Linear-Combined-Code-Based Unambiguous Code Discriminator Design for Multipath Mitigation in GNSS Receivers

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Abstract. Unambiguous tracking and multipath mitigation for Binary Offset Carrier (BOC) signals are two important requirements of modern Global Navigation Satellite Systems (GNSS) receivers. A GNSS discriminator design method based on optimization technique is proposed in this paper to meet these requirements. Firstly, the discriminator structure based on a linear-combined code is given. Then the requirements of ideal discriminator function are converted into the mathematical constraints and the objective function to form a non-linear optimization problem. Finally, the problem is solved and the local code is generated according to the results. The theoretical analysis and simulation results indicate that the proposed method can completely remove the false lock points for BOC signals and provide superior multipath mitigation performance compared with traditional discriminator and high revolution correlator (HRC) technique. Moreover, the proposed discriminator is easy to implement for not increasing the number of correlators.

Keywords

GNSS, multipath, BOC, HRC.

1. Introduction

In the Global Navigation Satellite System (GNSS) receivers, the code discriminator, which is the core unit of the delay lock loop, is responsible for the synchronization of the local and received codes in the presence of multipath [1]. Code discriminator design determines the tracking accuracy of the GNSS, which is affected by numerous factors. Among these factors, the multipath is a dominant source of tracking error [2]. Multipath errors are generated by the corruption the line-of-sight signal that is attributed to the delayed reflected signal. In modern GNSS receivers, multipath errors result in approximately a 10m tracking error. Numerous multipath mitigation techniques have been proposed to enhance multipath mitigation performance through the improvement of code discriminator structure. These techniques can be classified as the parameter estimation, the correlation processing, the correlation combination and shaping techniques.

The parameter estimation techniques use the statistical method to estimate the multipath parameters including the number of multipath paths, multipath amplitude and delay. The most representative technique of this class is the multipath estimating delay lock loop (MEDLL) [3]. The techniques that belong to this class are the most effective for multipath mitigation compared with other classes. However, such techniques are too complicated to implement in the GNSS receiver [4]. The correlation processing techniques process correlation values to determine the lineof-sight signal delay. The early-late-slope (ELS) [2] and the differential correlation (DC) methods [3] are two representative techniques in this class. These methods do not increase hardware complexity but they are sensitive to the noise in the signal. The correlation combination and shaping techniques use additional correlators or local linearcombined-codes to shape the correlation or the discriminator function. The high resolution correlator (HRC) [5] and the code correlation waveform (CCRW) [6] are the representative techniques of this class. These methods exhibit effective multipath mitigation performance with a minimal increase in complexity. Moreover, [7] has proven that the linear-combination of several correlators equally correlates the received signal by using a local linear-combined-code, thus, the complexity can be reduced in further.

In this paper, we will focus on correlation shaping techniques. The key of these techniques is to generate a discriminator function (S-curve) with enhanced multipath mitigation performance by using optimum linear-combined-codes (or equally by using linear-combined-correlators). Several S-curve design methods have been proposed, such as by [8], which is a design method aimed at minimizing the multipath error, but it doesn't consider the ambiguity problem. The ambiguity problem can be attributed to multiple side peaks in the autocorrelation function of Binary Offset Carrier (BOC) modulated signals which are widely used in modern GNSS to achieve spectral separability [9]. The ambiguity problem induces a false lock which may result in a large and irreparable tracking error. Therefore, the ambiguity problem must first be solved in modern GNSS receivers. The design objectives of the unambiguous discriminator as well as the calculation of the combination weights of the correlators are presented in [10]. However, this approach only limits the multipath-induced error into a certain bound instead of enabling such error to reach the minimum value. Furthermore, this approach uses a multicorrelator structure, thus the optimization is limited by hardware complexity.

The goal of this article is to propose a code discriminator design method that is based on the linear-combinedcode structure. In this article, we present the constraints of the unambiguous S-curve and the objective function to obtain the minimized multipath error. These constraints and function form a non-linear optimization problem. The linear-combined weights can be calculated after solving this problem. These weights are used to generate a local code, which is implemented in the code discriminator to obtain the optimum S-curve.

The remainder of this paper is organized as follows: In Section 2, the code discriminator model is analyzed and a discriminator structure that is based on linear-combined code is discussed. Section 3 presents the requirements of the ideal S-curve and builds a non-linear optimization problem. Section 4 discusses a number of key points in solving the optimization problem. The design results for BOC(1,1) and BOC(10,5) are presented in Section 5. Finally, the conclusion is drawn in Section 6.

