Modeling and Analysis of Sub-Banding the Secondary Users' Channel in Cognitive Radio Networks

Samira HOMAYOUNI, Seyed Ali GHORASHI

Cognitive Telecommunication Research Group, Department of Electrical Engineering,
Faculty of Electrical and Computer Engineering, Shahid Beheshti University, G. C, Tehran, Iran

s.homayooni@gmail.com, a_ghorashi@sbu.ac.ir

Abstract. *Modern cognitive radio systems employ dynamic spectrum access techniques and these networks allow more efficient usage of scarce spectrum. Secondary users are supposed to operate in primary users' channels without interfering them. In this paper, we propose model and analyze a channel sub-banding scheme for an opportunistic spectrum sharing system in which two secondary users are allowed to coexist in a channel. In the proposed sharing scheme when all the channels are occupied by the primary and/or secondary users, the channels of the secondary users are divided into two sub bands allowing both of the secondary users exploit the same channel concurrently. The proposed model is analyzed through a two-dimensional Markov chain model and evaluated by different derived metrics including blocking probability and dropping rate. Although using the proposed scheme reduces the quality of service for secondary users because of sub-banding, simulation results show significant reduction in the blocking probability and dropping rate of the secondary users.*

Keywords

Dynamic spectrum access, cognitive radio, channel sub-banding, Markov chain, blocking probability, dropping rate.

1. Introduction

Due to the growing telecommunication services and the increasing need for high data rate transmissions, users' demand for scarce frequency spectrum resources has been increased. Conventional wireless networks use the stationary spectrum allocation methods. In these methods, spectrum allocation is performed by government agencies for licensed users. Nowadays, due to the increase of demand for spectrum, this procedure faces spectrum shortage in certain bands. Although a large portion of the spectrum is already allocated, it has been used sporadically; at special times and locations these bands are not used by the corresponding users. This causes a non-optimal use of the available spectrum resources and the amount of spectrum efficiency has been reported very low [1], [2]. Therefore, dynamic spectrum access techniques have been used to improve the efficiency of the scarce spectrum resources' usage [3], [4]. Also, cognitive radio networks with the ability to realize and monitor the environment for opportunistically and intelligently utilization of the spectrum are proposed to solve this spectral inefficiency problem [5], [6].

There are many studies on the processes of sharing and having access to the radio spectrum in cognitive radio networks. The fundamental basis in these dynamic spectrum access networks is the opportunistic spectrum access for secondary users. In these networks, secondary users can use the same spectral band without interfering with primary users, [7], [8]. They can detect unused spectrum holes and use them by utilizing different spectrum sensing techniques. In addition, due to the presence of primary users, these channels cannot continuously be used by secondary users. Therefore, a secondary user upon the detection of a primary user in its channel should empty it, immediately. In such situation, the secondary user reviews all of the other channels, and if there is any vacant channel available, it switches to that channel. We call this procedure as spectrum hand-over. Otherwise, the secondary user connection is cut-off (dropped).

Many studies are related to spectrum sharing issues in cognitive radio networks. In [9] the modeling of interference based on the listen before talk scheme in spectrum access is analyzed. [10] proposes a measurement-based model for statistically describing the idle and busy periods of a WLAN. Spectral efficiency can be raised by allowing to reuse the empty bands by the secondary users [11], which proposed an opportunistic spectrum sharing model. Since in practical systems, spectrum sensing mechanism is associated with errors, the authors have generalized their model to the case of unreliable spectrum sensing in [12] and [13].

In [14] a network including N secondary users with different traffic types that coexist with a primary user was developed. Authors in [15] used the channel aggregation methods in cognitive radio networks in order to increase the spectral efficiency of the network. Using these methods, a secondary user can take advantage of several separate or adjacent parts of the spectrum to be used as a channel, simultaneously. In [16] two channel aggregation methods (fixed and variable) are studied. [15] and [16] indicate that channel aggregation for secondary users increases their throughput and makes more utilization of the spectrum. [17] and [18] suggest channel reservation methods for the primary and secondary users in order to prevent call dropping, and [19] introduces the use of backup channels for the secondary users. There are some other performance evaluation models for dynamic channel allocation schemes. The constrained spectrum sharing model based on competitive price game is established in [20]. Authors in [20] proposed a spectrum sharing strategy which deals with the constraints from primary users and available licensed spectrum resources by defining profit function to measure the impact of shared spectrum price strategies on the system profit.

In this paper we propose, model and analyze an spectrum sharing in cognitive radio networks in which sub-banding concept is utilized. Unlike [17], [18] and [19] that use additional resources or tools such as additional bandwidth or queues (buffers) to reduce the dropping rate of secondary users, in this paper we introduce a new method of channel sub-banding that does not require additional resources. The proposed method is such that if all the channels are being occupied by the primary and/or other secondary users, while a new request of secondary user arrives, then secondary users' channels are divided into two sub-bands, and two secondary users use a sub-band, simultaneously (i.e., both secondary users coexist in the same channel). The channel allocation models proposed in [11]- [13] were limited to the case that the number of primary and secondary users in the system are equal. In contrast, our spectrum sharing model is a general one that can be easily extended to different scenarios with different number of users. For performance evaluation, we have derived metrics such as blocking probability, dropping rate, channel utilization, user efficiency as well as individual and average throughput, which is fully explained in the next sections. Some of the initial results of using the proposed sharing scheme has been published by authors in [21] and this paper actually is an extension of that work with a Markov Chain model of sub-banding idea and more comprehensive simulation results.

2. Model and Assumptions

2.1 System Model

In order to explain the proposed method for utilizing the spectral holes, which temporarily have not been used in the licensed spectrum bands, we consider a simple diagram of a network in Fig. 1. In this cognitive radio network, which consists of primary and secondary users (PUs and SUs) who operate at the same spectrum bands, spectrum holes are used by SUs opportunistically. The primary system Access Point (AP1), manages all the N channels in a given service area. PUs have the priority to access the spectrum bands. Therefore, their activities should not be affected by the SUs' activities.

