

# Attenuation in Rectangular Waveguides with Finite Conductivity Walls

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**Abstract.** *We present a fundamental and accurate approach to compute the attenuation of electromagnetic waves propagating in rectangular waveguides with finite conductivity walls. The wavenumbers  $k_x$  and  $k_y$  in the  $x$  and  $y$  directions respectively, are obtained as roots of a set of transcendental equations derived by matching the tangential component of the electric field ( $E$ ) and the magnetic field ( $H$ ) at the surface of the waveguide walls. The electrical properties of the wall material are determined by the complex permittivity  $\epsilon$ , permeability  $\mu$ , and conductivity  $\sigma$ . We have examined the validity of our model by carrying out measurements on the loss arising from the fundamental  $TE_{10}$  mode near the cutoff frequency. We also found good agreement between our results and those obtained by others including Papadopoulos' perturbation method across a wide range of frequencies, in particular in the vicinity of cutoff. In the presence of degenerate modes however, our method gives higher losses, which we attribute to the coupling between modes as a result of dispersion.*

## Keywords

Attenuation, rectangular waveguides, finite conductivity, electrical properties.

## 1. Introduction

Propagation of electromagnetic waves in circular waveguides has been widely investigated, for waveguides with lossy [1] and superconducting walls [2], [3], unbounded dielectric rod [4], bounded dielectric rod in a waveguide [5], and multilayered coated circular waveguide [6]. The computations given by these authors were based on a method suggested by Stratton [7]. The circular symmetry of the waveguide allows the boundary matching equations to be expressed in a single variable which is the radial distance  $r$ . The eigenmodes could therefore be obtained from a single transcendental equation. This ap-

proach cannot be implemented in the case of rectangular symmetry where a 2D Cartesian coordinate system must be used.

Rectangular waveguides are employed extensively in microwave and millimeter wave receiver [8], [9], [10] since they are much easier to manipulate than circular waveguides (bend, twist) and also offer significantly lower cross polarization component. Despite that, not much has recently been published on analyzing the guided propagation of electromagnetic signals in lossy or superconducting rectangular waveguides.

The approximate power-loss method has been widely used in analyzing wave attenuation in lossy rectangular waveguides as a result of its simplicity and because it gives reasonably accurate result, when the frequency of the signal is well above cutoff [7], [11], [12], [13]. In this method, the field expressions are derived assuming perfectly conducting walls, allowing the solution to be separated into TE and TM modes. To calculate the attenuation, ohmic losses are assumed to exist due to small field penetration into the conductor walls. The power-loss method however fails near cutoff, as the attenuation obtained using this method diverges to infinity when the signal frequency  $f$  approaches the cutoff frequency  $f_c$ .

Bladel [14], and Robson [15] discussed degenerate modes propagation in lossy rectangular waveguides, but neither was able to compute the attenuation values accurately near cutoff. Like the power-loss method, their theories predict infinite attenuation at cutoff. An expression valid at all frequencies is given by Kohler and Bayer [16] and reiterated by Somlo and Hunter in [17]. This expression however is only applicable to the  $TE_{10}$  dominant mode.

The perturbation solution developed by Papadopoulos [18] shows that the propagation of a mode does not merely stop at  $f_c$ . Rather, as the frequency approaches  $f_c$ , transition from a propagating mode to a highly attenuated mode takes place. The propagation of waves will only cease when  $f=0$ . Papadopoulos' perturbation method (PPM) shows that the attenuation at frequencies well above  $f_c$  remains in

close agreement with that computed using the power loss method for non-degenerate modes. Because of this reason, PPM is perceived as a more accurate technique in computing the loss of waves traveling in waveguides. A similar solution has been derived by Gustincic using the variational approach in [12], [19].

