

# BPSK Receiver Based on Recursive Adaptive Filter with Remodulation

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**Abstract.** *This paper proposes a new binary phase shift keying (BPSK) signal receiver intended for reception under conditions of significant carrier frequency offsets. The recursive adaptive filter with least mean squares (LMS) adaptation is used. The proposed receiver has a constant, defining the balance between the recursive and the non-recursive part of the filter, whose proper choice allows a simple construction of the receiver. The correct choice of this parameter could result in unitary length of the filter.*

*The proposed receiver has performance very close to the performance of the BPSK receiver with perfect frequency synchronization, in a wide range of frequency offsets (plus/minus quarter of the signal bandwidth).*

*The results obtained by the software simulation are confirmed by the experimental results measured on the receiver realized with the universal software radio peripheral (USRP), with the baseband signal processing at personal computer (PC).*

## Keywords

Binary Phase Shift Keying, adaptive signal processing, frequency offset compensation.

## 1. Introduction

Mobile, terrestrial and especially satellite communications are subject to Doppler shift due to relative movement of the transmitter and receiver. The Doppler shift causes a significant performance degradation in any communication system, particularly in coherent M-ary phase shift keying (MPSK) systems. In order to improve the performance of a system in these conditions, the Doppler shift needs to be estimated and corrected [1]. The algorithm that provides the estimation and correction of the Doppler shift should have the following properties. The first of all, it should use no *a priori* knowledge of neither channel nor signals involved in the transmission. The second property should be a quick convergence. Finally, the algorithm should not be numerically complex due to implementation

in low power mobile devices. Having in mind all of the mentioned properties, several algorithms were developed based on the decision directed approach. There are adaptive [2]-[4] or non-adaptive [5], [6] algorithms. Digital delay locked loop (DDL) or a linear phase locked loop (PLL) [7]-[11] may also be used for this purpose. The drawbacks of these algorithms are often complex structure, slow convergence and performance degradation in the presence of large Doppler shift.

Decision directed approach is also used in [12] where two simple adaptive algorithms were proposed for the Doppler shift estimation and correction. These algorithms use phasor and split phasor LMS algorithms. Both algorithms have very quick convergence and show good performance in additive white Gaussian noise (AWGN) channel. However, these algorithms are developed based on the arrangements of the reference pilot symbols.

In this paper we consider a coherent BPSK system in the presence of frequency offset. The receiver contains a decision feedback estimator that is based on an adaptive transversal filter with feedback and remodulation with the introduction of a constant  $A$ . Value  $(1 - A)$  defines a part of the output signal that is returned to the input. The proposed receiver does not use reference pilot symbols and has performance very close to that of the ideal one in a wide frequency offsets range.

The results obtained by the software simulation are confirmed by the experimental results measured on the receiver realized with the USRP1 [13], with the baseband signal processing at PC.

## 2. System Model

Block diagram of the proposed BPSK signal receiver is shown in Fig. 1.

Signal at the input of the receiver is:

$$r(t) = s(t) + n(t) \quad (1)$$

where  $s(t)$  is the useful BPSK signal:

$$s(t) = m(t) \cos \hat{\omega}_c t \quad (2)$$

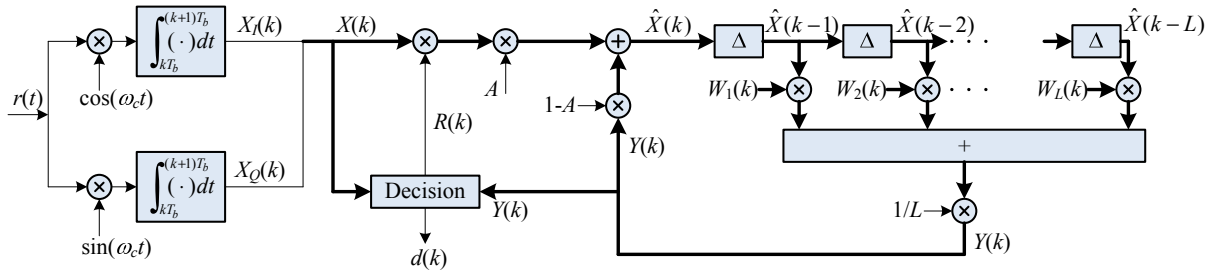


Fig. 1. Block diagram of the proposed BPSK signal receiver.

$m(t)$  is independent information stream with bit interval  $T_b$ ,  $\hat{\omega}_c = \omega_c + \Delta\omega$  is the carrier frequency,  $\omega_c$  is the locally generated fixed reference carrier frequency,  $\Delta\omega$  is the frequency offset, and  $n(t)$  is white Gaussian noise with power spectrum density  $N_0$ .