2. Code Discriminator Model

The S-curve comes from the correlation between the received signal and the local signal. The received BPSK or BOC modulated signal can be modeled as:

$$r(t) = \sqrt{Pd(t)m(t)\cos(\omega t + \theta_0)}$$

+
$$\sum_{k=1}^{M} \alpha_k \sqrt{Pd(t - d_k)m(t - d_k)\cos(\omega t + \theta_k)}$$

+
$$n(t)$$
(1)

where *P* is the power of the received signal. d(t) represents the data code, m(t) is the spreading code after subcarrier modulation or not for BOC and BPSK respectively. ω denotes the signal frequency. The first term is the direct line-of-sight signal with the phase θ_0 . The second term represents *M* signals of multipath, and each has the attenuation a_i , the relative delay d_k and the phase θ_k . The third term is the noise which can be ignored at the design stage. In the code tracking process, the received signal r(t) is multiplied by local I-phase and Q-phase carrier replicas and it becomes.

$$r_{l}(t) = \sqrt{P}c(t)\cos\phi_{0} + \sum_{k=1}^{M}\alpha_{k}\sqrt{P}c(t-d_{k})\cos\phi_{k},$$
 (2)

$$r_{\mathcal{Q}}(t) = \sqrt{P}c(t)\sin\phi_0 + \sum_{k=1}^M \alpha_k \sqrt{P}c(t-d_k)\sin\phi_k, \quad (3)$$

where c(t) denotes the modulated data code d(t)m(t), ϕ is the carrier phase error. After the carrier stripping, the signals of *I* and *Q* arms will correlate with the local replica code $c(t-jT_R)$ where jT_R is the correlator position and T_R is a constant which represents the time resolution of the delay, then the *j*th I-phase and Q-phase correlators output are

$$I_{j} = P\Lambda(\tau - jT_{R})\cos\phi_{0} + \sum_{k=1}^{M} a_{k}P\Lambda(\tau - jT_{R} - d_{k})\cos\phi_{k},$$
(4)
$$Q_{j} = P\Lambda(\tau - jT_{R})\sin\phi_{0} + \sum_{k=1}^{M} a_{k}P\Lambda(\tau - jT_{R} - d_{k})\sin\phi_{k}$$
(5)

where Λ is the autocorrelation of the modulated date code c(t). The $\Lambda(\tau)$ can be expressed as

$$\Lambda(\tau) = E[c(t) \otimes c(t)] \tag{6}$$

where $E[\bullet]$ represents the expectation operator, \otimes is the convolution operator.

There are two kinds of discriminator namely coherent and noncoherent discriminator can be implemented to track the GNSS signals. When the carrier phase information is available, which means $\phi_0 = 0$, the coherent discriminator can be used to acquire a better antinoise performance [1]. The output of coherent discriminator is

$$\tilde{D}(\tau) = \sum_{j=-N}^{N} w_j I_j \tag{7}$$

where w_j is the linear combination weight of correlator which should be determined. Substituting (4) and (6) into (7), we can derive

$$\tilde{D}(\tau) = \sum_{j=-N}^{N} w_j \{PE[c(t) \otimes c(t-jT_R)] + \sum_{k=1}^{M} a_k PE[c(t-d_k) \otimes c(t-jT_R)] \cos \phi'_k \}$$

$$= P\{E[c(t) \otimes \sum_{j=-N}^{N} w_j c(t-jT_R)] + \sum_{k=1}^{M} a_k E[c(t-d_k) \otimes \sum_{j=-N}^{N} w_j c(t-jT_R)] \cos \phi'_k \}$$
(8)

where ϕ'_k represents the multipath delay phase. From (8), we can discover that letting a new linear combined code

$$l(t) = \sum_{j=-N}^{N} w_j c(t - jT_R) .$$
(9)

It means that the linear-combined correlators can be changed into a linear-combined code. In other words, instead of using multiple correlators, the same discriminator output can be obtained by correlating the received signal with a local generated linear combined code l(t) [7]. The discriminator structure based on linear-combined code is illustrated in Fig. 1.

