A Separation Algorithm for Sources with Temporal Structure Only Using Second-order Statistics

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Abstract. *Unlike conventional blind source separation (BSS) deals with independent identically distributed (i.i.d.) sources, this paper addresses the separation from mixtures of sources with temporal structure, such as linear autocorrelations. Many sequential extraction algorithms have been reported, resulting in inevitable cumulated errors introduced by the deflation scheme. We propose a robust separation algorithm to recover original sources simultaneously, through a joint diagonalizer of several average delayed covariance matrices at positions of the optimal time delay and its integers. The proposed algorithm is computationally simple and efficient, since it is based on the second-order statistics only. Extensive simulation results confirm the validity and high performance of the algorithm. Compared with related extraction algorithms, its separation signal-to-noise ratio for a desired source can reach 20 dB higher, and it seems rather insensitive to the estimation error of the time delay.*

Keywords

Blind source separation (BSS), blind source extraction (BSE), linear autocorrealtion, joint diagonalization, second-order statistics.

1. Introduction

Recent decades have witnessed increasing attention in blind signal processing, especially the simultaneous blind source separation (BSS) and sequential blind source extraction (BSE), with high potential for applications in various areas of speech and image processing [1], [2], biomedical signal processing [3], wireless communication [4], [5] and so on. It denotes such a technique to recover latent sources from observed mixtures without available knowledge of the sources and the mixing process. In fact, we do not carry out the process in a completely blind manner, since some statistical properties of original sources can be utilized as prior information, e.g., non-Gaussianity [5], [6], sparseness [7], smoothness [8], nonstationarity [9], temporal structure [10-15], etc. As a physical condition in real world, natural image, speech and biomedical signals have significant temporal structures.

Using the temporal characteristics of a desired source (source of interest), researchers have developed efficient objective functions and a number of BSE algorithms, based on the generalized autocorrelations, i.e., linear or nonlinear autocorrelations. An important and primary work is done by Barros and Cichocki [10]. They propose a simple extraction algorithm with only one time delay used, which requires the precise estimation of an optimal time delay. Zhang et al. furthers the work and proposes another extraction algorithm (called Zhang's algorithm here) [11], based on eigenvalue decomposition of several delayed covariance matrices. And its performance is unaffected by the estimation error of the time delay if the error is not too large. Later, a fixed-point algorithm named MACBSE [13] has been proposed. Unfortunately, those algorithms' performances for temporally correlated sources with similar autocorrelation values at the time delay are not satisfying. Besides, they strongly depend on the estimation of the optimal time delay. What should be mentioned is that, though BSE can extract source signals one by one in a specific order with deflation techniques [16], which eliminate the already extracted sources from their mixtures. However, it will result in inevitable cumulated errors.

Therefore in this paper we propose a BSS algorithm based on linear autocorrelations and joint diagonalization (LAJD), to realize the simultaneous separation from mixtures of temporal correlated source signals. Nowadays, there is a trend to favor algorithms based on second-order statistics, such as AMUSE [17] and SOBI [18], due to its low computation load and fast processing speed. And our proposed LAJD algorithm also relies merely on second-order statistics. The idea of it is close to SOBI, that is, both of them are based on a joint diagonalization of a set of delayed covariance matrices. What makes LAJD efficient is that only covariance matrices around the optimal time delay and its integers are selected. And its sensitivity to the time delay is improved by averaging covariance matrices near the same time delay. Moreover, the LAJD algorithm is especially suitable for image sources with similar autocorrelation values at the time delay, where other BSE algorithms behave poorly.

The rest of the paper is organized as follows. Section 2 introduces the signal model. In Section 3, related BSE algorithms are briefly described and the proposed LAJD algorithm is presented. Simulation results are provided in Section 4. At the end of the paper, a concise conclusion is given.