The input signal is multiplied by the fixed frequency reference carrier and passed through the integrate and dump circuit. Signals at in-phase and quadrature branches are

$$X_I(k) = \int_{kT_b}^{(k+1)T_b} r(t) \cos(\omega_c t) dt$$

$$X_Q(k) = \int_{kT_b}^{(k+1)T_b} r(t) \sin(\omega_c t) dt$$

(3)

where  $k$  is discrete time at which there is output of the integrate and dump circuit.

The complex baseband signal at the input of the adaptive filter can be expressed as

$$X(k) = X_I(k) + jX_Q(k).$$

(4)

After the remodulation, we get

$$\tilde{X}(k) = R(k) \cdot X(k)$$

(5)

where  $R(k) \in \{+1, -1\}$  is the remodulation signal. The remodulation signal is an estimation of the modulation data. If the noise is neglected and  $R(k)$  is correctly estimated then  $\tilde{X}(k)$  represents the reference carrier, i.e. the input signal with the modulation removed.

Signal  $\tilde{X}(k)$  is processed by the proposed modified recursive adaptive transversal filter with remodulation.

The operation of a recursive adaptive filter is defined, in general, by the equation

$$Y(k+1) = \frac{1}{L_1} \sum_{l=0}^{L_1-1} a_l \cdot \tilde{X}(k-l) + \frac{1}{L_2} \sum_{l=0}^{L_2-1} b_l Y(k-l)$$

(6)

where  $a_l$  and  $b_l$  are adaptive weights.

Having in mind that the output signal should be as close to the input signal as possible, we propose one adaptive weight  $W_l$  that will be adjusted by the LMS algorithm. In order to provide stability during the adaptation, we introduce the following constraints

$$L_1 = L_2 = L$$

$$a_l = A \cdot W_l$$

$$b_l = (1-A) \cdot W_l$$

(7)

where  $A$  is the introduced constant parameter ( $A \leq 1$ ). Value  $(1-A)$  defines a part of the output signal that is returned to the input.

Equation (6) may now be written as:

$$Y(k+1) = \frac{1}{L} \sum_{l=0}^{L-1} W_l(k) \cdot [A \cdot \tilde{X}(k-l) + (1-A)Y(k-l)].$$

(8)

If we introduce

$$\hat{X}(k) = A \cdot \tilde{X}(k) + (1-A) \cdot Y(k),$$

(9)

we obtain the receiver in the form from Fig. 1. The output signal may be written as:

$$Y(k+1) = \frac{1}{L} \sum_{l=0}^{L-1} W_l(k) \cdot \hat{X}(k-l).$$

(10)

$Y(k)$  represents the estimation of the reference carrier, just like  $\tilde{X}(k)$  and  $\hat{X}(k)$  were, with the difference that  $Y(k)$  is a better estimation (the noise level is lower due to filtering) of the reference carrier than the other two.

The adjustment of the weights  $W_l(k)$  may be performed by the different adaptive techniques such as recursive least-squares (RLS) or LMS. In this paper we used the normalized LMS (NLMS) as a good compromise between the adaptation speed and complexity. Using the NLMS, the weights are adjusted as [14], [15]:

$$\hat{W}_l(k) = W_l(k) + \frac{\mu E(k) \hat{X}(k-l)^*}{|\hat{X}(k)|^2}$$

(11)

where  $\mu$  is the adaptation factor,  $(\cdot)^*$  is the complex conjugate of  $(\cdot)$ , and  $E(k)$  is the error signal that the LMS algorithm is operating with. The error signal is defined later.

There is a possibility, especially in case of large frequency offset, that some of  $W_l$  weights do not have correct sign which may cause the error propagation. In order to avoid this situation, we need to define some constraints on the values of  $W_l$  weights:

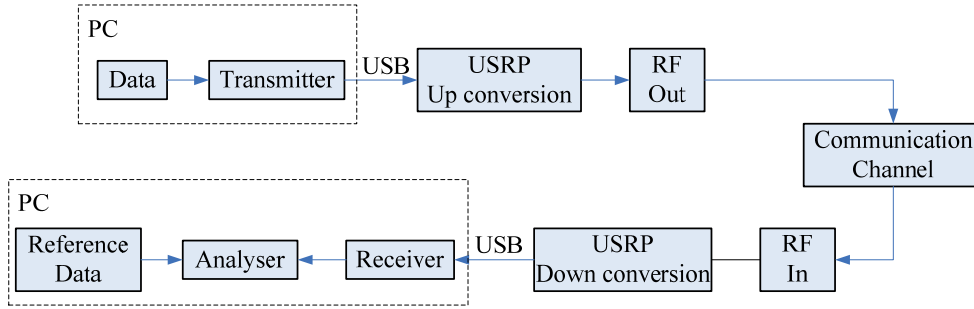


Fig. 2. Block diagram of the experimental setup.