In [20], we have introduced a novel accurate technique to compute the propagation constant of waves in rectangular walls with finite conductivity. The method has been applied to investigate the attenuation of the dominant mode. In this paper, we shall develop further the method in [20] to show the presence of mode coupling effects in degenerate modes. Here, in order to present a complete scheme, we outline the derivation of the transcendental equation in [20] for convenience. In our method, the solution for the attenuation constant is found by solving two transcendental equations derived from matching the tangential components of the electromagnetic field at the waveguide walls and making use of the surface impedance concept. The attenuation constants for the dominant non-degenerate TE<sub>10</sub> mode and the degenerate TE<sub>11</sub> and TM<sub>11</sub> modes are computed and compared with the power-loss method and the PPM. We will demonstrate that our method gives more realistic values for the degenerate modes since the formulation allows co-existence and exchange of power between these modes while other methods treat each one independently.

Finally, we would like to emphasize that significant deviation between the power loss and rigorous methods computations start to appear at frequencies well below cutoff where waveguides are used as filters and for other applications. At frequencies immediately below cutoff, the attenuation diverges at very high rate that at some stage power transmission in the waveguide becomes negligible. At these frequencies (very close to cutoff) the deviation in the computed results between our method and the power loss results is substantial. Experimentally measured attenuation confirms the integrity of our computation close to cutoff.

## 2. Formulation

### 2.1 Fields in Rectangular Waveguides

In a lossless waveguide, the boundary condition requires that the resultant tangential electric field  $E_t$  and the normal derivative of the tangential magnetic field  $\partial H_t / \partial a_n$  to vanish at the waveguide wall, where  $a_n$  is the normal direction to the waveguide wall. Due to the finite conductivity of the waveguide material, both  $E_t$  and  $\partial H_t / \partial a_n$  are not exactly zero at the boundary. To account for the presence of fields inside the walls, we have introduced two phase parameters;  $\phi_x$  and  $\phi_y$ , which we shall refer to as the field's penetration factors in the  $x$  and  $y$  directions, respectively.

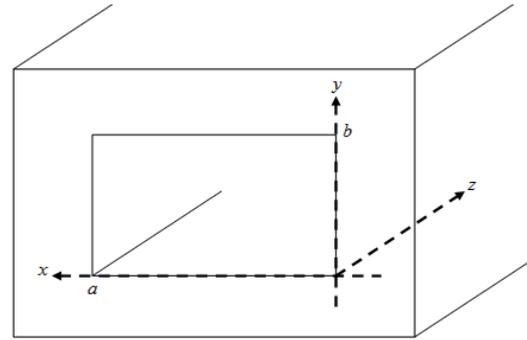


Fig. 1. A rectangular waveguide.

For waves propagating in a lossy rectangular waveguide, as shown in Fig. 1, a superposition of TM and TE waves is necessary to satisfy the boundary condition at the wall [3], [7]. The longitudinal electric and magnetic field components  $E_z$  and  $H_z$ , respectively, can be derived by solving Helmholtz homogeneous equation in Cartesian coordinate. Using the method of separation of variables [13], we obtain the following set of field equations:

$$E_z = E_0 \sin(k_x x + \phi_x) \sin(k_y y + \phi_y), \quad (1)$$

$$H_z = H_0 \cos(k_x x + \phi_x) \cos(k_y y + \phi_y) \quad (2)$$

where  $E_0$  and  $H_0$  are constant amplitudes of the fields and  $k_x$  and  $k_y$  are the wavenumbers in the  $x$  and  $y$  directions, respectively. A wave factor of form  $\exp[j(\omega t - k_z z)]$  is assumed but is omitted from the equations for simplicity. Here,  $\omega = 2\pi f$  is the angular frequency and  $k_z$  is the propagation constant.  $k_z$  for each mode will be found by solving for  $k_x$  and  $k_y$  and substituting the results into the dispersion relation:

$$k_z = \sqrt{k_0^2 - k_x^2 - k_y^2}. \quad (3)$$

Here,  $k_0$  is the wavenumber in free space.  $k_x$ ,  $k_y$ , and  $k_z$  are complex and may be written as:

$$k_x = \beta_x - j\alpha_x, \quad (4)$$

$$k_y = \beta_y - j\alpha_y, \quad (5)$$

$$k_z = \beta_z - j\alpha_z \quad (6)$$

where  $\beta_x$ ,  $\beta_y$  and  $\beta_z$  are the phase constants and  $\alpha_x$ ,  $\alpha_y$  and  $\alpha_z$  are the attenuation constants in the  $x$ ,  $y$ , and  $z$  directions, respectively.

