Efficient Algorithm for Power Allocation in Relay-based Cognitive Radio Network

N. FERDOUS, M. AHMED, M. A. MATIN, U. HABIBA

Dept. of Electrical Engineering and Computer Science, North South University, Dhaka, Bangladesh

m.a.matin@ieee.org

Abstract. *This paper addresses a cognitive radio (CR) network scenario where a relay is assigned to mitigate interference to primary users (PUs). We develop an average probability of successful secondary transmission (PSST) to introduce relay in the CR network. The power allocation is done using dual domain concept to maximize the system throughput as well as maintaining interference to an acceptable level and this approach is implemented in our paper that has a higher convergence rate. Furthermore, we propose an alternative approach that maintains a high throughput and at the same time reduces the computational complexity significantly. A detailed analysis is done before simulation. The simulated results validate the theoretical analysis.*

Keywords

Cooperative cognitive radio system, power allocation, dual domain, spectrum balancing algorithm, system throughput.

1. Introduction

The available spectrum that can be allocated to new emerging wireless service is getting scarcer, whereas a recent survey made by Federal Communications Commission (FCC) has indicated that the actual licensed spectrum is largely underutilized in vast temporal and geographic dimensions [1]. In order to solve the conflicts between spectrum scarcity and spectrum underutilization, cognitive radio (CR) technology has been recently proposed. CR is an intelligent radio that can first perceive its radio environment through wide-band spectrum sensing and then adapts its transmission and reception parameters such as the operating frequency, modulation scheme, code rate, and transmission power in real time with two primary objectives in mind: highly reliable communication whenever and wherever needed and efficient utilization of radio spectrum [2]. It can improve the spectrum utilization by allowing secondary (unlicensed) users to borrow unused radio spectrum from primary (licensed) users or to share the spectrum with the primary users [3].

Some of the major functions of CR are spectrum sensing, Dynamic Spectrum Management (DSM) and transmit-power control. To enhance the performance of the CR system various methods like cooperation and introduction of relays have been proposed [4]. By exploiting the benefit of cooperative relay, throughput of the system can be greatly increased as cooperative transmission between CR users improves both spatial and spectrum diversity [5]. There has been some recent work in the field of resource allocation in cooperative CR network. In Jia's paper [6] the power allocation and relay selection has been done by selfish optimization of resource to maximize system throughput without considering potential interference with primary users. In [7] the authors propose a power allocating algorithm using amplified-forward (AF) cooperative protocol to maximize SNR at the destination of the CR system. [8] considers the interference limits caused to PUs and propose an alternative cooperative protocol which uses Maclaurin series to formulate the problem to enhance the system throughput. In this paper, keeping interference limitation in mind, we have investigated power allocation, using a new approach which maintains high throughput and lowers computational complexity.

The rest of the paper is organized as follows. In Section 2, problem formulation of resource allocation and system throughput is shown. Section 3 holds an optimal approach of maximizing throughput. A near-optimal approach with reduced complexity is discussed in Section 4. Then in Section 5, simulation results are presented to show the performance comparison of the proposed approaches. Finally, conclusion is drawn in Section 6.

2. Problem Formulation and System Throughput

In the cooperative cognitive radio system depicted as in Fig.1, we have considered a simple three node relay model where transmission takes place. There is a pair of cognitive transmitter and receiver surrounded by P number of primary users and C number of CR users available as possible relay stations. Each CR user is with directional transmitting and omnidirectional receiving abilities. Direction of transmission is assumed to be random.

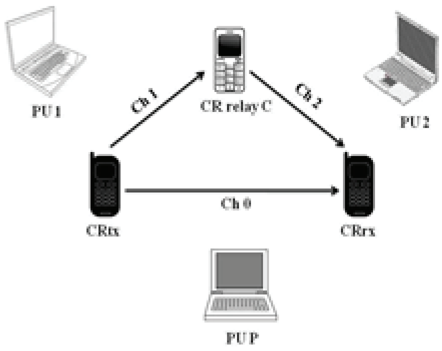

Fig.1. Cooperative cognitive radio network with relay.