2. The Signal Model

Let observations $\boldsymbol{x}(t)=[x_1(t),...,x_N(t)]^T$ be linear combinations of zero-mean and unit-variance source signals $\boldsymbol{s}(t)=[s_1(t),...,s_N(t)]^T$ as:

$$\boldsymbol{x}(t)=\mathbf{A}\boldsymbol{s}(t) \tag{1}$$

where $\mathbf{A}$ is an $N\times N$ unknown mixing matrix. A prewhitening strategy is often adopted to transform the observed signals $\boldsymbol{x}$ into $\tilde{\boldsymbol{x}}=\mathbf{V}\boldsymbol{x}$, $\boldsymbol{E}\left\{\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^T\right\}=\mathbf{I}$, where $\mathbf{V}$ is a prewhitening matrix [5]. Hence, the target of BSS is to find such an $N\times N$ unmixing matrix $\mathbf{W}=(\mathbf{w}_1,...,\mathbf{w}_N)^T$ that $\boldsymbol{y}(t)=[y_1(t),...,y_N(t)]^T$ are the estimated source signals, which can be written in matrix formation as:

$$\boldsymbol{y}(t)=\mathbf{W}\tilde{\boldsymbol{x}}(t) \ . \tag{2}$$

Correspondingly, the target of BSE is to find an $1\times N$ unmixing vector $\mathbf{w}_i^T$ so that $y_i(t)=\mathbf{w}_i^T\tilde{\boldsymbol{x}}(t)$.

We assume that the source signals have linear autocorrelation, satisfying the following (3) for a positive integer time delay τ^*:

$$\begin{cases} E\left\{s_i s_{i,\tau^*}\right\}=E\left\{s_i(t)s_i(t-\tau^*)\right\}>0 \\ E\left\{s_j s_{j,\tau^*}\right\}=E\left\{s_j(t)s_j(t-\tau^*)\right\}>0 \quad \forall j\neq i \\ E\left\{s_i s_{j,\tau^*}\right\}=E\left\{s_i(t)s_j(t-\tau^*)\right\}=0 \end{cases} \tag{3}$$

Note that if the source signals is periodic with a fundamental period τ_0, then τ^* can be set as $\tau^*=r\tau_0$, where r is a non-zero integer. The basic objective function for most BSE algorithms becomes:

$$J(\mathbf{w})=E\left\{y(t)y(t-\tau^*)\right\}=E\left\{(\mathbf{w}^T\tilde{\boldsymbol{x}}(t))(\mathbf{w}^T\tilde{\boldsymbol{x}}(t-\tau^*))\right\} . \tag{4}$$

When $J(\mathbf{w})$ reaches a maximum, $y(t)$ is the estimation of $s_i(t)$ with maximal autocorrelation, i.e., $E\left\{s_i s_{i,\tau^*}\right\}>E\left\{s_k s_{k,\tau^*}\right\}(\forall k\neq i)$. Otherwise, when $J(\mathbf{w})$ reaches a minimum, $y(t)$ is the estimation of $s_j(t)$ with minimal autocorrelation, i.e.,

$$E\left\{s_j s_{j,\tau^*}\right\}<E\left\{s_k s_{k,\tau^*}\right\}(\forall k\neq j) .$$

3. The Proposed LAJD Algorithm

3.1 The Zhang's Algorithm

Under the constraint $\|\mathbf{w}\|=1$, the Zhang's algorithm [11] is given as:

$$\begin{cases} \mathbf{R}_{\tilde{x}}(\tau_0)=E\left\{\tilde{\boldsymbol{x}}(t)\tilde{\boldsymbol{x}}(t-\tau_0)^T\right\} \\ \mathbf{w}=EIG\left(\sum_{m=1}^{M}(\mathbf{R}_{\tilde{x}}(m\tau_0)+\mathbf{R}_{\tilde{x}}(m\tau_0)^T)\right) \end{cases} \tag{5}$$

where $EIG(\cdot)$ denotes the eigenvalue decomposition operator. The normalized eigenvectors corresponding to the maximal eigenvalue and the minimal eigenvalue are the unmixing vector for the desired source with maximal autocorrelation and minimal autocorrelation respectively.

3.2 The MACBSE Algorithm

The MACBSE algorithm [13] can be written as:

$$\begin{cases} \mathbf{w}\leftarrow E\left\{\tilde{\boldsymbol{x}}(t)\tilde{\boldsymbol{x}}(t-\tau_0)^T+\tilde{\boldsymbol{x}}(t-\tau_0)\tilde{\boldsymbol{x}}(t)^T\right\}\mathbf{w} \\ \mathbf{w}\leftarrow \mathbf{w}/\|\mathbf{w}\| \end{cases} \tag{6}$$

and the initiation of **w** decides whether the algorithm converges to the desired source with maximal autocorrelation or minimal autocorrelation.