We should also remark that l(t) is a multi-level code instead of a binary one, this will make receiver a little more



Fig. 1. Discriminator structure based on linear-combined code.

complex, however, multi-level code generation is not very difficult in modern GNSS receiver [7]. The most important advantage of this structure is that N is unlimited because the large N will not increase the hardware complexity of the receiver. The output of this coherent discriminator can be obtained as

$$\tilde{D}(\tau) = E[r_l(t) \otimes l(t)].$$
(10)

In the absence of multipath, $\tilde{D}(\tau)$ can be simplified to $D(\tau)$ as

$$D(\tau) = P \sum_{j=-N}^{N} w_j \Lambda(\tau - jT_R) . \qquad (11)$$

Similar formulations can also be derived for the noncoherent scheme which is implemented when the phase cannot be accurately locked. The output of noncoherent discriminator is defined as

$$\tilde{D}_{nc}(\tau) = \sum_{j=-N}^{N} w_j (I_k^2 + Q_k^2) .$$
 (12)

Substituting (4), (5) and (6) into (12), the output can be rewritten as

$$\tilde{D}_{nc}(\tau) = \left(\Lambda^{2}(\tau - jT_{R}) + \sum_{k=1}^{M} a_{k}^{2} \Lambda^{2}(\tau - jT_{R} - d_{k}) + 2\Lambda(\tau - jT_{R}) \sum_{k=1}^{M} a_{k} \Lambda(\tau - jT_{R} - d_{k}) \cos \phi_{k}' + 2\sum_{k \neq m} a_{k} a_{m} \Lambda(\tau - jT_{R} - d_{k}) \Lambda(\tau - jT_{R} - d_{m}) \cos \phi_{m}' \right)$$
(13)

where ϕ'_k represents the multipath delay phase. However, since the discriminator function is nonlinear, there is no way to use a linear-combined code to generate this output. The noncoherent discriminator can only be multi-correlators structure. Under this structure, the correlators' number is limited for the hardware complexity and some optimum design is done in [1]. Hereafter in this paper, we will only discuss coherent discriminator based on linear-combined code structure.

3. Code Discriminator Design Requirements

Typically, the design requirements of the discriminator are as follows.

1) In the absence of noise and multipath, the estimation of the time delay is unbiased.

2) The discriminator sensitivity is greater than a prescribed value.

3) The S-curve is non-ambiguous for BOC modulated signals in pull-in range.

4) The code tracking error in the presence of multipath is as small as possible.

These requirements can be fulfilled by designing the weights w_j . In the following, these requirements will be converted into the constraints and the objective function to build a non-linear optimization problem.

In the absence of noise and multipath, the requirement 1 implies that when the time error τ is zero, the discriminator output is zero too. This can be expressed as:

$$D(0) = 0$$
. (14)

Substituting (11) into (14), we have

$$\sum_{j=-N}^{N} w_j \Lambda(-jT_R) = \boldsymbol{w}^T \boldsymbol{g} = 0$$
(15)

where the vector **w** is defined as $\mathbf{w} = [w_{-N} \ w_{-N+I} \ \cdots \ w_N]^T$ and **g** is the $(2N+1) \times 1$ vector whose (j+N+I)th entry is $\Lambda(-jT_R)$.

The sensitivity of the discriminator is defined as the derivative of $E(D(\tau))$ with respect to τ , evaluated at $\tau = 0$ [11]. The sensitivity represents the gain of the discriminator. The high sensitivity means a small residual error. The requirement 2 indicates that the sensitivity should be higher than a prescribed value \tilde{s} , which means

$$\left. \frac{dE(D(\tau))}{d\tau} \right|_{\tau=0} > \tilde{s} \ . \tag{16}$$

The value \tilde{s} can be selected according to the tracking loop design needs. (16) can be rewritten as

$$P\sum_{j=-N}^{N} w_{j} \frac{\Lambda(\tau - jT_{R})}{d\tau} \bigg|_{\tau=0} > \tilde{s}$$

$$\Rightarrow \sum_{j=-N}^{N} w_{j} \frac{\Lambda(\tau - jT_{R})}{d\tau} \bigg|_{\tau=0} > \frac{\hat{s}}{P}$$

$$\Rightarrow \boldsymbol{w}^{T} \boldsymbol{h} > s$$
(17)

where s denotes as \tilde{s} / P named normalized sensitivity, **h** is the $(2N+1) \times 1$ vector whose (j+N+1)th entry is $\Delta(\tau - jT_R)$

$$\frac{d\tau}{\tau}$$

The non-ambiguity constraint (requirement 3) can be stated as that there is no zero-crossover inside the pull-in range except the zero point, which means

$$\begin{pmatrix} D(\tau) > 0, & 0 < \tau \le \overline{\tau} \\ D(\tau) < 0, & 0 > \tau \ge \tau \end{cases}$$

