. Therefore, it is quite necessary to design the transparent device using numerical method.

Recently, there has been an explosion of interests in the study of numerical design of transformation media [14-16]. However, these works only focus on invisible cloak or acoustical cloak. In this paper, we extend our previous work [9] to design arbitrary transparent device using numerical method. Laplace's equation is adopted to establish the mapping between the original and the transformed coordinates. Since the design of the transparent device is completely converted to numerical solution of Laplace's equation with proper boundary conditions, and the coordination transformation does not need to be known in advance, it is a kind of inverse method different from that of Qiu et al. [17]. To show the effectiveness of the design method, the protection of a horn antenna is taken as an example, and the transparent device is extended to 3D case. Full wave simulations based on finite element method verified the design method.

2. Theoretical Model

According to the coordinate transformation method, under a space transformation from the original coordinates (x_1, x_2, x_3) to the new coordinates $(x_1'(x_1,x_2,x_3), x_2'(x_1,x_2,x_3), x_3'(x_1,x_2,x_3))$ the permittivity ε' and the permeability μ' in the transformed space are given by [18]

$$\varepsilon' = \mathbf{A}\varepsilon\mathbf{A}^T / \det\mathbf{A}\,, \tag{1a}$$

$$\mu' = \mathbf{A}\mu\mathbf{A}^T / \det\mathbf{A} \tag{1b}$$

where ε and μ are the permittivity and permeability of the original space. $\mathbf{A}$ is the Jacobian transformation matrix with components $A_{ij} = \partial x_i'/\partial x_j$. It is the derivative of the transformed coordinates with respect to the original coordinates. det $\mathbf{A}$ is the determinant of the matrix. The determination of matrix $\mathbf{A}$ is the key issue for designing the transformation mediums.

Fig. 1 shows the scheme for constructing an arbitrary transparent device. The region with boundary b in the original space is stretched to the region with boundary c' in the transformed space, while keeping the outer boundary of the region fixed. Thus, the region bounded between b and d is relatively compressed to the region between c' and d'. This procedure can be expressed by $U'\big|_{x=a} = a', U'\big|_{x=b} = c'$ for the stretching region, and $U'\big|_{x=b} = c', U'\big|_{x=d} = d'$ for the compressive region. The operator U' is the new coordinate for a given point during the transformation. Under this boundary condition, we can build the Laplace's equation which indicates the relationship between the original coordinate x and the transformed coordinates x' as

$$\begin{cases} \left(\frac{\partial^2}{\partial x_1{}^2} + \frac{\partial^2}{\partial x_2{}^2} + \frac{\partial^2}{\partial x_3{}^2}\right) x_i' = 0 \qquad i = 1,2,3 \\ U'\big|_{x=a} = a', U'\big|_{x=b} = c' \qquad a < x < b \\ U'\big|_{x=b} = c', U'\big|_{x=d} = d' \qquad b < x < d \end{cases} \tag{2}$$

By solving the Laplace's equation, the coordinate relationship $x_i'(x_1,x_2,x_3)$ can be obtained. In order to keep from the singular solution of Laplace's equation, we can use the inverse form as

$$\begin{cases} \left(\frac{\partial^2}{\partial x_1'^2} + \frac{\partial^2}{\partial x_2'^2} + \frac{\partial^2}{\partial x_3'^2}\right) x_i = 0 \qquad i = 1,2,3 \\ U\big|_{x'=a'} = a, U\big|_{x'=c'} = b \qquad a' < x' < c' \\ U\big|_{x'=c'} = b, U\big|_{x'=d'} = d \qquad c' < x' < d' \end{cases} \tag{3}$$

From the solution of (3), we can calculate the inverse Jacobian matrix elements $A'_{ij} = \partial x_i'/\partial x_j$, then determine the Jacobian transformation tensor via equation $\mathbf{A}=(\mathbf{A}')^{-1}$. The transformed material parameter can be calculated according to (1).

