Resource Allocation for Downlink Multi-Cell OFDMA Cognitive Radio Network Using Hungarian Method

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Abstract. *This paper considers the problem of resource allocation for downlink part of an OFDM-based multi-cell cognitive radio network which consists of multiple secondary transmitters and receivers communicating simultaneously in the presence of multiple primary users. We present a new framework to maximize the total data throughput of secondary users by means of subchannel assignment, while ensuring interference leakage to PUs is below a threshold. In this framework, we first formulate the resource allocation problem as a nonlinear and non-convex optimization problem. Then we represent the problem as a maximum weighted matching in a bipartite graph and propose an iterative algorithm based on Hungarian method to solve it. The present contribution develops an efficient subchannel allocation algorithm that assigns subchannels to the secondary users without the perfect knowledge of fading channel gain between cognitive radio transmitter and primary receivers. The performance of the proposed subcarrier allocation algorithm is compared with a blind subchannel allocation as well as another scheme with the perfect knowledge of channel-state information. Simulation results reveal that a significant performance advantage can still be realized, even if the optimization at the secondary network is based on imperfect network information.*

Keywords

Cognitive radio, OFDMA, radio resource allocation, bipartite graph, Hungarian method.

1. Introduction

With the fast development of wireless communications technology the scarcity of spectrum has become a serious issue, however, a large portion of the assigned spectrum in different networks is still used sporadically and this yields to under-utilization of a significant amount of spectrum [1]. This has inspired the concept of cognitive radio (CR) technology which is originally proposed by Mitola in [2]. CR can improve spectrum utilization by finding the spectrum holes which are unused by licensed primary users (PUs) and allocating those unutilized spectrum bands to unlicensed secondary users (SUs) who require more spectrum resources. Spectrum underlay [3] is one of the existing spectrum sharing techniques that allow the SUs to transmit simultaneously with the PUs in the same frequency band. Therefore, a CR network should be designed in a way that the qualities of service (QoS) requirements of the SUs are met, while ensuring that the interference power constraint of primary network (PRN) is satisfied.

Multi-carrier systems can further help in optimizing the usage of spectrum as they allow placing the carriers in a noncontiguous way. An overview and a comparison of the various multicarrier modulation schemes with respect to the potential use in CR networks can be found in [4]. The features and the ability of the Orthogonal Frequency Division Multiple Access (OFDMA) scheme makes it fit for the CR based transmission system. The main idea of OFDMA technology is that the available bandwidth will be divided into many orthogonal subchannels and allows multiple users to transmit simultaneously on the different subchannels. Combining cognitive radio and OFDMA is one of the best candidates for the future mobile networks due to its flexibility in allocating resource among CR users. A good survey on radio resource allocation in OFDMA-based systems can be found in [5] and the references therein. In some literature [6]-[10], different versions of Hungarian method [23] are proposed for solving resource allocation problem in an OFDM-based network. Most of these approaches have used Hungarian algorithm as an optimal solution which is considered as a reference upper bound of the system performance and is not suggested for use in practice. Also, these approaches can't be easily extended to CR network because the interference to the primary network (PRN) needs to be taken into account.

Even though the basic idea of CR is simple, the efficient design of CR systems imposes the new challenges compared to conventional wireless systems. Compared with the resource allocation methods in traditional OFDM based systems, the mutual interference between PUs and SUs has to be considered in the OFDM-based CR systems. The analysis of secondary system interference for cognitive radio in slow fading environment has been studied in [11]. Also a good summary of the state of the art in CR networks is provided by Zhang et al. in [12]. Several studies have been done on resource allocation for OFDM-based CR networks but most of them assume a special scenario for coexistence of primary and secondary system. For example, some resource allocation techniques for OFDM-based CR networks have been reported in [13]-[15] while assuming that there is only one primary base station and one secondary base station in the network. Also some studies which have applied Hungarian algorithm for resource allocation in CR networks, consider a simple system model. For example the authors in [16] proposed a subcarrier allocation algorithm based on Hungarian algorithm for multiuser OFDM-based CR systems. However, they only considered one primary user. The work in [17] proposed the Hungarian algorithm for maximizing the signal to interference-and noise ratio (SINR) with the fairness consideration and mainly focused on the uplink subchannel assignment issue. The problem of joint resource allocation and modulation and coding scheme (MCS) selection for a group of users with unicast video transmission in OFDMA networks have been formulated in [18]. To solve the formulated problem, the authors proposed a two-stage algorithm, where the Hungarian algorithm is used in the first stage to optimally assign the minimum resource units required for individual users based on their channel conditions.