Before transmitting, the CR transmitter senses the spectrum band for the coverage of its directional antenna by varying the direction of transmission randomly [9]. In case of idle sensing result, if the CR receiver is within the coverage of CR transmitter, a direct link between them can be established. However if that's not the case then relay stations can be used to assist the CR communication.

Cooperative relays are also used to increase the throughput of the system by exploiting diversity. The position of PUs and the density of the CR users determine whether a CR link can be assisted by a relay or not.

CR users have to sense the available spectrum bands before relay selection and data transmission. This is done by varying the antenna beamwidth. Antenna beamwidth is referred to as the angle θ of the antenna beam over which the relative power is at or above 50 % of the peak power. Hence we can say that normalized beamwidth, ς is $\theta/360^{\circ}$. It denotes the probability that PU is within the coverage of CR transmitter. The probability that CR transmitter can access the spectrum coverage is $(1-\varsigma)$. Therefore for P number of PUs, the probability of idle sensing result at CR transmitter can be obtained by

$$pr_{idle} = (1-\varsigma)^{P} . \quad (1)$$

Hence the probability of the one-hop direct communication between the CR transmitter and receiver is given by

$$pr_1 = \varsigma(1-\varsigma)^{P} . \quad (2)$$

Using the concept of moment generating function [10] the average probability of successful secondary transmission (PSST) of two hop relay can be obtained by

$$pr_2 = pr_{idle} . pr_{relay} = (1-\varsigma)^{L}[1-(1-\varsigma+\varsigma(1-pr_1))^{C}] \quad (3)$$

where pr_{relay} is the probability that there exists at least one relay in the coverage area of the CR transmitter antenna beamwidth. Since pr_1 and pr_2 are independent, the PSST of a maximum of 2-hop relay communication, is given as

$$\xi = 1-(1-pr_1)(1-pr_2). \quad (4)$$

As shown in Fig. 2 PSST increases with the number of candidate CR relays.

CR user needs to satisfy certain conditions to be able to act as a cooperative relay. CR transmitter should send signal to the cognitive radio relay (CRR) without interfering with PU (i.e. relay should be closer to the transmitter than to PU) and CRR should forward signal to CR receiver without interfering with PU (i.e. relay should be closer to receiver than the PU). Interference to PU can be reduced if CR is at a short transmission range, is with a low transmit power and/or has directional/multiple antennas with beamforming.

In the three node channel, each node uses orthogonal channels [6]. A CR user is assigned as relay to each CR transmitter and receiver. The CR transmitter sends data simultaneously via Ch 0 and Ch 1 to CR receiver and relay respectively. The relay then in turn forwards the data to the CR receiver via Ch 2. Channel gains are assumed to be known by the CR source and relays, and are constant during a data frame. The relays use amplify-and-forward (AF) protocol. Unit channel bandwidth is considered for simplicity. The received signal is corrupted by additive white Gaussian noise (AWGN) with zero mean and σ^2 variance for each channel.

After choosing a relay while allocating power, potential interference should be taken into account so that no harmful interference occurs with the PU. The interference with PU happens either due to misdetection of PU or during coexistence with PU using limited transmission power. In order to prevent interference the following conditions regarding the power of the three nodes must be satisfied:

$$P_{Ct} \,|\, h_{Ct,P} \,|^2 \leq \gamma \quad (5a)$$

where P_{Ct} is the transmission power from the transmitter to the receiver over Ch 0, $h_{Ct,P}$ is the channel gain between transmitter and PU for Ch 0, and γ is acceptable interference power of PUs.

$$P_{CR} \,|\, h_{CR,P} \,|^2 \leq \phi \quad (5b)$$

where P_{CR} is the transmission power from the transmitter to the relay over Ch 1, $h_{CR,P}$ is the channel gain between transmitter and PU for Ch 1, and ϕ is acceptable interference power of PUs.

$$P_{RELAY} \,|\, h_{RELAY,P} \,|^2 \leq \varpi \quad (5c)$$

where P_{RELAY} is the transmission power from the relay to the receiver over Ch 2, $h_{RELAY,P}$ is the channel gain between relay and PU for Ch 2, and ϖ is acceptable interference power of PUs.