3.3 The proposed LAJD Algorithm

Utilizing the source assumptions and the temporal characteristics shown in (3), it is obvious that $E\left\{\boldsymbol{s}\boldsymbol{s}^T\right\}=\mathbf{I}$ and $\mathbf{R}_s(\tau^*)=E\left\{\boldsymbol{s}\boldsymbol{s}_{\tau^*}^T\right\}$ is diagonal. After the prewhitening mentioned in Section 2, we get:

$$\begin{aligned} &\boldsymbol{E}\left\{\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^T\right\}=\mathbf{I} \\ &\Rightarrow \mathbf{V}\mathbf{A}\boldsymbol{E}\left\{\boldsymbol{s}\boldsymbol{s}^T\right\}\mathbf{A}^T\mathbf{V}^T=\mathbf{I} \\ &\Rightarrow (\mathbf{V}\mathbf{A})(\mathbf{V}\mathbf{A})^T=\mathbf{I} \end{aligned} \tag{7}$$

Investigating the whitened convariance matrix $\mathbf{R}_{\tilde{x}}(\tau^*)$ at $\tau^*\neq 0$, we find:

$$\mathbf{R}_{\tilde{x}}(\tau^*)=\mathbf{V}\mathbf{R}_x(\tau^*)\mathbf{V}^T=\mathbf{V}\mathbf{A}\mathbf{R}_s(\tau^*)\mathbf{A}^T\mathbf{V}^T=\mathbf{Q}\mathbf{R}_s(\tau^*)\mathbf{Q}^T . \tag{8}$$

Since $\mathbf{R}_{\tilde{x}}(\tau^*)$ can be calculated approximately from observation samples, the right side of (8) can be realized through an orthogonal diagonalizer. As it is said that, $\tau^*=r\tau_0\,(r\in\mathbb{Z}^+)$ we modify the solution to the orthogonal matrix $\mathbf{Q}$ into the implementation of joint diagonalization [19] at K different time delays, i.e., $\tau_0<2\tau_0<K\tau_0$.

Then the mixing matrix can be estimated as $\hat{\mathbf{A}} = \mathbf{V}^{-1}\mathbf{Q}$ and the sources as $\hat{\boldsymbol{s}}(t) = \mathbf{Q}^T\mathbf{V}\boldsymbol{x}(t)$.

The corresponding autocorrelation functions of three image sources which will appear in later simulations are shown in Fig. 1. One can see that: i) the autocorrelation functions present several peaks and the peaks are similar for each autocorrelation functions; ii) due to the effect of finite samples, the autocorrelation values of source signals at neighboring time delays around peaks are generally non-zero, and some of them are so considerable that they should be not be neglected. Hence, it is reasonable to carry out the following averaging operation:

$$\overline{\mathbf{R}}_{\tilde{x}}(\tau^*) = \frac{1}{L}\sum\nolimits_{l=1}^{L} E\left\{\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}_{\tau_l}^{\ T}\right\}, \left(\tau_l = \tau^* - \frac{L-1}{2},...,\tau^*,...,\tau^* + \frac{L-1}{2}\right). \tag{9}$$

Simulation results validate that (9) brings an additional benefit, that is, it can enhance the insensitivity of LAJD to the estimation of optimal time delays.

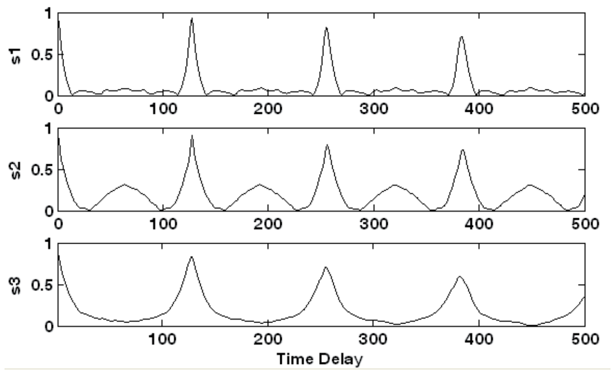

Fig. 1. Autocorrelation functions of the three image sources 1-3 in Fig. 2.

As for how to estimate the time delay τ^* in practical, we can calculate the autocorrelation $E\left\{x_i x_{i,\tau}\right\}$ of observation signals as a function of the time delay τ and detect the one corresponding to the peak within a specified interval of interest [10].