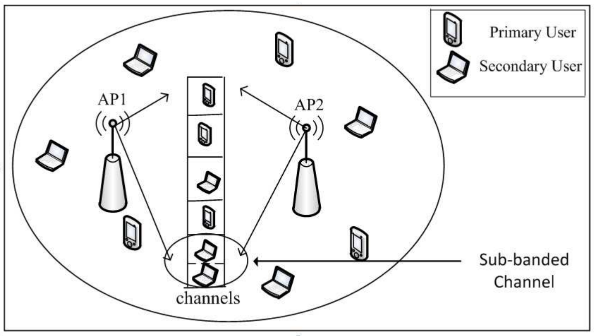

Fig. 1. General model of spectrum sharing with channel sub-banding.

Hence, the spectrum sharing process and the amount of the spectrum accessed by the SUs are affected by parts of the spectrum which are used by PUs. The cognitive wireless networks may be infrastructured or infrastructurless. Although we assumed that the PUs and SUs both operate on an infrastructured network which have BTS or AP in the service area, it can be possible to extend the proposed model to the infrastureless ad hoc networks. Also, we assume that the SUs are equipped with cognitive radio equipment's so that they have the capability to detect the PUs' activities; i.e. they can recognize the presence of a PU or its departure from a channel. In other words, the SUs can detect the in-used channels, and switch to other idle channels without interfering with the PUs. Furthermore, the SUs' access to channels is assumed to be controlled by a secondary access points/base station (AP2). The SUs can distinguish whether the spectrum is occupied by the SUs and/or PUs [22] (by using spectrum sensing methods). When a PU appears on a given SUs channel, the SUs should stop their transmission on the channel and set their parameters; i.e. they have to reduce their power transmission or vacate the channel and attempt to access other available bands. Note that the figure shown here (Fig. 1), is only a simple scenario for better insight and can be easily extended to other scenarios.

2.2 Spectrum Sharing with Sub-Banding

In this Section we propose a new opportunistic spectrum sharing scheme in cognitive radio networks. In this scheme, if all channels are occupied by PUs and/or SUs, then the SUs' channels are divided into two sub-channels. In this way both SUs can use a spectrum band simultaneously, if the channel is not occupied by the PUs. Obviously, the data transmission rate will be reduced (at least by half). However, in many scenarios (such as emergency calls in disaster situations), to have a lower data rate communication link between many nodes (with a low dropping rate) is much more beneficial than having a few high data rate links between some selected nodes while experiencing a high level of dropping rate by the nodes community.

In order to explain the proposed spectrum sharing scheme in detail, we consider a typical scenario shown in Fig. 1. In this scenario when a PU appears in the SU chan-

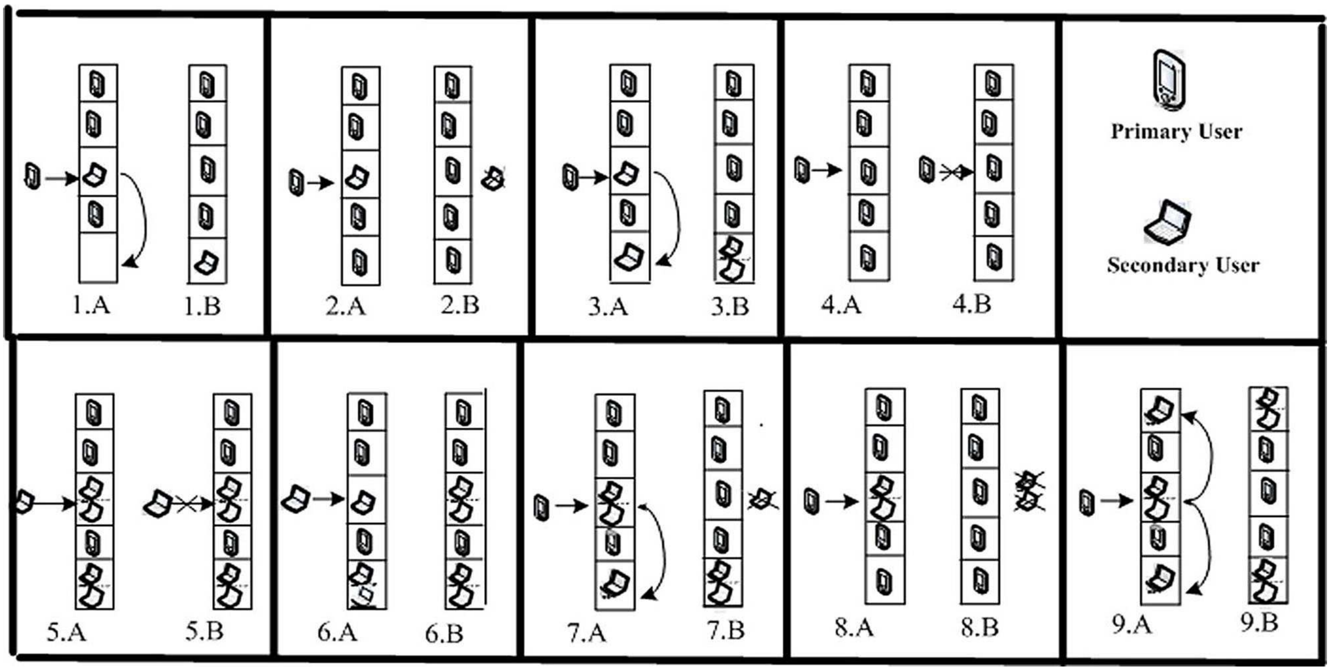

Fig. 2. All the possible cases in channel sub-banding process – (A) before service request, (B) after service request.

nel, the SU should stop its data transmission on this channel and switch to any other available channel. If there is not any idle channel in the system, SUs' service will be forced to stop (i.e. the SU is dropped). The SUs have the capability of detecting the channel status without causing interference with PUs. Therefore, when an ongoing SU detects the presence of a PU in its current channel, SU releases its channel to PU. By considering the spectrum hand over, SU switches to one of other idle channels if it is possible (Fig. 2, block 1). If all of the channels are busy by other PUs and there is no SU in system, the PU achieves the channel with a priority in accessing the channel and the ongoing SU will be dropped (Fig. 2, block 2). If at that time, all channels are busy and there is at least another SU in the system, the SU channel is divided into two sub-bands and both SUs use the same channel simultaneously with a lower data rate (Fig. 2, block 3). A PU's request will be blocked only if all the channels are busy by other PUs (Fig. 2, block 4). When a SUs' request arrives to the system and all the channels are busy and also all the SUs' sub-bands have been occupied, the SU's request will be blocked (Fig. 2, block 5). But if there is at least one SU in the system, the channel is divided to two sub-bands and the new SU coexists with ongoing SU in the same channel (Fig. 2, block 6).