The constraint of the battery capacity of the transmitter and relay should also be considered

$$P_{Ct} + P_{CR} \le \Psi , \tag{6}$$

$$P_{RELAY} \le \Theta \tag{7}$$

where Ψ is the overall power limit for the CR transmitter and Θ is the maximum transmission power allowed by each relay.

The received power signal, noise and hence SNR at the receiver from relay can be given as follows [11]:

$$P_S = \frac{P_{RELAY} P_{CR} \,|\, h_{T,RELAY} \,|^2 |\, h_{RELAY,R} \,|^2}{2 P_{RELAY} \,|\, h_{T,RELAY} \,|^2 + \sigma^2} , \tag{8a}$$

$$P_n = \left(\frac{P_{RELAY} \,|\, h_{RELAY,R} \,|^2}{P_{CR} \,|\, h_{T,RELAY} \,|^2 + \sigma^2} + 1 \right) \sigma^2 , \tag{8b}$$

$$SNR = \frac{P_S}{P_n} = \frac{P_{RELAY} P_{CR} \,|\, h_{T,RELAY} \,|^2 |\, h_{RELAY,R} \,|^2}{\left(P_{RELAY} \,|\, h_{RELAY,R} \,|^2 + P_{CR} \,|\, h_{T,RELAY} \,|^2 + \sigma^2 \right) \sigma^2} \tag{9}$$

where $h_{T,RELAY}$, $h_{T,R}$ and $h_{RELAY,R}$ are the channel gains between the transmitter and the relay, the transmitter and the receiver, and the relay and the receiver respectively.

Therefore, the overall system throughput can be shown as,

$$T(P_{Ct}, P_{CR}, P_{RELAY}) = (1-\alpha)\log_2 \left(1 + \frac{P_{Ct} \,|\, h_{T,R} \,|^2}{\sigma^2} \right) +$$

$$+(1-\alpha)\log_2 \left(1 + \frac{P_{RELAY} P_{CR} \,|\, h_{T,RELAY} \,|^2 |\, h_{RELAY,R} \,|^2}{\left(P_{RELAY} \,|\, h_{RELAY,R} \,|^2 + P_{CR} \,|\, h_{T,RELAY} \,|^2 + \sigma^2 \right) \sigma^2} \right) \tag{10}$$

where α is the misdetection probability for spectrum sensing.

3. Throughput Optimization

In this section we maximize the system throughput using an optimal approach [12] known as *optimal spectrum balancing* (OSB) and later modify the optimal approach. We start our work by first expressing the problem as a constrained optimization problem:

Maximize

$$T(P_{Ct}, P_{CR}, P_{RELAY}) \tag{11a}$$

subject to

$$P_{Ct} + P_{CR} \le \Psi , \tag{11b}$$

$$P_{RELAY} \le \Theta , \tag{11c}$$

$$0 \le P_{Ct} \,|\, h_{Ct,P} \,|^2 \le \gamma , \tag{11d}$$

$$0 \le P_{CR} \,|\, h_{CR,P} \,|^2 \le \phi , \tag{11e}$$

$$0 \le P_{RELAY} \,|\, h_{RELAY,P} \,|^2 \le \varpi . \tag{11f}$$

In order to solve this, Lagrangian function [11] is used,

$$L(P_{Ct}, P_{CR}, P_{RELAY}, \lambda_1, \lambda_2, \lambda_3) = (1-\alpha)\log_2 \left(1 + \frac{P_{Ct} \,|\, h_{T,R} \,|^2}{\sigma^2} \right) +$$