Equations (1) and (2) must also apply to a perfect conductor waveguide. In that case  $E_z$  and  $\partial H_z / \partial a_n$  are either at their maximum magnitude or zero at both  $x = a/2$  and  $y = b/2$ , therefore:

$$\sin\left(\frac{k_x a}{2} + \phi_x\right) = \sin\left(\frac{k_y b}{2} + \phi_y\right) = \pm 1 \text{ or } 0. \quad (7)$$

Solving (7), we obtain,

$$\phi_x = \frac{(m\pi - k_x a)}{2}, \quad (8)$$

$$\phi_y = \frac{(n\pi - k_y b)}{2} \quad (9)$$

where the integers  $m$  and  $n$  denote the number of half cycle variations in the  $x$  and  $y$  directions, respectively and every combination of  $m$  and  $n$  defines a possible  $TE_{mn}$  and  $TM_{mn}$  modes. For waveguides with perfectly conducting wall,  $k_x = m\pi/a$  and  $k_y = n\pi/b$ , (8) and (9) result in zero penetration and  $E_z$  and  $H_z$  in (1) and (2) are reduced to the fields of a lossless waveguide. To take the finite conductivity into account we allow  $k_x$  and  $k_y$  to take complex values yielding non-zero penetration of the fields into the waveguide material. This in turn results in complex value for the propagation constant of the waveguide  $k_z$  (see (3)) which yields loss in propagation.

Substituting (1) and (2) into Maxwell's source-free curl equations and expressing the transverse field components in terms of  $E_z$  and  $H_z$  [13], we obtain:

$$H_x = \frac{j}{h^2} [k_z k_x H_0 + \omega \varepsilon_0 k_y E_0] \sin(k_x x + \phi_x) \cos(k_y y + \phi_y) \quad (10)$$

$$H_y = \frac{j}{h^2} [k_z k_y H_0 - \omega \varepsilon_0 k_x E_0] \cos(k_x x + \phi_x) \sin(k_y y + \phi_y) \quad (11)$$

$$E_x = -\frac{j}{h^2} [k_z k_x E_0 - \omega \mu_0 k_y H_0] \cos(k_x x + \phi_x) \sin(k_y y + \phi_y) \quad (12)$$

$$E_y = -\frac{j}{h^2} [k_z k_y E_0 + \omega \mu_0 k_x H_0] \sin(k_x x + \phi_x) \cos(k_y y + \phi_y) \quad (13)$$

where  $\mu_0$  and  $\varepsilon_0$  are the permeability and permittivity of free space, respectively, and  $h^2 = k_x^2 + k_y^2$ . These expressions show that the field is a superposition of TE and TM modes.

## 2.2 Constitutive Relations for TE and TM Modes

Using Maxwell equations it can be shown that the ratio of the tangential component of the electric field to the surface current density at the conductor surface is given by [21]

$$\frac{E_t}{H_t} = \sqrt{\frac{\mu}{\varepsilon}} \quad (14)$$

where  $\mu$  and  $\varepsilon$  are the permeability and permittivity of the wall material, respectively, and  $\sqrt{\mu/\varepsilon}$  is the intrinsic impedance of the wall material. The subscript  $t$  in (14) denotes tangential fields. The dielectric constant is complex and  $\varepsilon$  may be written as

$$\varepsilon = \varepsilon_0 - j \frac{\sigma}{\omega} \quad (15)$$

where  $\sigma$  is the conductivity of the wall.