When the PU's request arrives to sub-bands which are used by SUs (e.g. A and B) and if there are at least two other SUs (e.g. C and D) in a system whose channels are not divided to sub-bands yet, each of the two SUs (A and B) joins the other SU channels (C and D) and PU occupies the initial channel (Fig. 2, block 9). If there is only one SU in the system (e.g. A) that uses non-divided channel, one of the ongoing SUs (B and C) coexists with the SUs' channel (A) and the other SU is dropped (Fig. 2, block 7). If at the arrival of the PU's request there are only sub-banded channels of SUs, then PU occupies one of those channels and the corresponding two SUs will be dropped (Fig. 2, block 8).

3. Performance Analysis and Channel Allocation Process

In this Section, we analyze the performance of the primary and secondary networks in the service area in which the spectrum is shared by using our proposed scheme. We assume a cellular network in which all cells are statistically identical. We further assume that our service area contains a cell and we analyze our model for a single cell. Suppose that there are N channels in a cell to serve both primary and secondary traffics. In each cell, the arrival times of PU and SU calls are independent Poisson processes with rates λ_p and λ_s, respectively.

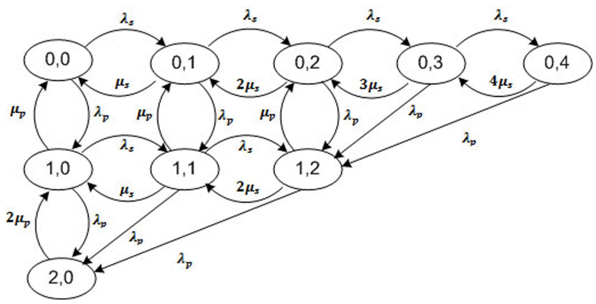

Fig. 3. Transition diagram for a channel sub-banding with $N = 2$ and $0 \le k_1 \le 2, 0 \le k_2 \le 4$.

The holding times (service duration) of both PU and SU calls follows an exponentially distributed with mean $1/h_p$ and $1/h_s$, respectively. The resident times of the PU and SU calls in the area under the service is exponentially distributed with means $1/r_p$ and $1/r_s$ respectively. We assume that the channel holding time is equal to the minimum holding time of calls and residence time in the given service area. Therefore, channel holding time for the PU and SU calls is exponentially distributed with mean $\mu_P^{-1} = 1/(h_p+r_p)$ and $\mu_S^{-1} = 1/(h_s+r_s)$, respectively [23]. For the sake of simplicity, we assume that both the PU and SU calls occupy only one channel per call. We define $(X_1(t), X_2(t))$ as a 2-D Markov process with state spaces $S = \{(k_1,k_2)|0 \leq k_1 \leq N, 0 \leq k_2 \leq 2(N-k_1)\}$ for sub-banding case and $S = \{(k_1,k_2)|0 \leq k_1 \leq N, 0 \leq k_2 \leq N\}$ for no sub-banding case, in which $X_1(t)$ and $X_2(t)$ are the numbers of PUs and SUs in the system at time t, respectively. $T_{k_1,k_2}^{k_1',k_2'}$ is the transition rate from state (k_1,k_2) to (k_1',k_2').

We also define the indicator function of $1_{\{x\}}$, which is 1 if x is true and is 0 in other cases. Channel allocation process by the primary base station is uniform; i. e. a PU selects a channel randomly with equal probability among the channels. Hence, the PU can randomly choose an unused channel or a channel which is used by a SU or SUs. The corresponding state transition diagram is shown in Fig. 5. The transition rate $T_{k_1,k_2}^{k_1',k_2'}$ for the Markov model with sub-banding is obtained as follows:

$$\begin{aligned}
T_{k_1,k_2}^{k_1+1,k_2} &= \lambda_p 1_{\{0\leq k_1<N, 0\leq k_2<2(N-(k_1+1))\}},\\
T_{k_1,k_2}^{k_1-1,k_2} &= k_1\mu_p 1_{\{0\leq k_1\leq N, 0\leq k_2\leq 2(N-k_1)\}},\\
T_{k_1,k_2}^{k_1,k_2+1} &= \lambda_s 1_{\{0\leq k_1\leq N-1, 0\leq k_2\leq 2(N-k_1)-1\}},\\
T_{k_1,k_2}^{k_1,k_2-1} &= k_2\mu_s 1_{\{0\leq k_1\leq N-1, 0\leq k_2\leq 2(N-k_1)\}},\\
T_{k_1,k_2}^{k_1+1,k_2-1} &= \lambda_p 1_{\{0\leq k_1<N, k_2=2(N-k_1)-1\}},\\
T_{k_1,k_2}^{k_1+1,k_2-2} &= \lambda_p 1_{\{0\leq k_1<N, k_2=2(N-k_1)\}}.
\end{aligned} \tag{1}$$

Similarly, the transition rate $T_{k_1,k_2}^{k_1',k_2'}$ for the Markov model with no sub-banding is obtained as follows:

$$\begin{aligned}
T_{k_1,k_2}^{k_1+1,k_2} &= \lambda_p 1_{\{0\leq k_1<N-1, 0\leq k_2<N-(k_1+1)\}},\\
T_{k_1,k_2}^{k_1-1,k_2} &= k_1\mu_p 1_{\{0\leq k_1\leq N, 0\leq k_2\leq N-k_1\}},\\
T_{k_1,k_2}^{k_1,k_2+1} &= \lambda_s 1_{\{0\leq k_1\leq N-1, 0\leq k_2\leq N-k_1\}},\\
T_{k_1,k_2}^{k_1,k_2-1} &= k_2\mu_s 1_{\{0\leq k_1\leq N-1, 0\leq k_2\leq N-k_1\}},\\
T_{k_1,k_2}^{k_1+1,k_2-1} &= \bar{\delta}(k_2)\lambda_p 1_{\{0\leq k_1<N, k_2=N-k_1\}}
\end{aligned} \tag{2}$$

where $\bar{\delta}(k_2) = 0$ for every $k_2 = 0$ and $\bar{\delta}(k_2) = 1$ in other cases. A forced SU disconnection depends on the rest of channels' state and users in the system. Two transitions rates $T_{k_1,k_2}^{k_1+1,k_2-1}$ and $T_{k_1,k_2}^{k_1+1,k_2-2}$ represent the states in which a SU and two SUs are interrupted from the system, respectively.