$$+(1-\alpha)\log_2 \left(1 + \frac{P_{RELAY} P_{CR} \,|\, h_{T,RELAY} \,|^2 |\, h_{RELAY,R} \,|^2}{\left(P_{RELAY} \,|\, h_{RELAY,R} \,|^2 + P_{CR} \,|\, h_{T,RELAY} \,|^2 + \sigma^2 \right) \sigma^2} \right) +$$

$$+\lambda_1 \left(\Psi - P_{Ct} - P_{CR} \right) + \lambda_2 \left(\gamma - P_{Ct} \,|\, h_{Ct,P} \,|^2 \right) +$$

$$+\lambda_3 \left(\phi - P_{CR} \,|\, h_{CR,P} \,|^2 \right) \tag{12}$$

where λs are the Lagrangian multipliers.

The throughput T increases with P_{RELAY}, therefore instead of inserting it in the Lagrangian function we use it in the form

$$P^*_{RELAY} == \min \left\{ \frac{\phi}{|\, h_{RELAY,P} \,|^2}, \Theta \right\} .$$

The main idea here is to solve the constrained optimization problem (11) in the dual domain. For that we define a dual objective function, $g(\lambda_1, \lambda_2, \lambda_3)$, as an unconstrained maximization of Lagrangian,

$$g\left(\lambda_1, \lambda_2, \lambda_3 \right) = \max L(P_{Ct}, P_{CR}, \lambda_1, \lambda_2, \lambda_3) . \tag{13}$$

To obtain the optimal solution of (11) it is required to minimize $g(\lambda_1, \lambda_2, \lambda_3)$ directly by updating all components of the Lagrangian multipliers at the same time along some search direction. Since $g(\lambda_1, \lambda_2, \lambda_3)$ is convex, a gradient type search is guaranteed to converge to the global optimum. The problem arises when $g(\lambda_1, \lambda_2, \lambda_3)$ is non-differentiable. In such a case we bring a vector d in the scenario on the basis of which the search direction can be found. The vector d is a subgradient of $g(\lambda_1, \lambda_2, \lambda_3)$.

Differentiating (13) we can find

$$d = \begin{bmatrix} \Psi - P_{Ct} - P_{CR} \\ \gamma - P_{Ct} \,|\, h_{Ct,P} \,|^2 \\ \phi - P_{CR} \,|\, h_{CR,P} \,|^2 \end{bmatrix} . \tag{14}$$

Let P_{Ct} *and* P_{CR} be the optimizing variable in $g(\lambda_1, \lambda_2, \lambda_3)$ According to KKT conditions [13],

$$\frac{\partial L}{\partial P_{Ct}} = 0, \frac{\partial L}{\partial P_{CR}} = 0. \tag{15}$$

Thus differentiating the Lagrangian function (12) and solving for P_{Ct} *and* P_{CR} gives,

$$P_{Ct} = \left[\frac{1-\alpha}{\left(\lambda_1 + \lambda_2 \,|\, h_{Ct,P} \,|^2 \right) \ln 2} - \frac{\sigma^2}{|\, h_{T.P} \,|^2} \right]^+ , \tag{16}$$

$$P_{CR} = \left[\frac{\sqrt{P_{RELAY}^2 \left|h_{RELAY,R}\right|^4 + 4P_{RELAY}\left|h_{RELAY,R}\right|^2 \left|h_{T,RELAY}\right|^2 \Lambda}}{2\left|h_{T,RELAY}\right|^2} - \right.$$
$$\left. - \frac{P_{RELAY}\left|h_{RELAY,R}\right|^2 + 2\sigma^2}{2\left|h_{T,RELAY}\right|^2} \right]^+ \tag{17}$$

where

$$\Lambda = \frac{1-\alpha}{\ln 2\left(\lambda_1 + \lambda_3 \,|\, h_{CR,P} \,|^2\right)} \quad \text{and } [.]^+ = \max(\ .\ ,0).$$

λ basically gives the price of the resource i.e. power in our case. If the power constraint is exceeded, the price goes up, otherwise it decreases. Hence it plays a role in allocating the resource in the system and so a systematic update of λ is required.