At the surface of the waveguide in the  $x$ -direction at  $y = b$ ,  $E_z/H_z = -E_x/H_x = \sqrt{\mu/\varepsilon}$ . Substituting (1), (2), (10), and (12) into (14), we obtain:

$$\frac{-E_x}{H_z} = \frac{j}{h^2} \left( \frac{E_0}{H_0} k_z k_x - \omega \mu_0 k_y \right) \tan(k_y b + \phi_y) = \sqrt{\frac{\mu}{\varepsilon}}, \quad (16a)$$

$$\frac{H_x}{E_z} = \frac{j}{h^2} \left( \frac{H_0}{E_0} k_z k_x + \omega \varepsilon_0 k_y \right) \cot(k_y b + \phi_y) = \sqrt{\frac{\varepsilon}{\mu}}. \quad (16b)$$

Similarly, at the surface in the  $y$ -direction,  $x = a$ , we obtain  $E_y/H_z = -E_z/H_y = \sqrt{\mu/\varepsilon}$ . Substituting (1), (2), (11), and (13) into (14), we obtain:

$$\frac{E_y}{H_z} = \frac{-j}{h^2} \left( \frac{E_0}{H_0} k_z k_y + \omega \mu_0 k_x \right) \tan(k_x a + \phi_x) = \sqrt{\frac{\mu}{\varepsilon}}, \quad (17a)$$

$$\frac{-H_y}{E_z} = \frac{-j}{h^2} \left( \frac{H_0}{E_0} k_z k_y - \omega \varepsilon_0 k_x \right) \cot(k_x a + \phi_x) = \sqrt{\frac{\varepsilon}{\mu}}. \quad (17b)$$

By letting the determinant of the coefficients of  $E_0$  and  $H_0$  in (16) and (17) vanish we obtain the transcendental equations:

$$\left[ \frac{j\omega\mu_0 k_y \tan(k_y b + \phi_y)}{h^2} + \sqrt{\frac{\mu}{\varepsilon}} \right] \left[ \frac{j\omega\varepsilon_0 k_y \cot(k_y b + \phi_y)}{h^2} - \sqrt{\frac{\varepsilon}{\mu}} \right] = \left[ \frac{k_z k_x}{h^2} \right]^2, \quad (18a)$$

$$\left[ \frac{j\omega\mu_0 k_x \tan(k_x a + \phi_x)}{h^2} + \sqrt{\frac{\mu}{\varepsilon}} \right] \left[ \frac{j\omega\varepsilon_0 k_x \cot(k_x a + \phi_x)}{h^2} - \sqrt{\frac{\varepsilon}{\mu}} \right] = \left[ \frac{k_z k_y}{h^2} \right]^2. \quad (18b)$$

In the above equations,  $k_x$  and  $k_y$  are the unknowns and  $k_z$  can then be obtained from (3). A multi-variable root searching algorithm such as the Powell Hybrid root-searching algorithm in a NAG routine [22] can be used to find the roots of  $k_x$  and  $k_y$ . The routine requires initial guesses of  $k_x$  and  $k_y$  for the search. For good conductors, suitable guess values are clearly those close to the perfect conductor values. For  $TE_{10}$  mode,  $m$  and  $n$  are set to 1 and 0, respectively, hence the search starts with  $k_x = \pi/a$  and  $k_y = 0$ . For  $TE_{11}$  and  $TM_{11}$  modes,  $m$  and  $n$  are both set to 1 and the initial guess values are  $\pi/a$  and  $\pi/b$  respectively for both modes. It is worthwhile noting that when a search is started with exactly these values, the solution did not always converge to the required mode. It was often necessary to refine the initial values slightly in order to ensure convergence to the correct mode.