4. Simulation Results

In our simulations three 128×128 images shown in the top of Fig. 2 are used as original sources. Here an optimal time delay $\tau_0 = 128$ is chosen (actually, other optimal time delays are multiples of 128). We have $E\left\{s_1 s_{1,\tau_0}\right\} > E\left\{s_2 s_{2,\tau_0}\right\} > E\left\{s_3 s_{3,\tau_0}\right\}$, in fact, the linear autocorrelations of the source signals are 0.9314 (source 1), 0.9080 (source 2) and 0.8268 (source 3) respectively. That is to say, source 1 has the maximal autocorrelation while source 3 has the minimal autocorrelation. So these two sources can be extracted by BSE algorithms without the deflation scheme. In order to measure the extraction accuracy of some a desired source, we adopt the following signal-to-noise ratio (*snr*) as:

$$snr = 10\log_{10}\left(\frac{E\left\{s_n^{\ 2}\right\}}{E\left\{\left(s_n - \hat{s}_n\right)^2\right\}}\right)(dB), (n = 1,2,...,N). \tag{10}$$

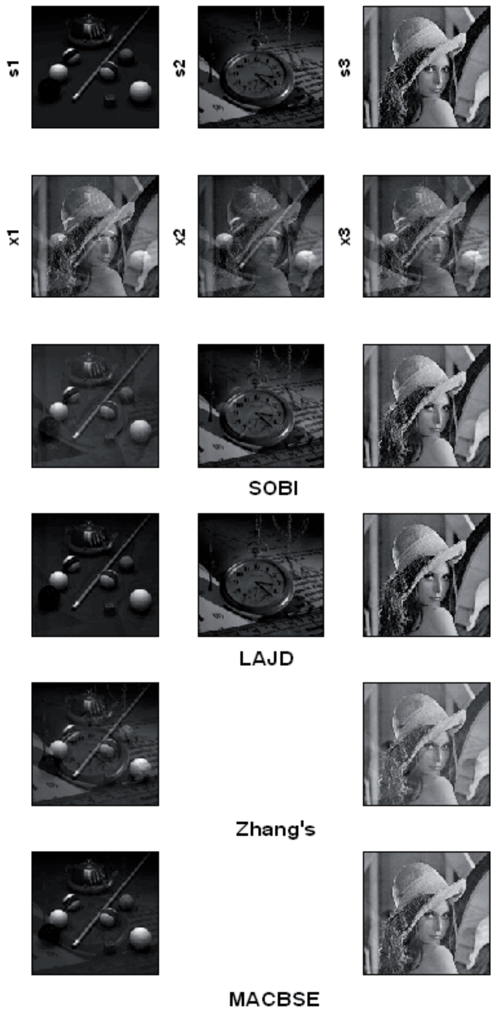

Fig. 2. Simulation results for mixtures of three 128×128 image signals. From top to bottom, the original image sources 1-3, the mixed image mixtures 1-3, the three separated images using the SOBI algorithm, the three separated images using the proposed LAJD algorithm, the extracted two images using the Zhang's algorithm and the extracted two images using the MACBSE algorithm.

Fig. 2 shows the extraction results of Zhang's and MACBSE ($M = 3$), as well as the separation results of SOBI ($K = 40$) and LAJD ($K = 1$, $L = 61$) in a typical run. We can see that the images have been separated successfully and clearly by the proposed LAJD algorithm. It obviously outperforms the other three algorithms, by which the estimated source 1 still mixes with source 2 to a degree. And the source 3 extracted by Zhang's and MACBSE seems to be much obscurer. Tab. 1 also presents the *snr* performance comparison between them, which agrees with the results in Fig. 1. LAJD achieves the best extraction accuracy, its *snr* for source 1 and source 3 can reach 20.2890 dB and 37.9092 dB respectively. And the Zhang's algorithm performs the worst, with *snr* for source 1 and source 3 being 5.2212 dB and 12.6558 dB respectively. The surprising gap between them is as large as 15 dB for source 1 and 25 dB for source 2.

Note that the linear autocorrelations of the former two images are so close that $E\left\{s_1 s_{1,\tau_0}\right\} \approx E\left\{s_2 s_{2,\tau_0}\right\}$. Therefore, it becomes difficult to distinguish source 1 from source 2 for Zhang's and MACBSE, since they extract signals according to the order determined by the autocorrelation values. It also explains why the *snr* for source 1 is lower than that of source 3. One reason for the inferiority of SOBI is that it chooses time delays arbitrarily, not making full use of the inherent temporal structures of sources.

	Zhang's	MACBSE	SOBI	LAJD
snr for source 1(dB)	5.2212	13.6559	10.7205	20.2890
snr for source 3(dB)	12.6558	16.0510	20.9724	37.9092

Tab. 1. The comparison of snr performance for source 1 and source 3 over 200 independent trials.