Different from all the aforementioned works, we consider a multi-cell downlink underlay CR network (CRN) where multiple cognitive links coexist with multiple primary links through OFDMA-based air-interface in each cell. The study on the spectrum underlay sharing for such multiple-SU multiple-PU scenario in multi-cell environment has received little attention in the literatures. Several technical difficulties appear for resource allocation in a multi-cell environment. The CRN/PRN coexistence involves inter-cell as well as intra-cell interferences. Therefore the CR network should control the interference to the PUs as well as the inter-cell interference within the CR network. In this paper, we address the problem of assigning subchannels in downlink for OFDMA based cognitive radio networks. The objective is to maximize the system throughput in such a way that the interference leakage to the PUs is below a specific threshold. In addition to the guarantee of tolerable interference limits induced by the CR network to the PUs, our proposed algorithm maximizes the sum rate of SUs in each cell by inter-cell interference management within the CR network and exploiting the multi-user diversity gains. Also in all the above literatures, a perfect spectrum-sensing functionality is assumed. However, due to fast fading component, it is impractical to track instantaneous the interference channel gain between CR transmitters and primary receivers. In this contribution, we assume that the CR system can access information of primary radio environments such as the location via radio environment map (REM) [19]. Thus in the proposed subchannel allocation method, we use an averaged interference channel gain such as path-loss with the aid of REM. Also we will propose a different way to efficiently protect PUs from SU interference based on a graphical framework for the downlink communication and the proposed approach can offer a global optimum to rate-sum maximization. In the proposed approach, first, we define two interference weight matrixes among each pair of SUs and between each pair of SU-PU, respectively. Then to exploit the multi-user diversity gains, we define another matrix based on SNR. Finally based on these matrixes, we represent the subchannel allocation problem as a maximum weighted bipartite matching problem and solve it by Hungarian algorithm.

The rest of the paper is organized as follows: Section 2 outlines the system model, followed by the formulation of the resource allocation problem. Then, the proposed resource allocation algorithm is presented in Section 3 and its performance is evaluated in Section 4. Finally, the paper is concluded in Section 5.

2. System Model and Problem Formulation

2.1 An OFDMA-based CR System

We consider a multi cell OFDM-based cognitive radio system as illustrated in Fig. 1. In each cell there are multiple PUs and SUs communicating with a primary base station (PBS) and a secondary base station (SBS). The directions of transmission in both PRN and CRN are assumed downward. The overall frequency band is subdivided into N orthogonal subchannels. In this context, we denote the PU, SU and subchannel sets as $m= \{1, 2, ... , M\}$, $k = \{1, 2, ... , K\}$ and $n = \{1, 2, … , N\}$, respectively. Furthermore, we assume that there are C cells in the network and the noise power density spectrum (PSD) on each subchannel is N_0. Also we use index i to refer the SBS in each cell ($i \in \{1, 2, … , C\}$).

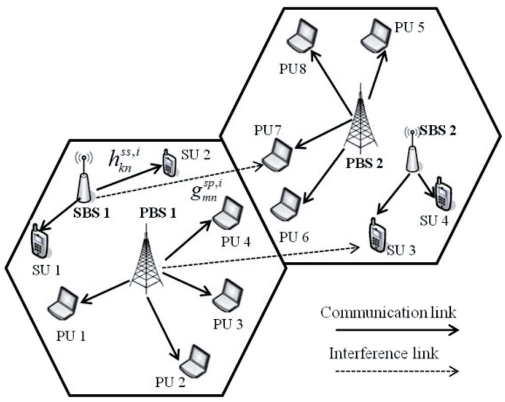


Fig. 1. System model.