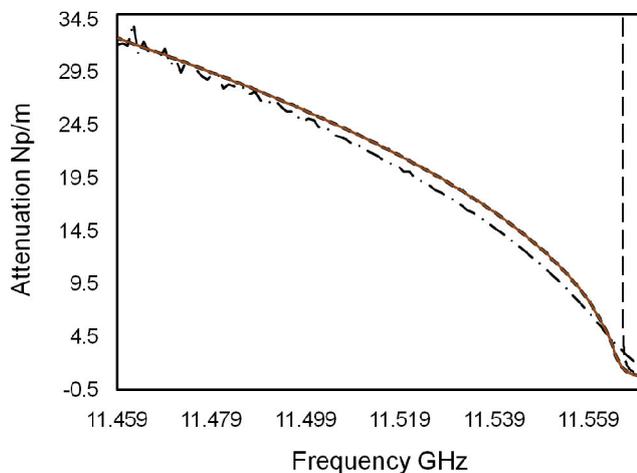
## 3. Results and Discussion

To validate the results experimentally, we measured the loss as a function of frequency for a 20 cm long rectangular waveguide using an Anritsu 37369C Vector Network Analyzer (VNA). The VNA was calibrated using the Thru-

Reflect-Line (TRL) method. The waveguide was made of copper and had dimensions of  $a = 1.30$  cm and  $b = 0.64$  cm. The loss was observed from the  $S_{21}$  parameter of the scattering matrix. The measurement was performed in the frequency range where only  $TE_{10}$  mode could propagate, while other higher order modes are evanescent.

We compared the attenuation of the  $TE_{10}$  mode below cutoff as predicted by our method, the conventional power-loss method, and the PPM as shown in Fig. 2. As can clearly be seen, the attenuation constant  $\alpha_z$  computed from the power-loss method diverges sharply to infinity, as the frequency approaches  $f_c$ , and is very different to the measured results, which show clearly that the loss at frequencies below  $f_c$  is high but finite. The attenuation curves computed using our method and the PPM in Fig. 2 match very well and in fact are indistinguishable on the plot. The figures for the loss between 11.47025 GHz and 11.49950 GHz computed by the two methods agree with measurement to within 5% which is comparable to the error in the measurement.

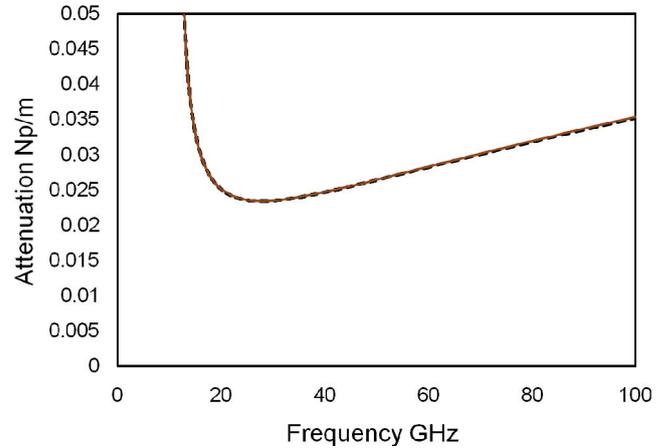
Fig. 3 shows the attenuation curve when the frequency is extended to higher values. Here, the loss due to  $TE_{10}$  alone could no longer be measured as higher-order modes, such as  $TE_{11}$  and  $TM_{11}$ , etc., start to propagate. At higher frequencies the loss due to  $TE_{10}$  predicted by the three methods, i.e. our method, the power-loss method, and the PPM are in very close agreement.