We define $I_{(k_1,k_2)}$ as a function which has two cases, 1 and 0; it is equal to 1 (with and without sub-banding), if $2k_1 + k_2 \leq 2N$ (for the case of sub-banding) and if $k_2 < N$ (for the case of without subbandig); and $I_{(k_1,k_2)} = 0$ otherwise. As the steady state probabilities are the probabilities of stationary states at time t, the sum of transition rates that originate from the state $X_i(t) \in S_{|i=1,2}$ are equal to the sum of transition rates that end to the state $X_i(t)$. Then, balanced state equation for two Markov models (with and without sub-banding) can be expressed as:

$$\begin{aligned}
&(T_{k_1,k_2}^{k_1+1,k_2} + T_{k_1,k_2}^{k_1-1,k_2} + T_{k_1,k_2}^{k_1+1,k_2-1} + T_{k_1,k_2}^{k_1+1,k_2-2} +\\
&T_{k_1,k_2}^{k_1,k_2+1} + T_{k_1,k_2}^{k_1,k_2-1}) I_{(k_1,k_2)}\, \pi(k_1,k_2)\\
&=\\
&T_{k_1+1,k_2}^{k_1,k_2} I_{(k_1+1,k_2)} \pi(k_1+1,k_2) + T_{k_1-1,k_2}^{k_1,k_2} I_{(k_1-1,k_2)}\\
&\pi(k_1-1,k_2) + T_{k_1,k_2+1}^{k_1,k_2} I_{(k_1,k_2+1)} \pi(k_1,k_2+1) +\\
&T_{k_1,k_2-1}^{k_1,k_2} I_{(k_1,k_2-1)} \pi(k_1,k_2-1),
\end{aligned} \tag{3}$$

and we have:

$$\begin{aligned}
&\sum_{k_2=0}^{N}\sum_{k_1=0}^{N} \pi(k_1,k_2) I_{(k_1,k_2)} = 1, \text{ without sub-banding},\\
&\sum_{k_2=0}^{2N}\sum_{k_1=0}^{N} \pi(k_1,k_2) I_{(k_1,k_2)} = 1, \text{ with sub-banding}
\end{aligned} \tag{4}$$

where $\pi(k_1,k_2)$, is the steady state probability for the state (k_1,k_2). By computing the steady state probabilities (π)s expressed above, we can obtain different required metrics as described in the next Section. Fig. 3 shows a simple model of the transition diagram in a channel sub-banding case for $N = 2$ and $0 \leq k_1 \leq 2, 0 \leq k_2 \leq 4$. The extended diagram for the case that the number of channels is N, and $0 \leq k_1 \leq N, 0 \leq k_2 \leq 2N$ is shown in Fig. 4. In order to distinquish between the steady states in sub-banding and no-sub-banding scenarios, π_{SB} and π notations will be used for sub-banding and no-sub-banding, respectively.

4. Performance Metrics

In this Section we describe different measures of interest, such as channel utilization, individual throughput, user

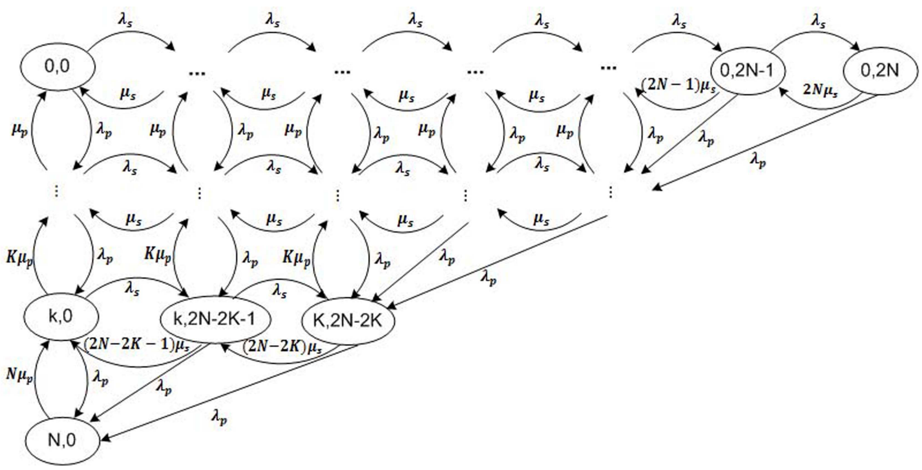

Fig. 4. Transition diagram for a channel sub-banding with N channels and $0 \le k_1 \le N$ and $0 \le k_2 \le 2N$.

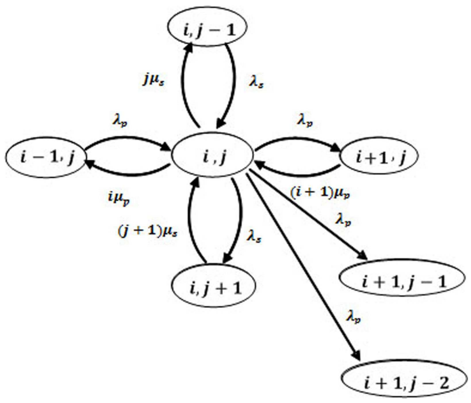

Fig. 5. Transition diagram for a channel sub-banding case.

efficiency and average throughput as well as blocking probability and dropping rate, in order to evaluate the performance of the proposed sub-banding algorithm.