We assume spectrum underlay approach in which the SUs can utilize the whole spectrum being used by the PUs as long as they do not exceed the interference threshold at the PUs. Therefore, the whole frequency band used by PUs is also available to SUs. We define the channel gain matrix $H^{SS,i}$, where $H^{SS,i} = \{h^{SS,i}{}_{kn}\}$ and $h^{SS,i}{}_{kn}$ denotes the channel gain of the communication link from the i^{th} SBS to the k^{th} SU on the n^{th} subchannel. We also define the channel gain matrix $G^{SP,i}$, where $G^{SP,i} = \{g^{SP,i}{}_{mn}\}$ and $g^{SP,i}{}_{mn}$ represents the channel gain of the interference link from the i^{th} SBS to the m^{th} PU on the n^{th} subchannel for $i = 1, 2, \ldots, C$. Also for the sake of simplicity, the maximum transmit power budget of each SBS (P_{max}) is evenly divided among it's SU in each cell, i.e.

$$P_S = \frac{P_{\max}}{|k_c|} \qquad \forall k \in k_c,\ c \in \{1, 2, \ldots, C\}\,. \tag{1}$$

In (1), k_c denotes the set of SUs which are located in cell c. Also we assume that a subchannel cannot be shared by more than one SU in each cell and all subchannels which are used by PUs in each cell are unavailable for those SUs that are located in that cell. We define $a_{k,n}$ as a subchannel allocation indicator which can only be either 1 or 0, indicating whether the n^{th} subchannel is allocated to the k^{th} SU or not. Also, throughout the paper, we will use the following notation. Ω_n: the set of SBSs in CRN that communicate with their SUs on subchannel n, Λ_n: SUs set that receive signal from their serving SBSs on subchannel n, and finally Ψ_n: the set of PUs that receive signal from their serving PBS on subchannel n. From a practical point of view, we assume that the tolerable interference limit of PRN can be broadcasted, before the start of the communication, by the PBS. Due to the coexistence of PUs and SUs, there are two types of interference in this system: first, the total interference introduced by CRN to the m^{th} PU that is denoted by I_m^{total} and also second, the total interference introduced by PRN to the k^{th} SU that is denoted by J_k^{total}. Suppose that the m^{th} PU receives its signal from corresponding BS on subchannel n and at same time, a set of SBSs in CRN communicate with their SUs on subchannel n (as mentioned before, this set is denoted by Ω_n). If we denote the power spectral density of the n^{th} subchannel's signal by $\phi_n(f)$, then the interference imposed by the i^{th} SBS ($i \in \Omega_n$) on the m^{th} PU in subchannel n can be written as:

$$I^i(m,n) = \int_{f \in F_m} \left| g_{mn}^{SP,i} \right|^2 \phi_n(f)\, df \quad , i \in \Omega_n \tag{2}$$

where F_m denotes the frequency band licensed to the m^{th} PU. The total interference at the m^{th} PU should be smaller than the tolerable interference limit. Thus, the interference constraint for m^{th} PU can be written as:

$$I_m^{total} = \sum_{i \in \Omega_n} I^i(m,n) \le I_{th} \ . \tag{3}$$

2.2 Problem Formulation

In an OFDMA-based CRN, different subchannels may experience different channel gains. The received signal-to interference-plus-noise ratio (SINR) for the k^{th} SU on subchannel n can be written as:

$$\gamma_{k,n} = \frac{P_S\, h_{kn}^{SS,i}}{\sum_{\substack{\ell \in \Omega_n \\ \ell \ne i}} P_S\, h_{kn}^{SS,\ell} + J_k^{total} + N_0} \tag{4}$$

where the index i refers to serving SBS of the k^{th} SU. The achievable throughput for the k^{th} SU is given by Shannon capacity formula as follows:

$$R_k = \sum_{n=1}^{N} a_{k,n} \log_2(1 + \gamma_{k,n}) \qquad \forall k\,. \tag{5}$$