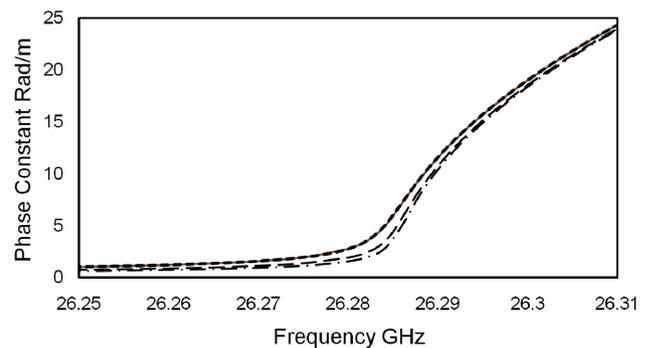
**Fig. 2.** Loss of  $TE_{10}$  mode in a hollow rectangular waveguide below cutoff. — our method, - - - the power-loss method, ..... the Papadopoulos' Perturbation method, and - · - · - the measurement result.

Next, we compared the propagation constants  $k_z$  of  $TE_{11}$  and  $TM_{11}$  degenerate modes, which have equal phase constants  $\beta_z$  in the lossless case. Here the power-loss method can only give  $\alpha_z$  whereas both the PPM and our method give both  $\beta_z$  and  $\alpha_z$ . Fig. 4 shows that the phase constant  $\beta_z$  for  $TE_{11}$  mode computed using our method is in good agreement with that computed using the PPM. For  $TM_{11}$  mode however, the results differ slightly. Unlike that

of the lossless case, the values of  $\beta_z$  differ slightly for the different modes in a lossy waveguides due to dispersive effects.



**Fig. 3.** Loss of  $TE_{10}$  mode in a hollow rectangular waveguide below cutoff. — our method, - - - the power-loss method, ..... the Papadopoulos' Perturbation method.



**Fig. 4.** Phase constant  $\beta_z$  of  $TE_{11}$  and  $TM_{11}$  in a rectangular waveguide. ..... and — indicate  $\beta_z$  of  $TE_{11}$  computed using the PPM and our method, respectively. - · - · - and - - - indicate  $\beta_z$  of  $TM_{11}$  computed using the PPM and our method, respectively.

The behavior of the degenerate  $TE_{11}$  and  $TM_{11}$  modes is illustrated in Fig. 5 to Fig. 8, both near cutoff and in the propagating region. In Fig. 5 and Fig. 6,  $\alpha_z$  computed by the PPM and our method, agree very well near cutoff. However, Fig. 7 and Fig. 8 show that when the frequency increases beyond 28.5 GHz for  $TE_{11}$  and 27.0 GHz for  $TM_{11}$ , the results start to disagree significantly.

To explain this disagreement we recall that power losses of a number of modes that propagate simultaneously in a waveguide is not simply additive [23]. The cross product terms between the different modes gives rise to additional dissipation, making the total loss greater than the one obtained from the addition of loss in independent propagation of single modes. This is because the product of the average power density,  $P_{av} = \frac{1}{2} \text{Re}(\mathbf{E}_1 \times \mathbf{H}_2^*)$  of the electric field of mode 1  $\mathbf{E}_1$  and magnetic field of mode 2  $\mathbf{H}_2$ , when integrated along the boundary, is not zero and the

current induced by  $\mathbf{H}_2$  will deliver power to mode 1, and vice versa. In this case, there will be coupling of power between multiple propagating modes, which give rise to power loss as a result of the change in the amplitude distribution of the fields across the area of the waveguide [23]:

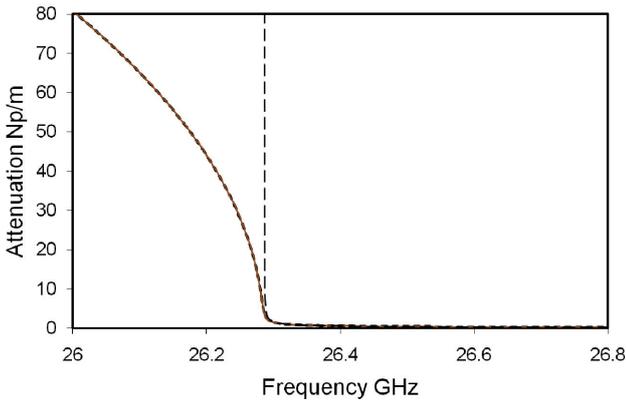
$$\begin{aligned}
 P_L = & \frac{1}{2} R \\
 & \left\{ \sum_{m=1}^M \sum_{n=1}^M A_m^{(TE)} A_n^{(TE)*} \oint_c [H_{mc}^{(TE)} H_{nc}^{(TE)*} + H_{nz}^{(TE)} H_{nz}^{(TE)*}] dc \int_0^l \exp[j(\beta_m^{(TE)} - \beta_n^{(TE)})z] dz \right. \\
 & + \sum_{m=1}^{M'} \sum_{n=1}^{M'} A_m^{(TM)} A_n^{(TM)*} \oint_c [H_{mc}^{(TM)} H_{nc}^{(TM)*}] dc \int_0^l \exp[j(\beta_m^{(TM)} - \beta_n^{(TM)})z] dz \\
 & + \sum_{m=1}^M \sum_{n=1}^{M'} A_m^{(TE)} A_n^{(TM)*} \oint_c [H_{mc}^{(TE)} H_{nc}^{(TM)*}] dc \int_0^l \exp[j(\beta_m^{(TE)} - \beta_n^{(TM)})z] dz \\
 & \left. + \sum_{m=1}^{M'} \sum_{n=1}^M A_m^{(TM)} A_n^{(TE)*} \oint_c [H_{mc}^{(TM)} H_{nc}^{(TE)*}] dc \int_0^l \exp[j(\beta_m^{(TM)} - \beta_n^{(TE)})z] dz \right\}. \tag{19}
 \end{aligned}$$

Here,  $A^{(TE)}$  and  $A^{(TM)}$  are arbitrary amplitude coefficients for the TE and TM modes respectively,  $R$  is the surface resistance,  $c$  is the contour around the inner surface of the waveguide, which is also normal to the propagating  $z$  axis. The subscript  $c$  represents the component of the transverse field tangential to the contour  $c$ .  $M$  is the number of different TE propagating modes, and  $M'$  is the number of different TM propagating modes.

It turns out that mode coupling increases the interaction between the propagating power and the waveguide walls, making the attenuation dependent on the axial distance from the source. Integrating the exponential terms in (19), the factor that determines coupling between modes can be written as [23]:

$$F = \frac{\exp[j(\beta_m^{TE} - \beta_n^{TM})l] - 1}{[j(\beta_m^{TE} - \beta_n^{TM})l]} \tag{20}$$

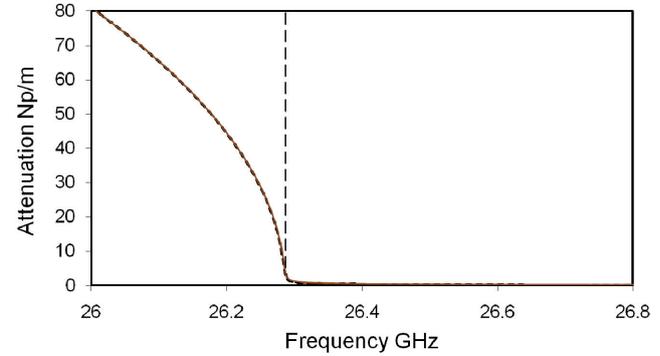
where  $\beta_m$  and  $\beta_n$  are the phase constants of 2 different modes which could be either TM or TE, while  $l$  is the length of the waveguide.