$$(18)$$

where $\overline{\tau}$ is a positive number and $\underline{\tau}$ is a negative number and the pull-in range is from $\underline{\tau}$ to $\overline{\tau}$. Let **m** be the $(2N+1) \times 1$ vector whose (j+N+1)th entry is $\Lambda(\tau-jT_R)$, (18) can be rewritten to (19),

$$\begin{pmatrix} \boldsymbol{w}^{T}\boldsymbol{m} > \boldsymbol{0}, & \boldsymbol{0} < \tau \leq \overline{\tau} \\ \boldsymbol{w}^{T}\boldsymbol{m} < \boldsymbol{0}, & \boldsymbol{0} > \tau \geq \underline{\tau} \end{pmatrix}.$$
(19)

Typically, the criterion to evaluate the multipath mitigation performance of a discriminator is the multipath error envelope (MEE). It commonly uses two paths, both inphase and out-of-phase to calculate MEE [12]. The MEE curve has no analytical expression and the points of this curve is related to the solutions of the following equations

$$\tilde{D}(\tau, d_k) = 0$$
 $(d_k \in [0, 1.5]chips)$. (20)

For simplify, we will use the function name $MEEcurve(\tau)$ to denote the curves of MEE hereafter. An example of the MEE of traditional early minus late (EML) discriminator for BOC(1,1) signal is illustrated in Fig.2, the amplitude of the second-path is 6 dB lower than the one of the line-in-sight path. The early-late spacing Δ is set to 0.1 chips.



Fig. 2. The MEE of EML discriminator for BOC(1,1) signal. The multipath amplitude $a_1=0.5$, $\Delta = 0.1$ chips.

From Fig. 2 we can discover that there is an enclosed area between the absolute value of the upper MEE and the absolute value with minus sign of the lower MEE [8]. Here we use the unit m-chip to indicate this area and the area is defined as

$$area = \int |MEEcurve(\tau)| d\tau .$$
 (21)

It can be seen as a nonlinear function of w. The enclosed area of traditional EML discriminator for BOC(1,1) is

0.0315 m·chip. The minimized multipath error criterion (requirement 4) is equivalent to minimize the enclosed area of MEE.

On the whole, a non-linear optimization problem can be built from above objective function and constraints. For coherent discriminator, the problem can be expressed as

$$\min_{\mathbf{w}} area$$

$$\begin{cases} \mathbf{w}^{T} \mathbf{g} = 0 \\ \mathbf{w}^{T} \mathbf{h} > s \\ \mathbf{w}^{T} \mathbf{m} > 0, \quad 0 < \tau \le \overline{\tau} \\ \mathbf{w}^{T} \mathbf{m} < 0, \quad 0 > \tau \ge \tau \end{cases}$$
(22)

4. Optimization Problem Solution

The problem (22) is established by a nonlinear objective function and some linear constraints of w. This nonlinear problem can be solved by the sequential quadratic programming (SQP) method. This method is suitable for the problems involving linear and nonlinear with equality and inequality constraints. SQP method can find an optimum solution from arbitrary starting point and it will have fewer evaluations and gradients compared. The detail of this method can be found in [13].

The requirements (16)-(19) are all related to the continuous variable τ . In order to use computer to get the solution, we must sample this one to a discrete variable. Set $[-\alpha, \alpha]$ as the chip delay field of interest of discriminator function and the number of sampled points is *K*. Then the sampled delay point τ_i for discriminator function is

$$\tau_i = i \frac{2\alpha}{K-1} - \alpha \quad i = 0, 1, \cdots, K-1.$$
 (23)

Moreover, the MEE should also be discretized to make objective function discretization. Set $[0, \beta]$ as the chip delay field of interest of MEE and the number of sampled points is *L*. Then the sampled delay point τ_i for MEE curve is

$$\tau_i = i \frac{\beta}{L-1}$$
 $i = 0, 1, \cdots, L-1$. (24)

The area (21) can be rewritten as

$$area = \frac{\beta}{L-1} \sum_{i=0}^{L-1} \left| MEEcurve(\tau_i) \right|.$$
(25)

The start point of w which is denoted as w_0 should also be chosen before solution. Although in theory it can be set arbitrarily, a suitable start point will reduce the calculation steps and acquire better result. There is an Scurve shaping method proposed in [7] for coherent discriminator. We can use this method to set w_0 corresponding to EML discriminator function. A system of linear equations is obtained as

$$\Lambda \mathbf{w}_0 = \mathbf{D}_{\text{target}} \tag{26}$$

where is the $K \times (2N+1)$ vector whose (i+1, (j+N+1))th entry is $\Lambda(\tau_i - jT_R)$, $D_{\text{target}} = (D_{EML}(\tau_i))_{K\times 1}$ and D_{EML} is the S-curve of EML discriminator. The least squares solution with the smallest norm of this system can be used as the start point w_0 . This solution can generate a discriminator curve nearly the same as D_{target} .