Based on the above explained system model, the goal of this paper is to devise a subchannel assignment policy that maximizes the expected sum throughput of all of SUs while satisfying constraints on the tolerable interference level of each individual primary user. Specifically, the design problem of interest can be formulated as follows:

$$\max_{\{a\}} \sum_{k=1}^{K} \sum_{n=1}^{N} a_{k,n} \log 2(1 + \gamma_{kn})\,, \tag{6}$$

$$s.t. \quad P_S \ge 0 \quad , \tag{6a}$$

$$I_m^{total} = \sum_{i \in \Omega_n} I^i(m,n) \le I_{th} \quad \forall m, n\,, \tag{6b}$$

$$a_{k,n} \in \{0,1\} \quad \forall k, n\,, \tag{6c}$$

$$\sum_{k=1}^{K} a_{k,n} \le 1 \quad \forall n\,. \tag{6d}$$

The objective function in (6) represents the total throughput of the secondary system. The constraint in (6b) ensures that the interference leakage to each PU is always below a given threshold, I_{th}. Constraints (6c) and (6d) enforce a disjoint subchannel assignment in secondary OFDMA system, that is, one subchannel is permitted to be assigned to at most one SU in each cell. The optimization problem represented in (6) is NP-hard and as noted in [20], the computational complexity needed to directly resolve this type of combinatorial problems, at least, increases exponentially with the number of subchannels N. Thus, obtaining a globally optimal solution for it is not straight forward. In the following section, we will propose a suboptimal technique to solve this problem.

3. Subchannel Allocation Algorithm

The main goal of the proposed subchannel allocation algorithm in this section is the management and mitigation of interference introduced by CRN to PUs and SUs. In this algorithm we propose a method for determining a specific weight between each pair of SU_k, $k \in \{1, 2, \ldots, K\}$ in CRN and subchannel n($n \in \{1, 2, \ldots, N\}$). This weight is determined based on two metrics: first the interference level that imposed by SU_k on PUs and other SUs if subchannel n is allocated to SU_k , and second the channel quality between SU_k and its serving BS on subchannel n. Then based on these weights, we represent the problem of subchannel allocation in CRN as a weighted bipartite graph and solve it by Hungarian algorithm. Our solution to the subchannel allocation problem has three steps that are described as follows:

3.1 Step 1) Definition of the Interference Weight Matrixes

We define two matrixes W_{SS} and W_{SP} to represent the level of interference of secondary to secondary and secondary to primary users, respectively.

$$W_{SS} = \begin{bmatrix} 0 & w_{ss}^{12} & w_{ss}^{13} & \dots & w_{ss}^{1K} \\ w_{ss}^{21} & 0 & w_{ss}^{23} & \dots & w_{ss}^{2K} \\ w_{ss}^{31} & w_{ss}^{32} & 0 & \dots & w_{ss}^{3K} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{ss}^{K1} & w_{ss}^{K2} & w_{ss}^{K3} & \dots & 0 \end{bmatrix}_{K\times K}, \quad (7)$$

$$W_{SP} = \begin{bmatrix} w_{sp}^{11} & w_{sp}^{12} & w_{sp}^{13} & \dots & w_{sp}^{1K} \\ w_{sp}^{21} & w_{sp}^{22} & w_{sp}^{23} & \dots & w_{sp}^{2K} \\ w_{sp}^{31} & w_{sp}^{32} & w_{sp}^{33} & \dots & w_{sp}^{3K} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{sp}^{M1} & w_{sp}^{M2} & w_{sp}^{M3} & \dots & w_{sp}^{MK} \end{bmatrix}_{M\times K} \quad (8)$$