$$W_l(k+1) = \begin{cases} \hat{W}_l(k), & \text{Re}\{\hat{W}_l(k) \cdot W_{l-1}^*(k+1)\} \geq 0 \\ -\hat{W}_l(k), & \text{Re}\{\hat{W}_l(k) \cdot W_{l-1}^*(k+1)\} < 0 \end{cases} \quad (12)$$

$l = 1, 2, \dots, L, W_0(k+1) = 1$

The error signal is

$$E(k) = R(k) \cdot X(k) - Y(k) = \tilde{X}(k) - Y(k). \quad (13)$$

For the estimate of the remodulation signal  $R(k) \in \{+1, -1\}$  we choose  $R(k)$  that minimizes the function  $|R(k) \cdot X(k) - Y(k)|^2$ . In other words, we choose  $R(k)$  that minimizes the difference between the input signal  $X(k)$  and the output signal  $Y(k)$ . This minimization may be mathematically written as:

$$R(k) = \arg \min_{r \in \{+1, -1\}} |r \cdot X(k) - Y(k)|^2. \quad (14)$$

Since the information data is unipolar and  $R(k)$  is bipolar, the detected bit is:

$$d(k) = \begin{cases} 1, & R(k) = -1 \\ 0, & R(k) = +1 \end{cases} \quad (15)$$

### 3. Experiment

Performance of the algorithm described in the previous section is determined by Monte-Carlo simulation. Two types of simulation were performed. The first one is complete software simulation chain in PC, including transmitter, channel, and the receiver. In the second type of simulation, the baseband processing is performed in PC, up/down conversion is performed in USRP1 hardware, and the communication channel is real. The block diagram of this simulation is shown in Fig. 2. Blocks marked with PC are common for both simulation and experimental setup. The PC part of the setup runs on Linux and is written in C++. In order to be able to compare the received and transmitted data bits, block *Data* generates and repeats a pseudorandom sequence. The same sequence is generated within *Reference Data* block. Block *Transmitter* performs baseband processing and generates BPSK modulated signal. BPSK signal is then transferred to USRP via USB interface. The communication at USB interface is performed using *libusb* library. USRP receives data from USB

interface and performs digital to analog conversion and up-conversion to 2.4 GHz band. At the receiver chain, similar processing is performed. After down-conversion and analog to digital conversion in USRP, signal is transferred via USB interface to PC. *Receiver* block performs demodulation and baseband processing. The received data are compared to the sent data in *Analyser* block.

In order to avoid problems with the difference between theoretical fading channel model and an actual fading channel in the laboratory, we decided to consider an AWGN channel. Therefore, during the experiment, the transmitter and the receiver were connected by a coaxial cable.

### 4. Numerical Results

The results shown in the following figures are obtained by Monte-Carlo simulation with 10 million steps. Carrier frequency is  $f_c = 2.44$  GHz, and bitrate is  $v_T = 1/T_b = 100$  kb/s.

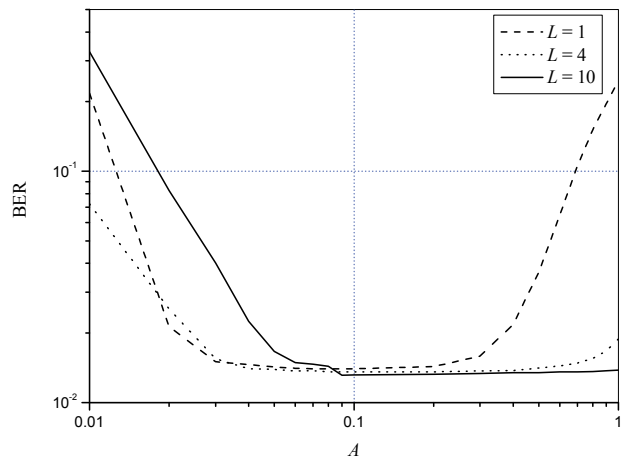


Fig. 3. Error probability as a function of feedback parameter  $A$

The influence of the proposed BPSK receiver parameter  $A$  on the system performance for different transversal filter lengths is shown in Fig. 3. For every value of transversal filter length there is the range of values of the parameter  $A$ , which provides the minimum error probability. Bearing in mind that with optimally chosen parameter  $A$  the filter length has very little effect on the value of the

error probability, because of simplicity we propose that the new receiver uses the transversal filter of unitary length.