4.1 Blocking Probability

- Blocking probability of SU

The blocking of the SUs (with sub-band) happens when all channels are busy by SUs and/or PUs, while an SU arrives in the system. At that time, there is no available subband for a new request, i.e. all channels of SUs have already divided and occupied. Blocking probability of the SUs is denoted by $PB_{SU_{SB}}$ that can be calculated by:

$$PB_{SU_{SB}} = \sum_{k_1=0}^{N} \pi_{SB}(k_1, 2(N-k_1)). \tag{5}$$

For the mode without sub-banding, blocking of the SUs happens when all channels are busy by other SUs and/or PUs at the arrival of an SU, while there is no idle channel for a new incoming request. PB_{SU} in this case is given by

$$PB_{SU} = \sum_{k_1=0}^{N} \pi(k_1, N-k_1). \tag{6}$$

- Blocking probability of PUs

A primary call request is blocked when upon the arrival of a primary call request, all the channels are busy by PUs and at that time there is no idle channel for a new primary call request. Blocking probability of the PUs is denoted by PB_{PU} and it does not depend on the channel sub-banding by SUs. Therefore, it is obtained in both cases of with and without sub-banding, as:

$$PB_{PU} = \pi(k_1 = N, k_2 = 0). \tag{7}$$

4.2 Dropping Rate

Upon the arrival of a PU, if all channels are busy by SUs and/or PUs, and also there is no idle channel for an ongoing SU or SUs in order to switch to it, then the user or users will be dropped. In this case all channels of SUs have already been divided and occupied. Also note that in such a case, at least one SU should be available in the system. Dropping rate of the SUs is denoted by PD_{SU} and can be modeled as follows, for the cases of without and with sub-banding, respectively.

$$PD_{SU} = \sum_{k_2=1}^{N} \pi(N-k_2, k_2) \tag{8}$$

and

$$PD_{SU_{SB}} = \sum_{k_2=1}^{2N} \sum_{k_1=0}^{N-1} (T_{k_1,k_2}^{k_1+1,k_2-1} \pi_{SB}(k_1,k_2) + T_{k_1,k_2}^{k_1+1,k_2-2} \pi_{SB}(k_1,k_2)) I_{(k_1,k_2)}$$
$$= \sum_{k_1=0}^{N-1} \pi_{SB}(k_1, 2(N-k_1)-1) + \pi_{SB}(k_1, 2(N-k_1)). \tag{9}$$

4.3 Channel Utilization

The ratio of the average number of busy channels to the number of all channels is defined as Total channel utilization. It is denoted by η and for the case of no sub-banding, it can be modelled as follows:

$$\eta = \frac{1}{N}\sum_{k_1=0}^{N}\sum_{k_2=0}^{N-k_1}((k_1+k_2)\pi(k_1,k_2))I_{(k_1,k_2)}. \quad (10)$$

In calculating this utilization, when a channel is used by two SUs, it is assumed that one channel is busy. For the case of with sub-banding, η_{SB} can be written as follows:

$$\eta_{SB} = \frac{1}{N}[\sum_{k_1=0}^{N}\sum_{k_2=0}^{2(N-k_1)}((k_1+k_2)\pi_{SB}(k_1,k_2))I_{(k_1,k_2)}1_{\{0\le k_1+k_2\le N\}} + \sum_{k_1=0}^{N}\sum_{k_2=N-k_1+1}^{2N}(N\,\pi_{SB}(k_1,k_2))I_{(k_1,k_2)}1_{\{k_1+k_2>N\}}]. \quad (11)$$

The Total Carried Traffic (TCT) is defined as the total traffic of PUs and SUs that is supported in a given service area. For the case of with sub-banding, TCT is obtained as follows:

$$TCT = \sum_{k_1=0}^{N}\sum_{k_2=0}^{2(N-k_1)}((k_1+k_2)\pi_{SB}(k_1,k_2))I_{(k_1,k_2)}1_{\{0\le k_1+k_2\le N\}} + \sum_{k_1=0}^{N}\sum_{k_2=N-k_1+1}^{2N}(N\,\pi_{SB}(k_1,k_2))I_{(k_1,k_2)}1_{\{k_1+k_2>N\}}. \quad (12)$$

4.4 Individual Throughput

The individual throughput, T_{ind} is defined as the ratio of the summation of individual users' throughout to the total number of users in channel sub-banding mode (which is normalized to the maximum data rate of a channel). For example, suppose that each channel allocated to a user has the maximum data rate of 24 kbps while if it is allocated to two users (sub-banding), the maximum data rate of each sub-band would be at most $24/2 = 12$ kbps. Also suppose that for example there are 3 channels to be shared by PUs and SUs. If two channels are allocated to the PUs, and the third channel is allocated to two SUs by sub-banding, then $T_{ind} = (24+24+2\times 12)/(4\times 24) = 0.75$ which is less than 1 for the case that 3 channels are allocated to 3 PUs or SUs. In general, T_{ind} can be determined as follows:

$$T_{ind} = \sum_{k_1=0}^{N}\sum_{k_2=0}^{N-k_1}\pi(k_1,k_2)I_{(k_1,k_2)} + \sum_{k_1=0}^{N-1}\sum_{k_2=N-k_1+1}^{2(N-k_1)}(\tfrac{N}{(k_1+k_2)}\pi_{SB}(k_1,k_2))I_{(k_1,k_2)} \quad (13)$$

where the first term is for the case of no sub-banded channels, and the second one is for the case of sub-banded channels. This equation is normalized to the maximum data rate of a channel (i.e., 24 kbps in the above example). The parameter N, in the numerator is the considered maximum rate and (k_1+k_2) in the denominator indicates the total number of users in the system.