where $w_{ss}^{kk'}$ represents the interference level between SU_k and SU k', $k,k' \in \{1, 2, \dots, K\}$ and w_{sp}^{mk} represents the interference level between PU_m , $m \in \{1, 2, \dots, M\}$ and SU_k. In order to make these two matrixes, two sets of A_k and B_k are defined for SUs, where A_k is the serving *SBS* set of SU_k and B_k is the neighbor *SBS* set of that user. The number of *SBSs* in set B_k depends on the location of SU_k with respect to its neighboring *SBS*s as well as a path-loss threshold. This means that if path-loss component in channel gain between SU_k and a SBS in CRN is lower than path-loss threshold then put this SBS as neighbor BS in B_k. Also two sets of A_m and B_m are defined for each PU, where A_m is the serving *PBS* set of PU_m and B_m is the neighbor *SBS* set of that user. Again, for each PU in PRN, the number of *SBSs* in set B_m is determined based on its location and path-loss threshold. The pseudo code of the proposed procedure to determine these interference matrixes is depicted in Tab. 1 and Tab. 2. In these algorithms, the interference weights are assigned to feasible interference links based on path-loss attenuation factor in a way that the higher value of path-loss component in channel gain between one user and an interferer base station, leads to assigning a smaller weight to this interference link. This concept is done by a descending mapping function such that the smaller values of path-loss are mapped to bigger interference weights. Also, if two SUs have the same serving SBS or a PU and SU are within the same cell, then they will have intra-cell interference with each other. Therefore, the intra-cell interference weight W_A should be assigned with a very large value because the intra-cell interference has to be avoided. Similarly, a no-interference weight W_N is set to a small value to allow the users to use a specific channel, simultaneously.

```
W_SS = zeros(K×K)
For k = 1: K-1
  For k′ = k+1: K
    1) If A_k ∩ A_k′ ≠ ∅ ⟶ inter-cell interference
          w_ss^{kk′} = w_ss^{k′k} = W_A , Go to 5.
    2) If A_k ∩ B_k′ ≠ ∅
          X= {path-loss attenuation of link between serving SBS
              of SU_k and SU_k′}
          weight_1 = mapping_ descending (X)
       else
          weight_1= W_N
       end
    3) If A_k′ ∩ B_k ≠ ∅
          X= {path-loss attenuation of link between serving SBS
              of SU_k′ and SU_k}
          weight_2 = mapping_ descending (X)
       else
          weight_2= W_N
       end
    4) w_ss^{kk′} = w_ss^{k′k} = max(weight_1, weight_2)
    5) Output w_ss^{kk′}
  end
end
```

Tab. 1. Proposed algorithm for construction of W_{SS} matrix.

```
W_SP = zeros(M×K)
For m = 1: M
  For k =1: K
    1) If A_m ∩ A_k ≠ ∅ ⟶ inter-cell interference
          w_sp^{mk} = W_A , Go to 3.
    2) If B_m ∩ A_k ≠ ∅
          X= {path-loss attenuation of link between serving SBS
              of SU_k and PU_m}
          w_sp^{mk} = mapping_ descending (X)
       else
          w_sp^{mk} = W_N
       end
    3) Output w_sp^{mk}
  end
end
```

Tab. 2. Proposed algorithm for construction of W_{SP} matrix.

Now we define the final interference weight matrix $W_{final} = \{w_f^{kn}\}$ as a $K \times N$ matrix where w_f^{kn} represents the interference imposed by SU_k to PRN and other SUs in CRN, if subchannel n is assigned to SU_k. Since Ψ_n denotes the set of PUs that receive signal from their serving PBSs on subchannel n, we can partition all PUs in PRN in to N groups of $\Psi_1, \Psi_2, \ldots, \Psi_n$.

Since the assignment of subchannel n to a given SU_k, $k \in \{1, 2, \ldots, K\}$ leads to impose interference on all the PUs who belong to Ψ_n, the elements of W_{final} matrix can deduce as follows:

$$w_f^{kn} = \sum_{m \in \Psi_n} w_{sp}^{mk} . \tag{9}$$

3.2 Step 2) Definition of Throughput Weight Matrix

Based on the channel gain of the communication link from a SBS to its SUs in each cell, we can define another $K \times N$ matrix $W_{SNR} = \{w_{snr}^{kn}\}$ where w_{snr}^{kn} represents the received signal to noise ratio (SNR) for the k^{th} SU on subchannel n that can be calculated as:

$$w_{snr}^{kn} = \text{ascending\_mapping } \left(\frac{P_S\, h_{kn}^{SS,i}}{N_0}\right) \tag{10}$$

where the index i refers to serving SBS of SU_k. On the other hand, the k^{th} row in this matrix represents the received SNR for SU_k on different subchannels. In this step, by means of an ascending function, we assign the throughput weight to each pair of SU_k and subchannel n, based on the received SNR of SU_k on subchannel n such that the higher value of SNR, leads to assigning a bigger weight to this pair.