After the discretization processing, the optimization problem can be solved and the linear-combined codes can be generated according to (9) by the computer. Although the solution processing of nonlinear optimization problem is very complicated, we should remark that the linear-combined codes can be generated only once in a computer and stored at the receiver side. So the proposed method will not increase the complexity of the receiver.

5. Code Discriminator Design Results

In this section, we will use the proposed design method to calculate the weight vector to form discriminator functions with minimized multipath error for modern GNSS signals. As a Galileo receiver is to be realized, the BOC(1,1) modulation is implemented. BOC(1,1) is also widely used in other modern GNSS [14]. So in this paper, we will firstly design a discriminator for BOC(1,1). Specific parameters that are used in the optimization process are as follows: N = 40, $T_R = 1/20$ chips, $\overline{\tau} = 1$ chips, $\underline{\tau} = -1$ chips, $\alpha = 1.5$ chips and $\beta = 1.5$ chips. The normalized sensitivity is required can be set to greater than 2 [10], which means s = 2.



Fig. 3. The start point w_0 and the optimum result w (for BOC(1,1) signal).

Fig. 3 and Fig. 4 show the optimum results of weights and S-curve comparing with the original ones. It can be seen that the original S-curve which is generated according to w_0 is indeed the same as the theoretic EML S-curve. Starting with w_0 , The optimum weight w is obtained and the result S-curve is generated according to (9) and (11). We can discover that the result S-curve is asymmetric around the origin and there is only one zero-crossover point. Moreover, the function values from delay -1 chips to -0.1 chips are not 0 but very small negative values because of the constraint (18).



Fig. 4. The EML, original and the optimum S-curve (for BOC(1,1) signal).

Fig. 5 shows the MEE for BOC(1,1) signal tracking, as well as for high-resolution correlator (HRC) technique for comparing. The HRC technique proposed in [15] is a typical multipath mitigation technique for modern GNSS signals. From Fig. 5, it can be seen that the optimum MEE came from proposed method is much more improved than the original MEE. Comparing with HRC, the difference between the proposed method and HRC is minimal at short multipath delays within 0 to 0.1 chips while the proposed method can nearly completely remove the multipath error around the medium and long multipath delay.



Fig. 5. MEE for original and optimum design and HRC (for BOC(1,1) signal).

BOC(10,5) is also a popular modulation in modern GNSS [14]. The higher modulation order make it more difficult to do unambiguous tracking and multipath mitigation than BOC(1,1). Next, we will design an optimum discriminator for BOC(10,5). The same parameters as above are used in this design.



Fig. 6. The start point w_0 and the optimum result w (for BOC(10,5) signal).



Fig. 7. The EML, original and the optimum S-curve (for BOC(10,5) signal).



Fig. 8. MEE for original and optimum design and HRC (for BOC(10,5) signal).

Fig. 6 to Fig. 8 demonstrate the design result of BOC(10,5) discriminator. From Fig. 7, it is obviously that the optimum S-curve is unambiguous while the original S-curve has six false lock points. From Fig. 8, it can be seen that the performance of HRC for BOC(10,5) is much worse than for BOC(1,1) because the S-curve of BOC(10,5) has

more peaks. But the performance of the proposed method doesn't decrease much comparing with BOC(1,1). In long delay range, the multipath restraint performance of proposed method is still much better than HRC. Even in short delay range within [0, 0.1] chips, the proposed method is slightly better than HRC.

6. Conclusions

In this paper, an optimum discriminator design method for GNSS receivers is presented. The aim of this optimum discriminator is to track BOC unambiguously and have minimized multipath error. The proposed method use a discriminator structure based on linear-combined code which is easily implemented in GNSS receives. An optimization problem is formed and solved to achieve the design aim. The design examples with respect to BOC(1,1)and BOC(10,5) signals demonstrate that the proposed method can completely remove the false lock points and provide much better performance for multipath mitigation than HRC technique. Besides, the proposed method gives a new way based on optimization to design discriminator. This approach may make the design reach the theoretical optimal if the objective function and constraints are perfect. Future works will focus on extending this method to noncoherent discriminator and improve the optimization objective function and constraints to acquire better results.

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