Fig. 4 illustrates the dependence of the error probability on the signal to noise ratio. It can be noticed that for  $E_b / N_0 > 2$  dB the error probabilities for any considered filter length are almost equal. Also, the error probability of the proposed receiver is very close to the error probability of the ideal coherent BPSK receiver.

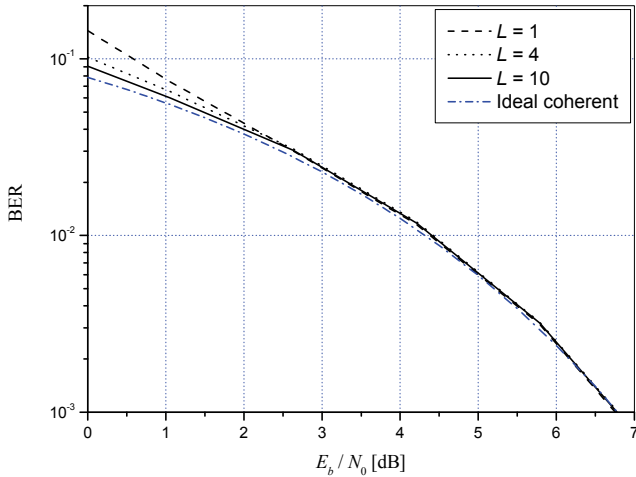


Fig. 4. Error probability as a function of  $E_b / N_0$ .

Fig. 5 illustrates the influence of frequency offset on the probability of error at different lengths of transversal filter. It can be seen that the filter length does not have influence on the frequency offset range width within there is no performance degradation. Also, the filter length again has very little influence on the error probability. This confirms the correctness of the decision on the use of unitary length transversal filter.

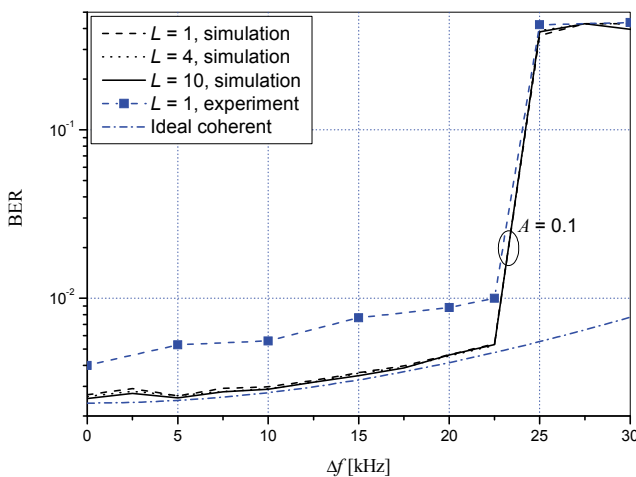


Fig. 5. Error probability as a function of carrier frequency offset  $\Delta f$ .

This figure also illustrates the error probability of the ideal BPSK receiver with perfect knowledge of frequency offset. The proposed receiver has performance very close to that of the ideal one in a wide frequency offsets range ( $\pm B / 4$ , where  $B$  is the bandwidth of the input signal).

The experimental results are also shown in this figure. The experimental results are obtained by measurement on the receiver realized with the USRP, with the baseband signal processing at PC.

Time synchronization is performed by second order DLL [16] with delay line gain  $K_{DLL} = 0.01$ , and the low-pass filter constant  $A_{DLL} = 0.01$ .

Both experimental and simulation results show that the proposed receiver has performance close to the performance for zero frequency offset in a very wide carrier frequency offsets range ( $\pm B / 4$ ).

There is a difference between the experimental and simulation results since the experimental results are obtained with the described DLL, and therefore there is timing jitter that depends on the time granularity of the DLL ( $1/10^{\text{th}}$  of  $T_b$  in our case). Simulation results are obtained under the assumption of perfect bit synchronization (perfect DLL).

### 5. Conclusion

A new BPSK signal receiver intended for the reception under conditions of significant carrier frequency offsets, such as in low Earth orbit (LEO) satellite and mobile communications, is proposed in this paper. By optimal choice of the receiver constant  $A$  it is possible to have a simple receiver that has performance very close to the performance of the BPSK receiver with perfect frequency synchronization, in a wide range of frequency offsets ( $\pm B / 4$ ).

The results obtained by the software simulation are confirmed by the experimental results measured on the receiver realized with the USRP, with the baseband signal processing at PC.

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