3.3 Step 3) Subchannel Allocation Using Hungarian Method

In this step, first we represent the subchannel allocation problem as a weighted bipartite graph [21]-[22] and solve it by finding maximum weighted bipartite matching. A bipartite graph $G = (U, V; E)$ is a graph whose vertices can be divided in two disjoint sets U and V such that each edge $(u_i, v_j) \in E$ connects two vertexes of $u_i \in U$ and $v_j \in V$. If each edge in graph G has an associated weight w_{ij}, the graph G is called a weighted bipartite graph. A matching M of graph G is a subset of E such that no two edges in M share a common vertex. If the graph G is a weighted bipartite graph, the maximum/minimum weighted bipartite matching is a matching whose sum of the weights of the edges is minimum/maximum. In 1955, Kuhn proposed an algorithm (called Hungarian) that finds maximum/minimum matching in a weighted bipartite graph within a time bounded by the order of the number of vertices and edges [23]. For subchannel allocation using Hungarian method, a weighted bipartite graph $G = (U, V; E)$ is assumed such that U vertices are used to denote N groups of $\Psi_1, \Psi_2, \ldots, \Psi_N$ and V vertices are used to denote all of SUs in CRN. Therefore, the cardinality of U vertices and V vertices are $|N|$ and $|K|$, respectively. If SU_k is located in cell c, the edge between two vertexes of $u_n \in U$ and $v_k \in V$ is associated with the weight w^{kn} which is determined as:

$$w^{kn} = \begin{cases} \dfrac{w_{snr}^{kn}}{w_f^{kn}} & if\ n \notin n_c \\ 0 & if\ n \in n_c \end{cases} \tag{11}$$

where n_c represents the set of subchannels that are used by PUs in cell c. Then the $K \times N$ weight matrix W is defined as $W = \{w^{kn}\}$. Note that this scheme sets all subchannels that are used by PUs in each cell, unavailable for SUs in that cell. Now, subchannel allocation for CRN is performed by applying Hungarian algorithm on this weighted bipartite graph and finding maximum matching, iteratively. Each iteration consists of the following steps:

a. A matching of N subchannels in U vertices to N SUs in V vertices is found by Hungarian algorithm whose sum of the weights of the edges is maximum.

b. For each assignment of subchannel n to SU_k in this matching, the interference that imposed by this assignment on PRN is estimated as:

$$I(m)_{[dB]} = P_S - \varphi_{i,m} \quad \forall m \in \Psi_n \tag{12}$$

where $\varphi_{i,m}$ represents the path-loss component in channel gain between serving SBS of SU_k (which is referred by i) and PU_m. If the constraint on the tolerable interference level of each individual PU_m, ($m \in \Psi_1$), is satisfied then set $a_{k,n} = 1$ and update the tolerable interference level of these PUs as follows:

$$I_{th} = I_{th} - I(m) \quad \forall m \in \Psi_n . \tag{13}$$

c. After each subchannel allocation, first update Λ_n:

$$\{\forall n, k\, , a_{kn} = 1 : \Lambda_n = \Lambda_n \cup k\} , \tag{14a}$$

then update each element of W as follows:

$$w^{k1} = w^{k2} = \cdots = w^{kN} = 0 \quad if\ \alpha_{kn} = 1, \tag{14b}$$

$$w^{k''n} = 0 \ ;\ k'' \in \ell_k \quad if\ \alpha_{kn} = 1 \tag{14c}$$

where ℓ_k is the set of SUs that are located in the same cell with SU_k.

d. Now each element of W that $w^{kn} \neq 0$ updates as follows:

$$w_f^{kn} = \sum_{m \in \Psi_n} w_{sp}^{mk} + \sum_{k' \in \Lambda_n} w_{ss}^{kk'} \Rightarrow w^{kn} = \frac{w_{snr}^{kn}}{w_f^{kn}} . \tag{14d}$$

By (14b) we ensure the SU which is assigned a subchannel in previous iteration, isn't selected by Hungarian algorithm again in the next iteration. Also (14c) guarantees that the intra-cell interference is avoided. When subchannel n is allocated to SU_k, then this subchannel is unavailable for all SUs which are located in the same cell with SU_k. Finally, in (14d), w_f^{kn} consists of two parts that indicate the interference level imposed on PRN and CRN, respectively, if subchannel n is allocated to SU_k.