3.1 Subgradient Method

In the λ update method discussed in [11] update is basically done in the subgradient direction on the basis of a step size sequence λ_1, λ_2 and λ_3 are obtained through the following iterations,

$$\lambda_1^{k+1} = \left[\lambda_1^k - s^k\left(\Psi - P_{Ct} - P_{CR}\right)\right]^+, \tag{18a}$$

$$\lambda_2^{k+1} = \left[\lambda_2^k - s^k\left(\gamma - P_{Ct} \,|\, h_{Ct,P} \,|^2\right)\right]^+, \tag{18b}$$

$$\lambda_3^{k+1} = \left[\lambda_3^k - s^k\left(\phi - P_{CR} \,|\, h_{CR,P} \,|^2\right)\right]^+ \tag{18c}$$

where k is the iteration number and s^k is the step size sequence.

3.2 Proposed λ Update Method

In this section we implement an update method that has a faster convergence than the method discussed in the previous section. The idea here is to localize the set of candidate λ's in a minimalized ellipsoid region. Then by evaluating the subgradient of $g(\lambda_1, \lambda_2, \lambda_3)$ at the center of the ellipsoid, roughly half of the ellipsoid is sliced away from the candidate set. The iteration continues as the size of the candidate set diminishes until it converges to an optimal set of λ. An ellipsoid can be defined as

$$E = \left\{x \,|\, (x-z)^T A(x-z) \le 1\right\} \tag{19}$$

where z is the center and A is a semidefinite matrix that gives the size and orientation of E.

The update algorithm is as follows,

$$d_{n+1} = \frac{d_n}{\sqrt{d_n^T A_n^{-1} d_n}}, \tag{20}$$

$$z_{n+1} = z_n - \frac{1}{N+1} A_n^{-1} d_{n+1}, \tag{21}$$

$$A_{n+1}^{-1} = \frac{N^2}{N^2-1}\left(A_n^{-1} - \frac{2}{N+1} A_n^{-1} d_{n+1} d_{n+1}^T A_n^{-1}\right). \tag{22}$$

The initial ellipsoid should be chosen so that it bounds all the λ's. By considering the KKT conditions and differentiating the Lagrangian function we get following equations (23a), (23b), and (23c),

$$\lambda_1 = \frac{1-\alpha}{\ln 2} \frac{|\, h_{T,R} \,|^2}{(\sigma^2 + P_{Ct} \,|\, h_{T,R} \,|^2)}, \tag{23a}$$

$$\lambda_2 = \frac{1-\alpha}{\ln 2} \frac{|\, h_{T,R} \,|^2}{(\sigma^2 + P_{Ct} \,|\, h_{T,R} \,|^2) \,|\, h_{Ct,P} \,|^2}, \tag{23b}$$

$$\lambda_3 = \frac{1-\alpha}{\ln 2} . \frac{\left(P_{RELAY} \,|\, h_{RELAY,R} \,|^2 |\, h_{T,RELAY} \,|^2 \sigma^2\right)\left(P_{RELAY} \,|\, h_{RELAY,R} \,|^2 + \sigma^2\right)}{\begin{bmatrix} \left(P_{RELAY} \,|\, h_{RELAY,R} \,|^2 + P_{CR} \,|\, h_{T,RELAY} \,|^2 + \sigma^2\right)\sigma^2 + \\ P_{RELAY} P_{CR} \,|\, h_{T,RELAY} \,|^2 |\, h_{RELAY,R} \,|^2 \end{bmatrix} |\, h_{CR,P} \,|^2}. \tag{23c}$$