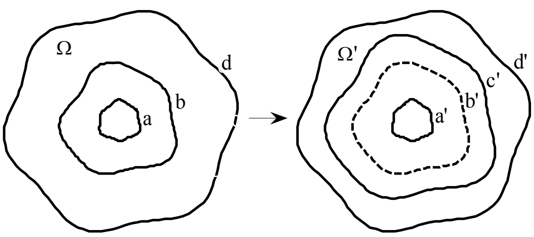

Fig. 1. The scheme for the construction of arbitrary transparent device.

3. Application to Circular Cylindrical Transparent Device

Firstly, let's consider the circular cylindrical transparent device, of which the radii of the inner and outer boundaries are r_1, r_2, r_3 and r_4. The Laplace's equation in the cylindrical coordinate system is expressed as

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial u_i}{\partial r'}\right) + \frac{1}{r'^2}\frac{\partial^2 u_i}{\partial \theta'^2} + \frac{\partial^2 u_i}{\partial z'^2} = 0 \tag{4}$$

where u_i (i = 1,2,3) denotes the original coordinate component r, θ, z, respectively. For two dimensional transparent device, if we set $\theta = \theta'$ and $z = z'$, then coordinate r satisfies the following equation

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial r}{\partial r'}\right) = 0 \tag{5}$$

With boundary conditions of $r(r' = r_1) = r_1$, $r(r' = r_3) = r_2$ for the stretching region, and $r(r' = r_4) = r_4$, $r(r' = r_3) = r_2$ for the compressive region, the solution of (5) is given by

$$r' = (\tau / r_3)^{\frac{r-\tau}{\tau - r_2}} + \tau \tag{6}$$

where $\tau = r_1$ for the stretching region ($r_1 < r' < r_3$), $\tau = r_4$ for the compressive region ($r_3 < r' < r_4$). Then, the principal stretches corresponding to the (6) are given by

$$\lambda_r = dr'/dr = r' \ln(\tau / r_3)/(\tau - r_2)\,, \tag{7a}$$

$$\lambda_\theta = r'/r = \frac{r'}{(\tau - r_2)\log_{\tau/r_3}(r'/\tau) + \tau}\,, \tag{7b}$$

$$\lambda_z = 1\,. \tag{7c}$$

And thus material parameters for the cylindrical transparent device can be obtained as

$$\varepsilon_r = \lambda_r / \lambda_\theta \lambda_z = \ln(r'/\tau) + \tau \ln(\tau / r_3)/(\tau - r_2)\,, \tag{8a}$$

$$\varepsilon_\theta = \lambda_\theta / \lambda_r \lambda_z = 1/\varepsilon_r\,, \tag{8b}$$

$$\varepsilon_z = \lambda_z / \lambda_\theta \lambda_r = \frac{(\tau - r_2)\log_{\tau/r_3}(r'/\tau) + \tau}{r'^2 \ln(\tau / r_3)/(\tau - r_2)}\,. \tag{8c}$$

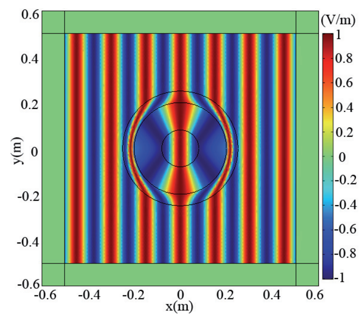

Fig. 2. Electric field distribution in the vicinity of the circular cylindrical transparent device. Material parameters are given by (8).

In order to validate the above material parameter equations, we make full-wave simulations on the cylindrical transparent device using the software COMSOL Multiphysics, which is based on finite element method. Fig. 2 shows the electric field distribution in the vicinity of the circular cylindrical transparent device with material parameters given by (8) under TE plane wave irradiation, of which the frequency is 2 GHz. The radii of the circular cylindrical transparent device are chosen as $r_1 = 0.08$,

$r_2 = 0.1$, $r_3 = 0.2$, $r_4 = 0.25$ in SI units. In the simulation, the whole computational region is surrounded by a perfectly matched layer (PML) that absorbs waves propagating outwards from the bounded domain. From Fig. 2, we can clearly observe that the field pattern of TE plane wave in the inner region ($r' < r_1$) of the transparent device is identical to that outside the transformation region ($r_1 < r' < r_4$). This result validates the performance of the circular cylindrical transparent device.