4.5 User Efficiency

We define user efficiency, η_u as the average number of users per channels in the case of channel sub-banding and it is determined as:

$$\eta_u = \frac{1}{N}[\sum_{k_1=0}^{N}\sum_{k_2=0}^{2N}(k_1+k_2)\pi_{SB}(k_1,k_2)I_{(k_1,k_2)}]. \quad (14)$$

4.6 Average Throughput

In practical systems there are many factors that make the average throughput to be less than the ideal throughput, T. Here, we introduce the average throughput as another important metric which is used in the channel sub-banding mode to show how the sub-banding reduces the ideal throughput. Here, we define the average throughput, T_{re}, as the ratio of the real throughput of system to the ideal one and we provide a formula for it. If we define $\acute{R}$ as the maximum rate per sub-band, for $\acute{R} = R/2$ the average throughput is equal to the ideal throughput, i.e. $T_{re} = T$. But, practically $\acute{R} < R/2$ (for example, because of the required guard frequency bands if FDMA), and consequently $T_{re} < T$. The average throughput is determined as:

$$T_{re} = \sum_{k_1=0}^{N}\sum_{k_2=0}^{N-k_1}\tfrac{1}{N}(k_1+k_2)\pi(k_1,k_2)I_{(k_1,k_2)} + \sum_{k_1=0}^{N-1}\sum_{k_2=N-k_1+1}^{2(N-k_1)}\tfrac{R.\chi+2\acute{R}(N-\chi)}{N.R}\pi_{SB}(k_1,k_2)I_{(k_1,k_2)} \quad (15)$$

where $\chi = 2N-(k_1+k_2)$, R is the transmitted data rate (e.g. 24 kbps in the above example) and $\acute{R}$ is the maximum rate per sub-band. The first term in (15) is for no sub-banded users and the second one corresponds to the sub-banded users. Lower $\acute{R} = R/2$ yields to more loss and therefore the average throughput is sensitive to the $\acute{R}/R$ ratio. This can be seen as the cost to be paid for sub-banding in practical systems.

5. Numerical and Simulation Results

In this Section, we present the numerical and simulation results to show the applicability and the performance of the proposed channel allocation method. In order to show the performance benefit of our proposed scheme, we also

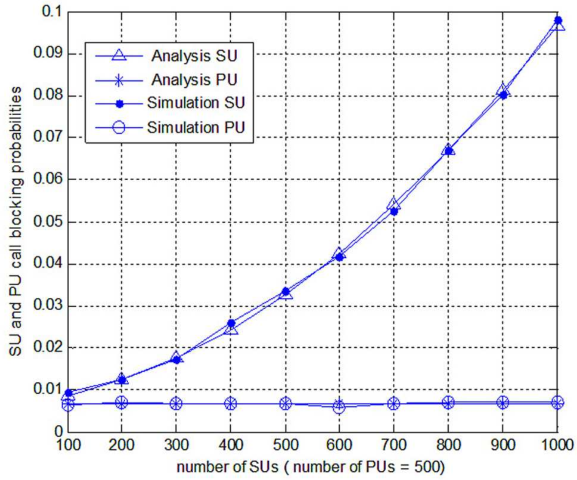

Fig. 6. Validation of PU and SU call blocking probabilities in the case of channel sub-banding ($PB_{SU_{SB}}$ and PB_{PU}).

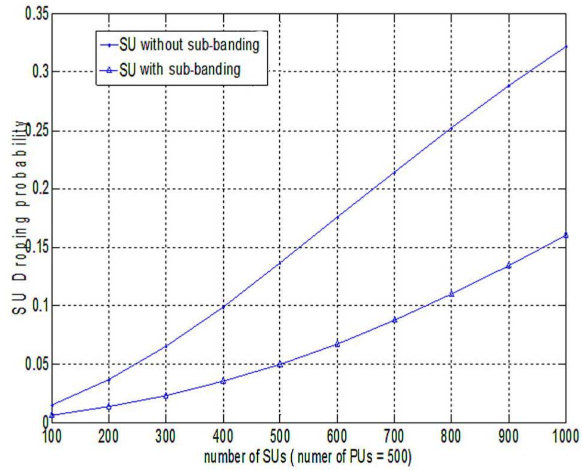

Fig. 8. SU call dropping rate, in the case of using and not using channel sub-banding ($PD_{SU_{SB}}$ and PD_{SU}).

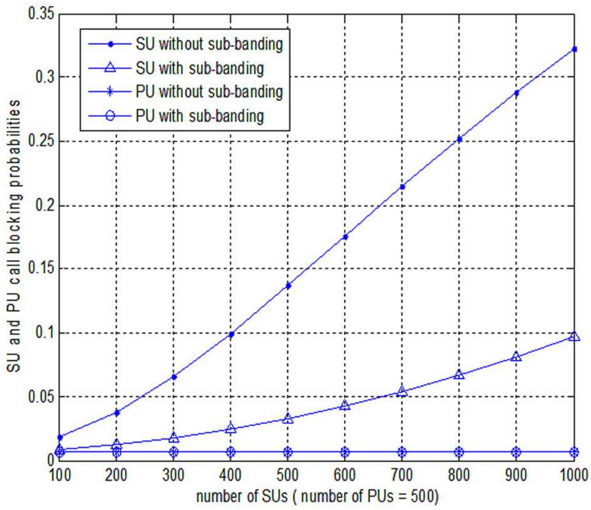

Fig. 7. SU and PU call blocking probabilities ($PB_{SU_{SB}}$, PB_{SU} and PB_{PU}).

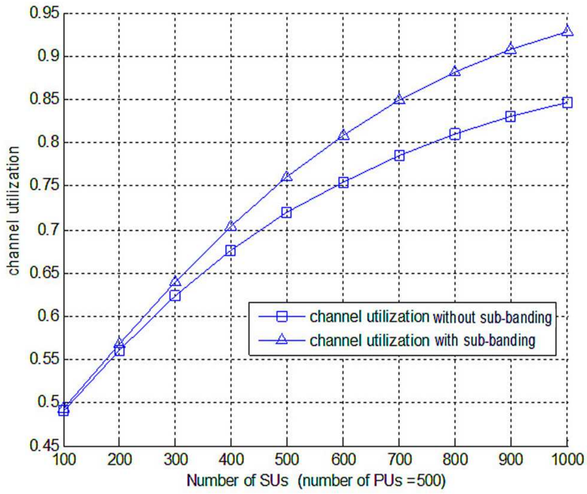

Fig. 9. Channel utilization, in the case of using and not using channel sub-banding (η_{SB} and η).

include the performance of an original system without sub-banding for comparison purposes. Also, we study the impact of channel sub-banding on blocking probability and dropping rate. To obtain numerical and simulation results, we consider a cellular system in which each cell has 10 channels available. We assume that the arrivals of both PU and SU calls follow a Poisson process with parameter $\lambda_p = \lambda_s =$ 1 call/hour, and the call time of both users follows an exponential distribution with the mean of 30 secs. We assume that the number of PUs per each cell is constant and it is equal to 500, while the number of SUs is variable to study the effect of users' number on the performance of PUs. All the simulations are performed by MATLAB software.