To enclose a region where the optimal λ may reside, we choose an initial ellipsoid using the above results,

$$A_0^{-1} = \begin{bmatrix} N\left(\frac{(1-\alpha)\lambda_1}{2}\right)^2 & 0 & 0 \\ 0 & N\left(\frac{(1-\alpha)\lambda_2}{2}\right)^2 & 0 \\ 0 & 0 & N\left(\frac{(1-\alpha)\lambda_3}{2}\right)^2 \end{bmatrix}, \tag{24}$$

$$z_0 = \begin{bmatrix} \frac{(1-\alpha)\lambda_1}{2} & \frac{(1-\alpha)\lambda_2}{2} & \frac{(1-\alpha)\lambda_3}{2} \end{bmatrix}^T. \tag{25}$$

This proposed method is a better update method than subgradient method because it converges λ to the optimal faster. However the computational costs of the two methods are similar.

4. Proposed Near–Optimal Approach with Reduced Complexity

In this section we have proposed a new scheme that solves the dual optimization problem (11) in a near-optimal fashion but with reduced computational complexity.

Here the main objective is to evaluate $g(\lambda_1, \lambda_2, \lambda_3)$ with a complexity that is linear to the number of users. This can be done by

$$\max L(P_{Ct}, P_{CR}, \lambda_1, \lambda_2, \lambda_3) \triangleq \max h\left(P_{Ct}, P_{CR}\right). \tag{26}$$

The optimization of $h(P_{Ct}, P_{CR})$ is done in an iterative water filling fashion via coordinate descent. Here, for each fixed set of $(\lambda_1, \lambda_2, \lambda_3)$ the power levels P_{Ct}, P_{CR}, P_{RELAY}

are optimized individually where in each case we keep the other power levels constant. The correct dual variables, λ's, are then used in the following search method,

$$\lambda_m^{k+1} = \left[\lambda_m^k - s^k d_m\right]^+, \quad \text{for } m = 1, 2, 3 \tag{27}$$

where s^k is the step size sequence and d_m denotes the m^{th} row in the subgradient matrix, d given in (14). The optimal approach, discussed in the previous section, provides significant performance improvement as compared to the pioneer spectrum balancing algorithm known as iterative water-filling (IW) [14]. However, unlike IW, the computational complexity of optimal approach [11] is exponential to the number of users in the system.

The proposed scheme is a middle ground between IW and OSB. This new scheme reaps the advantage of both the dual formulation of OSB and the low-complexity of IW. Therefore, unlike IW, the proposed scheme reaches a global optimum by optimizing the objective function that includes the joint rate of all users. Furthermore, it dualizes the power constraint in an optimal fashion. It can be shown that the total computational complexity of this scheme is lower than that of OSB. Here, in the evaluation of $h(P_{Ct}, P_{CR})$ each iteration has a computational complexity that is linear to the number of users. The number of iterations needed to evaluate each $h(P_{Ct}, P_{CR})$ is T_1 and the number of subgradient update needed in the OSB algorithm is T_2. Therefore the total computational com-plexity of our scheme is $O(T_1T_2BK)$, where B is the maximum number of bits per channel and K is the total number of users. On the other hand, the complexity of OSB scheme is exponential in K, the total computational complexity of OSB is $O(T_2B^K)$. The comparison between the optimal [11] and proposed scheme has been shown in Tab. 1.

Approach	Complexity	Comment
Optimal Approach used in [11]	$O(T_2B^K)$.	Exponential in K
Proposed Approach	$O(T_1T_2BK)$	Linear in K

Tab.1. Comaprison of existing and proposed scheme complexity.

The simulation results in the next section show that the proposed approach gives similar results to OSB yet it is of a much lower complexity.

5. Simulation Result

In this section, numerical results have been discussed. The maximum PSST, ξ versus normalized beamwidth, ς is illustrated in Fig. 2. Here, ξ increases with the number of candidate relays thus exploiting spatial diversity.