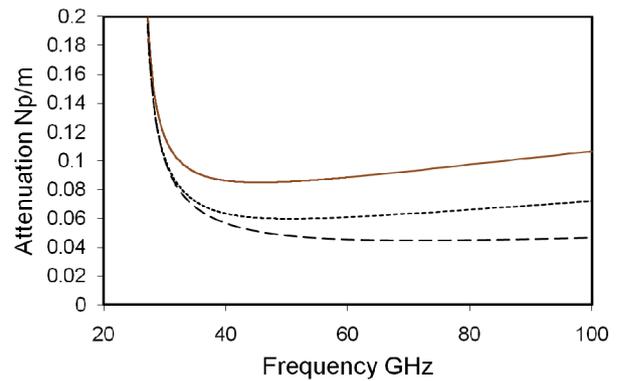
**Fig. 5.** Loss of TE<sub>11</sub> mode in a hollow rectangular waveguide near cutoff. — our method, - - - the power-loss method, ..... the PPM.

As expected, equation (20) shows that the cross coupling is significant when the difference between the phase

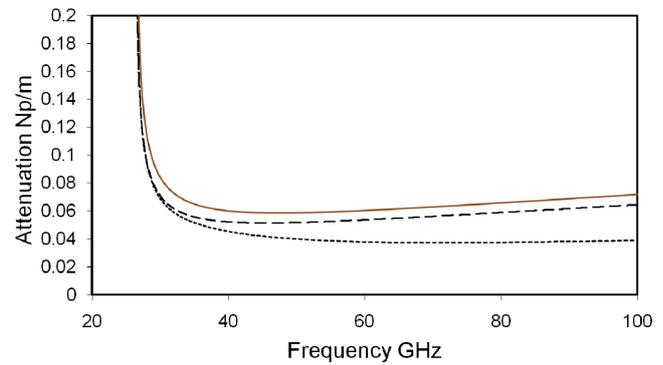
constants of the propagating modes that exist in the waveguide is small. Therefore, we expect that the coupling effect between TE<sub>11</sub> and TM<sub>11</sub> in a waveguide fabricated from a good conductor to be significant because the phase constants for TE<sub>11</sub> and TM<sub>11</sub> are very close as shown in Fig. 4.



**Fig. 6.** Loss of TM<sub>11</sub> mode in a hollow rectangular waveguide from 20 GHz to 100 GHz. — our method, - - - the power-loss method, ..... the PPM.



**Fig. 7.** Loss of TE<sub>11</sub> mode in a hollow rectangular waveguide from 20 GHz to 100 GHz. — our method, - - - the power-loss method, ..... the PPM.



**Fig. 8.** Loss of TM<sub>11</sub> mode in a hollow rectangular waveguide from 20 GHz to 100 GHz. — our method, - - - the power-loss method, ..... the PPM.

In Fig. 7 and Fig. 8 we plotted the attenuation constant for the TE<sub>11</sub> and the TM<sub>11</sub> modes at frequencies when

both of them can propagate simultaneously. It can clearly be seen that in this region, the computed attenuation using our method is significantly higher than the one computed using the power loss method. This is of course to be expected because the power loss method attenuation will exclude coupling losses. It is interesting to see however that in this range, the attenuation computed by PPM is even lower than that obtained by the power loss method, indicating that the PPM method under-estimates the loss significantly in degenerate mode propagation.

## 4. Conclusion

We have proposed a fundamental and accurate technique to compute the propagation constant of waves in a lossy rectangular waveguide. The formulation is based on matching the electric and magnetic fields at the boundary, and allowing the wavenumbers to take complex values. The resulting electromagnetic fields were used in conjunction with the concept of surface impedance to derive transcendental equations, whose roots give values for the wavenumbers in the  $x$  and  $y$  directions for different TE or TM modes. The wave propagation constant  $k_z$  could then be obtained from  $k_x$ ,  $k_y$ , and  $k_0$  using the dispersion relation.

Our computed attenuation curves are in good agreement with the PPM and experimental results for the case of the dominant TE<sub>10</sub> mode. An important consequence of this work is the demonstration that the loss computed for degenerate modes propagating simultaneously is not simply additive. In other words, the combined loss of two co-existing modes is higher than adding the losses of two modes propagating independently. This can be explained by the mode coupling effects, which is significant when the phase constants of two propagating modes are different yet very close.

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