First, we validate the analysis of blocking probabilities by comparing the simulation and analytical results of the presence of sub-banding for both PUs and SUs ($PB_{SU_{SB}}$ and PB_{PU}). Notice that sub-banding has no effect on PUs blocking probability as indicated in Fig. 6. This figure shows an exact compliance between simulation and analytical results of the proposed schemes for both PUs and SUs and for different SU numbers.

In Fig. 7, blocking probability for SUs and PUs ($PB_{SU_{SB}}$, PB_{SU} and PB_{PU}) are shown for two cases, with and without sub-banding, as a function of SUs number. We observe that the increase of SUs' population in both cases have no effect on PUs blocking probability. This shows that in an ideal cognitive radio system, the SUs' activities have no influence on PUs' performance. The SUs blocking probability for the case of no sub- banding is more than the case of using sub-banding. The reason is that, while all channels are busy, by dividing the channels to sub-bands, the number of available channels for incoming SU calls increases and the blocking probability of the SUs' decreases, consequently.

In Fig. 8, the dropping rate of the SUs for different populations of SUs, for two cases of using and not using sub-banding ($PD_{SU_{SB}}$ and PD_{SU}) are compared. According to this figure, if there are sub-bands, dropping rate of the SUs decreases considerably. The reason is, when SUs are going

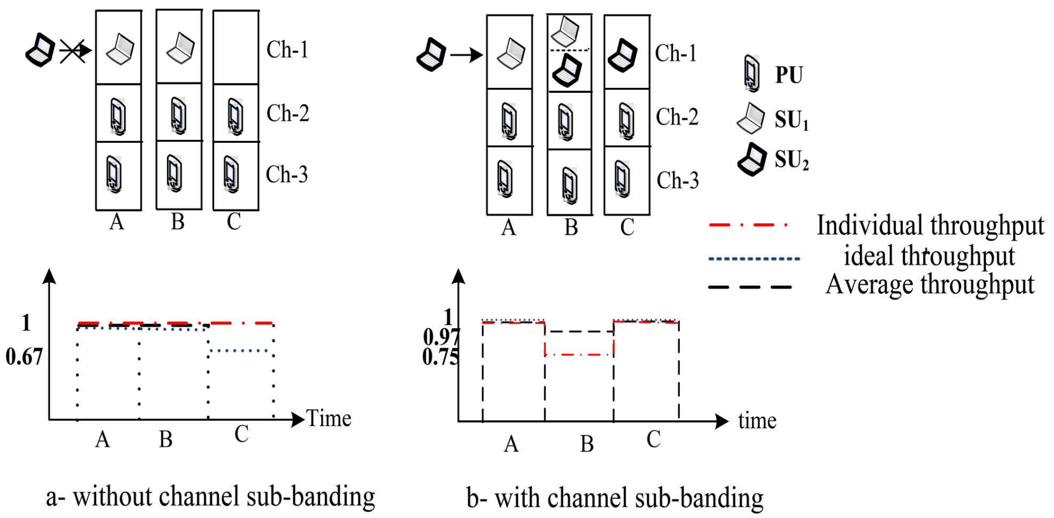

Fig. 10. Example of spectral access with $N = 3$ (upper: system model; lower: individual throughput). Total throughput of a channel increases with sub-banding.

to be dropped by arriving new PU calls, the channels of other SUs are divided into two bands, and two SUs use the same channel, simultaneously without dropping the ongoing SU calls. This decrease in dropping rate of SUs is the main advantage of the proposed sub-banding based channel sharing algorithm.

Fig. 9 compares the channel utilization for the two cases of without and with channel sub-banding (η_{SB} and η). As you can see from the figure, channel utilization increases as the traffic increases for both cases. This is due to the fact that the probability of busy channel increases as the number of secondary users grows up. Furthermore, channel utilization is higher in the case of sub-banding, and when the traffic is high, this difference is bigger. This can be explained as follows. Consider a snapshot of spectral access as demonstrated in Fig. 10. Let $N = 3$ and suppose that channels 2 and 3 are allocated to two PUs and channel 1 is allocated to an SU, called SU_1. The throughput vs. time in three consecutive snapshots for channel 1 is illustrated in this figure (Fig. 10a and 10b for two cases of without and with sub-banding, respectively).

In the case of without sub-banding, first SU_1 attains the channel. As the SU_1 is receiving service, SU_2's request for service arrives at the system. This request fails since there is no empty channel in the system. After finishing the SU_1 service time, channel 1 remains unused until a new request is received from users. In the second case, SU_1 first attains the spectral band and the SU_2's service request arrives at the system during the SU_1's service time. As there is no empty channel in the system in this case, SU_1 shares the spectral band with SU_2 that may lead to a lower throughput (utilization) for individual users. After SU_1's servicing is finished, SU_2 continues receiving service till its service time finishes or a PU enters to the system. Therefore, the probability of busy utilization) is higher in this case, particularly when there are more service requests and the average of busy channels to utilization) is higher in this case, particularly when there are more service requests and the average of busy channels to the total channels is large.

Fig. 11 compares the individual throughput T_{ind} (average throughput of single users) in addition to the users' efficiency η_u in the two cases. It is obvious that the user efficiency is higher in the second case in comparison with the first one, when there is no sub-band. It is demonstrated in Fig. 10 with a simple example, where in three snapshots of A, B and C, three, three, and two users in the first case, and three, four, and three users in the second one have occupied the channels, respectively. This increment is more significant in high traffics when the number of users grows up; of course, this is not costless.