We further provide simulation results to evaluate the performance of the proposed update method used for

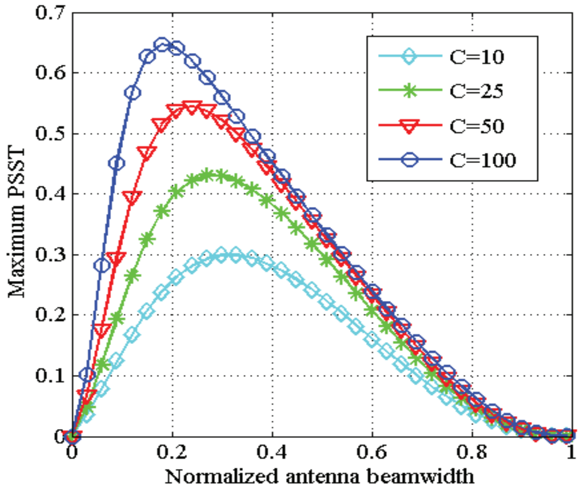

Fig. 2. PSST versus ς for different number of candidate CR relays, C.

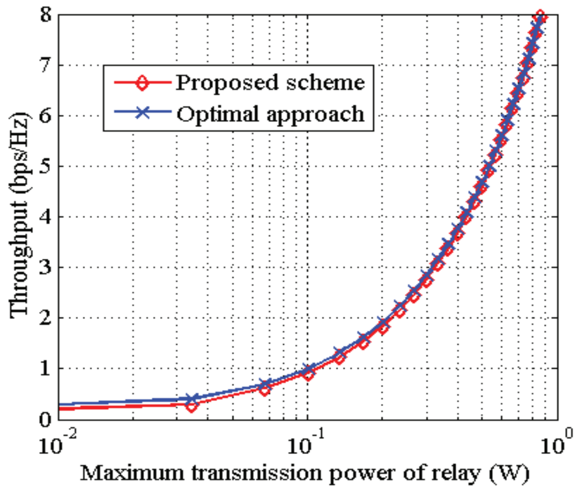

Fig. 3. Performance comparison of optimal approach and proposed scheme.

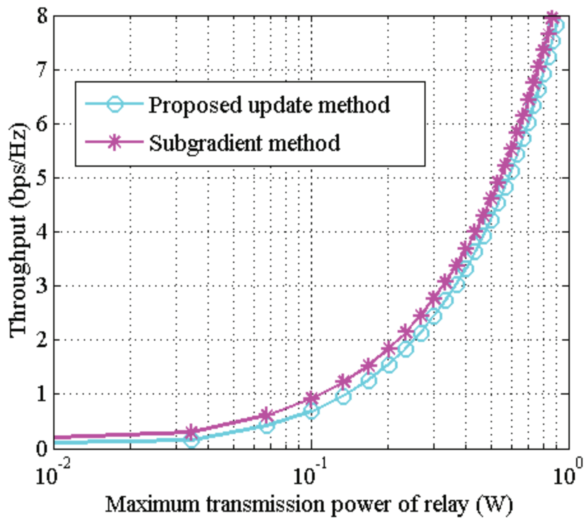

Fig. 4. Performance of subgradient method and proposed update method.

Lagrangian multiplier update and the proposed low-complexity scheme in the cognitive radio system. In Fig. 3 we see a comparison between optimal spectrum balancing approach and proposed low-complexity approach. In this simulation we have taken the values of the interference limits $\gamma = \phi = \varpi =$ 0.001 W. The total power limit of the cognitive transmitter, Ψ is taken as 0.5 W as done in [11]. The standard deviation of shadowing is taken to be 3.98.

As shown in Fig. 3, the low-complexity scheme implemented in CR system has actually achieved a performance very much similar to high-complexity OSB approach. We see that the throughput of the system increases exponentially with the increase in the maximum power of the relay, Θ. This shows that the throughput is dependent on the relay power when the total power of the CR transmitter is constant.