4. Application to Arbitrary Shaped Transparent Device

For an arbitrary transparent device, of which the boundary is difficult to be expressed in an analytical form, Laplace's equation must be solved numerically. In what follows, we show the design of the transparent device with arbitrary geometry with the help of the commercial software COMSOL Multiphysics.

Firstly, we depict four boundaries a', b', c' and d' for an arbitrary transparent device, of which the intermediate layer b' is set to be conformal with c' ($b' = kc'$, $k = 0.5$). In the Cartesian coordinate system, two Laplace's equations should be solved to get the coordinate transformation, since the coordinate has components x and y. This is achieved by adding two partial difference equation (PDE) modes provided by COMSOL. In the PDE modes of Laplace's equation, we set $U_x(c') = kx$, $U_y(c') = ky$ for boundary c', and $U_x(a') = x$, $U_x(d') = x$, $U_y(a') = y$, $U_y(d') = y$ for boundary a' and d'. After solving these Laplace's equations, the inverse Jacobian matrix elements $A'_{ij} = \partial x_i/\partial x_j'$ can be obtained. The Jacobian transformation tensor is determined via $\mathbf{A}=(\mathbf{A}')^{-1}$, and material parameters can be obtained according to (1). Then, to check the designed transparent device, the simulation model will be illuminated by the electromagnetic wave with the help of RF model in the same software. Fig. 3(a) shows the electric field distribution in the vicinity of the arbitrary shaped transparent device under TE plane wave irradiation. It is seen that the field pattern in the inner region of the transparent device recovers to the original wave fronts. Next, we show the performance of the transparent device under cylindrical wave irradiation. The line source with current of 10^{-3}A is located at (-0.4, 0), (0, 0.4), (-0.4, -0.4) and for Figs. 3(b)-(d), respectively. Apparently, the phenomenon of transparency can also be observed, and it is independent on the location of the line source.

Fig. 4 and Fig. 5 show the distributions of material parameters for the stretching region and the compressive region, respectively. It can be clearly observed that the values of permittivity and permeability in the transformation regions fluctuate with the contours of the device and are highly anisotropic. It must be realized with the metamaterial technology.

The proposed method can be used to design the arbitrary transparent device with many separated stretching regions. As an example, we show in Fig. 6 the simulation result of a square transparent device with four separated stretching regions. The circular stretching regions with radii of $r_1 = 0.05$, $r_3 = 0.1$are centered at $O_1(-0.15, 0.15)$,

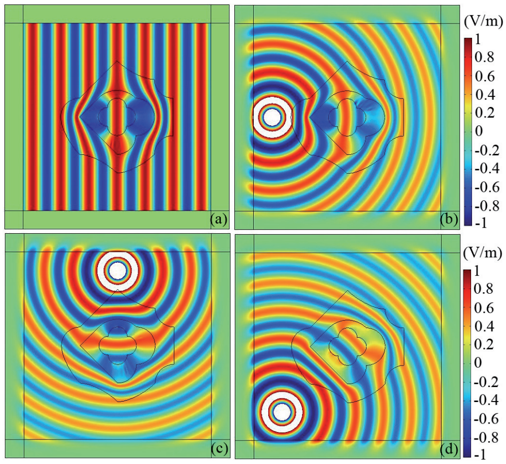

Fig. 3. The electric field distribution in the vicinity of the arbitrary shaped transparent device. (a) The TE plane wave is irradiated from the left. (b)-(d) Simulation results under cylindrical wave irradiation.