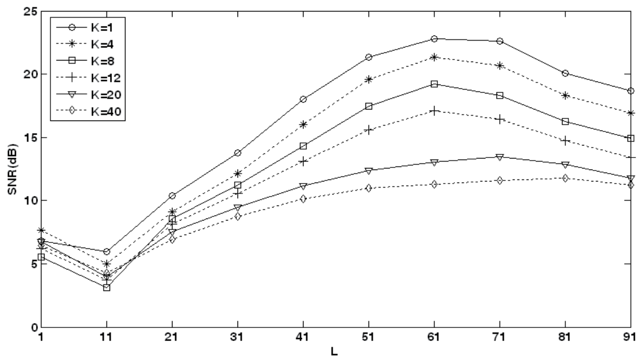

Fig. 3. The effects of the parameters K and L on SNR performance for the proposed LAJD algorithm over 200 independent trials.

Fig. 3 depicts how the selections of the parameters K and L affect the performance of the proposed LAJD algorithm. To make a fair balance, the vertical axis denotes the average separation *SNR* as:

$$SNR = 10\log_{10}\left(\frac{1}{N}\sum_{n=1}^{N}\frac{E\left\{s_n^{\ 2}\right\}}{E\left\{\left(s_n-\hat{s}_n\right)^2\right\}}\right)\text{[dB]}. \quad (11)$$

On one hand, we can see that under different K values, *SNR* is so poor that it stays between 5 dB and 8 dB when $L = 1$. As the L value increases, the *SNR* performance improves, and it reaches the peak approximately when $L = 61$. It tells us that the averaging operation in (9) helps greatly. On the other hand, it is easy to get that under different L values, the *SNR* performance preserves superiority when $K = 1$. This phenomenon implies that LAJD is not limited to the separation of periodic signals. As K value increase, *SNR* degrades gradually. Hence, $K = 1$ and $L = 61$ appears to be an appropriate option in this case.

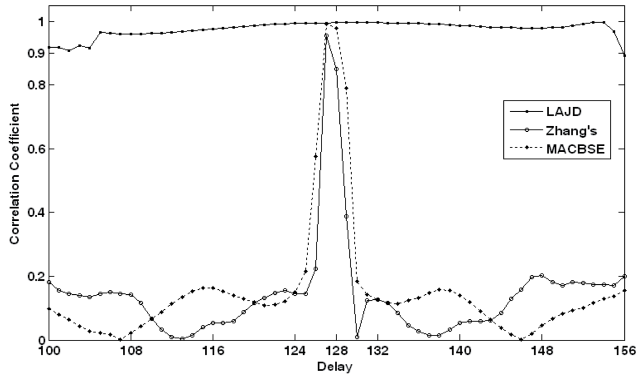

Fig. 4. Correlation coefficients between source 1 and its corresponding estimation at different time delay over 200 independent trials.

In practice, the estimation error of optimal time delay cannot be avoided. To evaluate the algorithm robustness, we compute the correlation coefficients at different time delay τ between source 1 and its corresponding estimation:

$$\rho = \frac{E\left\{s_1(t)\hat{s}_1(t)\right\}}{\sqrt{E\left\{s_1^{\ 2}(t)\right\}}\cdot\sqrt{E\left\{\hat{s}_1^{\ 2}(t)\right\}}} \quad (12)$$

where the correlation coefficient higher than 0.9 can be regarded as a good extraction level. The results are plotted in Fig. 4. Zhang' and MACBSE are very sensitive to the estimation error of the time delay. Their satisfactory extraction happens only at $\tau = 128$ and $\tau = 128, 129$, respectively. Small estimation error could lead to poor performance. However, our LAJD algorithm works well even if the range of estimation error is quite large, i.e., $\tau = 100, 101, \ldots, 156$.

5. Conclusions

Based on linear autocorrelations of the source signals, we propose the LAJD separation algorithm by a joint diagonalizer of several average delayed covariance matrices at optimal time delays. The proposed algorithm is fast since it utilizes the second-order statistics only. It is also robust, because it is much more insensitive to the estimation error of the time delay than the most well-known BSE algorithms. Besides, its performance is so superior that its separation signal-to-noise ratio for a desired source can reach 20 dB higher. The theoretical analysis of the selection of parameters K and L should be studied in future research.

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