In Fig. 4, a comparison between subgradient and proposed update method in updating Lagrangian multiplier λ_1, λ_2, λ_3 in the OSB approach has been demonstrated. It's seen that the two update methods give similar output. This graph shows that we can also use the update method instead of subgradient method without hampering the performance and at the same time reap the advantage of our proposed update method of having a faster con-vergence of the Lagrangian multipliers.

6. Conclusion

In this paper, the optimal power allocation for a relay-based CR system has been studied before presenting our new approach. First, a dual optimization problem is proposed for maximizing the system throughput under power constraints due to the co-existence of the primary users and then a near-optimal approach is implemented to reduce the computational complexity significantly.

References

[1] Federal Communications Commission. *Spectrum Policy Task Force 1/8, an interleaver Report, ET Docket No. 02-155 Nov 02*. 2002.

[2] HAYKIN, S. Cognitive radio: Brain-empowered wireless communications. *IEEE Journal on Selected Areas in Communications*, 2005, vol. 23, no. 2, p. 201 - 20.

[3] MITOLA, J., MAQUIRE, G. M. Cognitive radio: Making software radios more personal. *IEEE Personal Communications*, 1999, vol. 6, no. 4, p. 13 - 18.

[4] GANESAN, G., LI, Y. Agility improvement through cooperative diversity in cognitive radio networks. in *IEEE Global Telecommunications Conference GLOBECOM 2005*. St. Louis (USA), 2005, p. 2505 - 2509.

[5] ZHANG, Q., JIA, J., ZHANG, J. Cooperative relay to improve diversity in cognitive radio networks. *IEEE Communications Magazine*, 2009, vol. 47, no. 2, p. 111 - 117.

[6] JIA, J., ZHANG, J., ZHANG, Q. Cooperative relay for cognitive radio networks. in *Proceedings IEEE INFOCOM*. Rio de Janeiro (Brazil), 2009, p. 2304 - 2312.

[7] YAN, S., WANG, X. Power allocation for cognitive radio systems based on nonregenerative OFDM relay transmission. In *5th International Conference on Wireless Communications, Networking and Mobile Computing WiCom 2009*. Beijing (China), 2009, p. 1-4.

[8] LIU, Z., XU, Y., ZHANG, D., GUAN, S. An efficient power allocation algorithm for relay assisted cognitive radio network. In *International Conference on Wireless Communications and Signal Processing WCSP 2010*. Suzhou (Cina), 2010, p. 1 - 5.

[9] OLFAT, M., FARROKHI, F. R., LIU, K. J. R. Power allocation for OFDM using adaptive beamforming over wireless networks. *IEEE Transactions on Communications*, 2005, vol. 53, no. 3, p. 505 - 514.

[10] IRWIN, M. E. *Moment Generating Function.* 19 pages. [Online] Available at :

http://www.markirwin.net/stat110/Lecture/ Section45.pdf .

[11] LI, L., ZHOU, X., XU, H., LI, G. Y., WANG, D., SOONG, A. Simplified relay selection and power allocation in cooperative cognitive radio systems. *IEEE Transactions on Wireless Communications*, 2011, vol. 10, no. 1, p. 33 - 36.

[12] CENDRILLON, R., MOONEN, M., YU, M., VERLINDEN, J., BOSTOEN, T. Optimal multiuser spectrum balancing for digital subscriber lines. *IEEE. Transactions on Communications*, 2006, vol. 54, no. 5, p. 922 - 933.

[13] BOYD, S., VANDENBERGHE, L. *Convex Optimization.* Cambridge University Press, 2004.

[14] YU, D. D., CIOFFI, J. M. Iterative water-filling for optimal resource allocation in OFDM multiple access and broadcast channels. In *Proceedings of IEEE Global Telecommunications Conference Globecom 2006*. San Francisco (CA, USA), 2006.