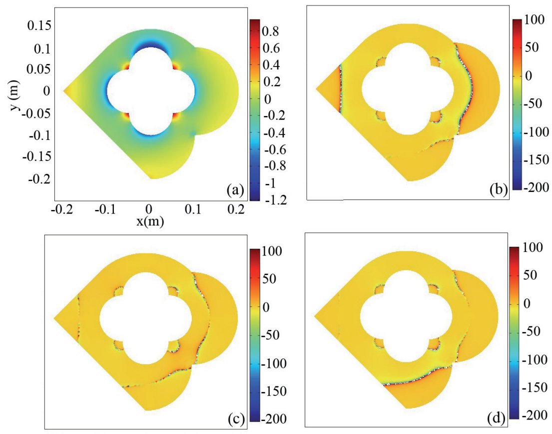

Fig. 4. Permittivity and permeability tensors for the transparent device in the stretching region. (a) ε_{zz}, (b) μ_{xx}, (c) μ_{xy}= (μ_{yx}), (d) μ_{yy}.

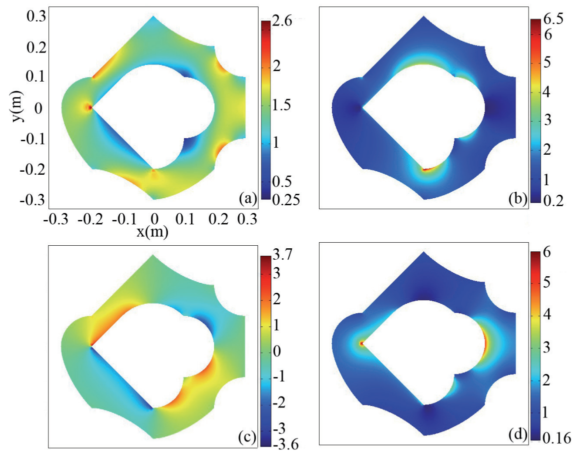

Fig. 5. Permittivity and permeability tensors for the transparent device in the compressive region. (a) ε_{zz}, (b) μ_{xx}, (c) μ_{xy}= (μ_{yx}), (d) μ_{yy}.

$O_2(0.15, 0.15)$, $O_3(-0.15, -0.15)$, $O_4(0.15, -0.15)$, respectively. The intermediate boundary *b'* is conformal with boundary *c'* with a ratio of $k = 2/3$. The boundary conditions for the numerical solution of Laplace's equation then become $U_x(c_i') = k(x + O_{ix})$, $U_y(c_i') = k(y + O_{iy})$ for boundary c_i', and $U_x(a_i') = x + O_{ix}$, $U_x(d_i') = x + O_{ix}$, $U_y(a_i') = y + O_{iy}$, $U_y(d_i') = y + O_{iy}$ for boundary a_i' and d_i' ($i = 1, 2, 3, 4$). Fig. 6(a) shows the electric field distribution in the computational domain. The TE plane wave is propagated along x axis. It is seen that the transparent effect can also be achieved in the device with multiple stretching regions. The same phenomenon can also be observed under cylindrical wave irradiation as shown in Fig. 6(b).

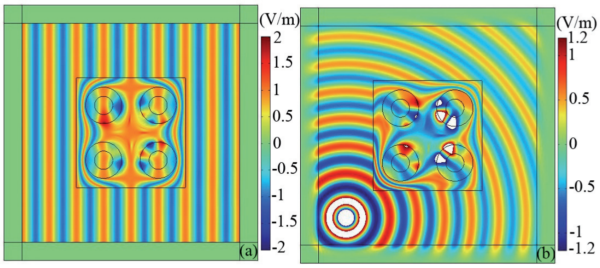

Fig. 6. The square transparent device with separated stretching regions. (a) The electric field distribution in the vicinity of the transparent device under TE plane wave irradiation. (b) The electric field distribution in the vicinity of the transparent device under cylindrical wave irradiation.