In Fig. 11, you can also see two other curves which show the individual throughput T_{ind} for the two discussed cases. As it can be seen, in high traffics, individual user's

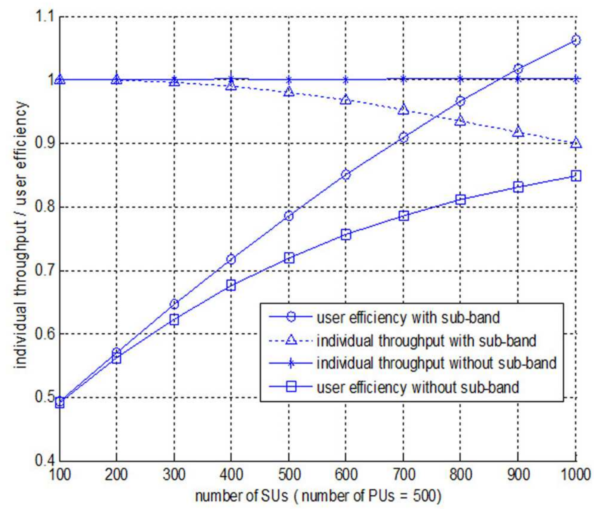

Fig. 11. Individual throughput T_{ind} and user efficiency η_u for two cases.

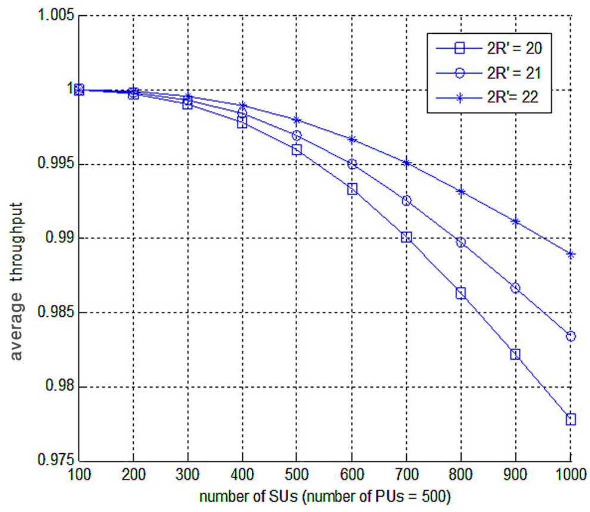

Fig. 12. The average throughput T_{re}.

throughput in the first case is higher than the second one. Suppose that each user has 24 kbps data rate to use the channel. In Fig. 10-a-A and B there are three 24 kbps channels for three users and each user can be assigned 24 kbps, individually. The same is correct for Fig. 10-b-A, but in Fig. 10-b-B there are three 24 kbps for four users. Therefore, each of the two users which use the channels 2 and 3 can be assigned 24 kbps channels and each of the two users using channel 1 can be assigned 12 kbps in the best case, i.e. $(2\times 24+2\times 12)/4$. Thus, data rate depends on the number of sub-banded channels. With a higher number of users, three 24 kbps channels will be shared among more users. Therefore, we can say sub-banding the channels leads to a lower individual throughput for single users and this is the first price paid for increasing the number of served users.

Fig. 12 demonstrates the amount of average throughput T_{re} in the case of sub-banding the channels, as a function of users' number. This amount depends on the maximum transmitted data rate per sub-band and is shown for three cases of $\acute{R}$ 10, 10.5 and 11 kbps. The less $\acute{R}/R$, the more cost paid for sub-banding in terms of throughput loss. In other words, if the $\acute{R}$ is less than R, this sub-banding method yields to some throughput loss. This is the second price paid for using the proposed method.

6. Conclusion

In this paper, we have analyzed an special opportunistic spectrum sharing model in cognitive radio networks. The proposed scenario is such that if all the channels are allocated to the PUs and/or SUs, then SUs' channels will be divided into two sub-bands and two SUs can use the same channel, simultaneously. In order to evaluate the performance of channel sub-banding strategy, by using Markov model we have derived some metrics such as blocking probability, dropping rate and channel utilization as well as user efficiency, individual and average throughput. Based on the derived metrics, the results for the case of using and not using sub-banding schemes have been compared. Finally, it has been observed that in a cognitive radio network, although channel sub-banding causes a lower quality of service for sub-banded users, it significantly reduces the blocking probability and dropping rates of the SUs. This phenomena can be useful in some situations such as emergency or disaster situations, in which we prefer to have more users connected even with lower quality of service level. The proposed sub-banding scheme can be easily generalized to the case of imperfect spectrum sensing and channel sub-banding to N channels.

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About Authors...

Samira HOMAYOUNI is currently a M.Sc. Graduated student in Communication Engineering from Shahid Beheshti University G.C., Tehran, Iran. She received her B.Sc. degree in electronic engineering in 2007 from University of Arak, Iran. Her research interests are on Cognitive Radio Networks and Wireless Communication Systems. She currently works on Cognitive Telecommunication Systems under the supervision of Dr. S. Ali Ghorashi in Cognitive Telecommunication Research Group, Shahid Beheshti University, G. C, Tehran, Iran. Besides these academic activities, she works expertly on programming, embedding and optimizing telecommunication systems and algorithms on Digital Signal Processors.

Seyed Ali GHORASHI is received his B.Sc. and M.Sc. degrees in Electrical Engineering from the University of Tehran, Iran, in 1992 and 1995, respectively. Then, he joined SANA Pro Inc., where he worked on modelling and simulation of OFDM based wireless LAN systems and interference cancellation methods in W-CDMA systems. Since 2000, he worked as a research associate at King's College London on "capacity enhancement methods in multi-layer WCDMA systems" sponsored by Mobile VCE. In 2003 He received his PhD at King's College and since then he worked at Kings College as a research fellow. In 2006 he joined Samsung Electronics (UK) Ltd as a senior researcher and now he is a faculty member of Cognitive Telecommunication Research Group, Department of Electrical Engineering, Shahid Beheshti University G. C., at Tehran, Iran, working on wireless communications.