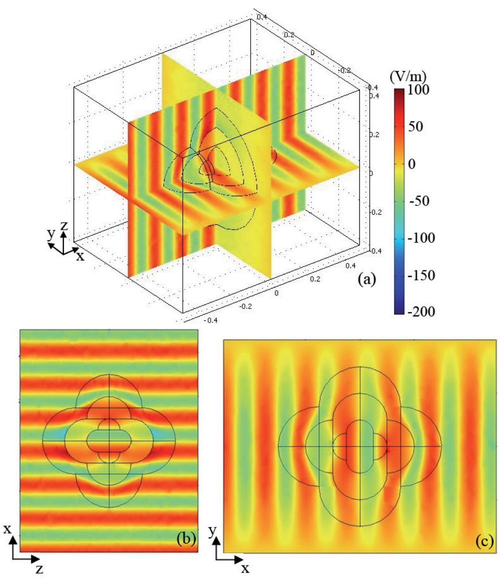

Fig. 7. Simulation results of the 3-D transparent device with arbitrary geometry. (a) 3-D profile of the electric field (Ez) distribution. (b) Electric field distribution in *y*=0 plane. (b) Electric field distribution in *z*=0 plane.

The design method can also be extended to three-dimensional case. The simulation results are shown in Fig. 7. The TE plane wave (2 GHz) with electric field polarized along z axis is propagated in x direction. It is seen that the device is transparent to the incoming electromagnetic waves. The electric field distribution in z=0 plane as shown in Fig. 7(c) is slightly fluctuated. This is believed to be induced by the numerical method due to discretization. It can be improved by fine mesh with the cost of memory consumption and computation time.

The above properties of the transparent device are useful for radome structures, and for the protection of electrical equipment such antenna and radar station. Generally, to design a traditional standard protection radome, its interaction with the inner antenna should not be ignored [19]. But with the coordinate transformation method, the antenna and the radome can be designed separately. For example, the shape of the radome can be dictated by aerodynamic considerations. Once the protection radome is designed, its performance will be independent on the antenna located inside. In the following discussion, we take the horn antenna as an example to show the transparent device's performance as a protection radome. In Fig. 8(a), the horn antenna is covered with the transparent device. Fig. 8(b) shows the radiation of the horn antenna in free space. As can be seen, the near field distributions of the antenna with and without the transparent device are almost the same. The normalized far field intensity profiles are shown in Fig. 8(c). We can observe that the far field intensity of the horn antenna covered with transparent device agrees well with the far field irradiation of the horn antenna in free space. It further demonstrates the effectiveness of the transparent device in antenna protection.

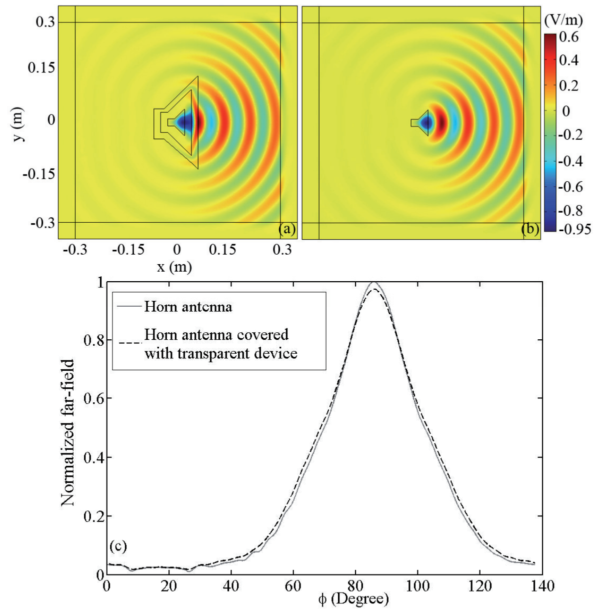

Fig. 8. (a) Near field distribution of the horn antenna covered with transparent device. (b) The irradiation of the horn antenna in free space. (c) Normalized far-field intensity.

5. Conclusions

In conclusion, we propose a general and flexible method for designing arbitrary transparent device based on Laplace's equation. Different from traditional analytical method, of which the coordinate transformation must be known in advance, this method is completely built on numerical solution of Laplace's equation with proper boundary conditions. The numerical method provides a powerful tool for designing transparent device with irregular shape and can be easily extended to 3-D case. It is expected that our works are helpful for designing new transparent devices for the protection of electrical equipment and contribute to more applications in electromagnetic field engineering.

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