To illustrate the procedure of the proposed algorithm, a simple example is described in the following. It is assumed that the network consists of $C = 4$ cells, there are two SUs and three PUs in each cell. Also the number of available subchannels is set to $N = 6$ for simplicity reasons. Subchannel allocation of PUs and the available subchannel for SUs in each cell have been described in Tab. 3. First, based on channel allocation of PRN, all PUs are partitioned to N sets, $\{\Psi_1, \Psi_2, \ldots, \Psi_6\}$. Then, we define the final interference weight matrix based on W_{SS} and W_{SP}. For instance, the elements of first row in final interference weight matrix in this network are:

$$\begin{cases} w_f^{11} = \sum_{m\in\Psi_1} w_{sp}^{m1} = w_{sp}^{4,1} + w_{sp}^{10,1} \\ w_f^{12} = \sum_{m\in\Psi_2} w_{sp}^{m1} = w_{sp}^{3,1} + w_{sp}^{5,1} \\ w_f^{13} = \sum_{m\in\Psi_3} w_{sp}^{m1} = w_{sp}^{6,1} + w_{sp}^{7,1} + w_{sp}^{12,1} \\ w_f^{14} = \sum_{m\in\Psi_4} w_{sp}^{m1} = w_{sp}^{8,1} + w_{sp}^{11,1} \\ w_f^{15} = \sum_{m\in\Psi_5} w_{sp}^{m1} = w_{sp}^{1,1} + w_{sp}^{9,1} \\ w_f^{16} = \sum_{m\in\Psi_6} w_{sp}^{m1} = w_{sp}^{2,1} \end{cases} \tag{15}$$

Number of cell	Number of PUs	Subchannel allocation of PRN	Number of SUs	Subchannel available for CRN
C = 1	1 , 2 , 3	5 , 6 , 2	1 , 2	1 , 3 , 4
C = 2	4 , 5 , 6	1 , 2 , 3	3 , 4	4 , 5 ,6
C = 3	7 , 8 , 9	3 , 4 , 5	5 , 6	1 , 2 , 6
C = 4	10 , 11 , 12	1 , 4 , 3	7 , 8	2 , 5 , 6

Tab. 3. Information of subchannel allocation in PRN and subchannel available in CRN.

Then, according to W_{final} and W_{SNR}, we determine the weight matrix for edges in bipartite graph as (16). For each pair of SU_k and subchannel n, when subchannel n is not available, we set $w^{kn} = 0$. The weighted bipartite graph for this example is shown in Fig. 2. Now maximum weighted bipartite matching can be found by Hungarian algorithm. Note that according to definition of matching, Hungarian algorithm selects only the best six secondary users (lower interference weight and upper throughput) among all the secondary users for assigning to subchannels. Therefore, for subchannel allocation of all SUs in CRN, Hungarian algorithm should be applied iteratively. Suppose that by applying Hungarian algorithm in the first iteration, six assignments have been found from which only two assignments are acceptable by considering the constraint on the tolerable interference level of PUs. For example, the subchannel $n = 4$ and $n = 1$ are allocated to SU_2 and SU_5, respectively.

$$W = \begin{bmatrix} \frac{w_{snr}^{11}}{w_f^{11}} & 0 & \frac{w_{snr}^{13}}{w_f^{13}} & \frac{w_{snr}^{14}}{w_f^{14}} & 0 & 0 \\ \frac{w_{snr}^{21}}{w_f^{21}} & 0 & \frac{w_{snr}^{23}}{w_f^{23}} & \frac{w_{snr}^{24}}{w_f^{24}} & 0 & 0 \\ 0 & 0 & 0 & \frac{w_{snr}^{34}}{w_f^{34}} & \frac{w_{snr}^{35}}{w_f^{35}} & \frac{w_{snr}^{36}}{w_f^{36}} \\ 0 & 0 & 0 & \frac{w_{snr}^{44}}{w_f^{44}} & \frac{w_{snr}^{45}}{w_f^{45}} & \frac{w_{snr}^{46}}{w_f^{46}} \\ \frac{w_{snr}^{51}}{w_f^{51}} & \frac{w_{snr}^{52}}{w_f^{52}} & 0 & 0 & 0 & \frac{w_{snr}^{56}}{w_f^{56}} \\ \frac{w_{snr}^{61}}{w_f^{61}} & \frac{w_{snr}^{62}}{w_f^{62}} & 0 & 0 & 0 & \frac{w_{snr}^{66}}{w_f^{66}} \\ 0 & \frac{w_{snr}^{72}}{w_f^{72}} & 0 & 0 & \frac{w_{snr}^{75}}{w_f^{75}} & \frac{w_{snr}^{76}}{w_f^{76}} \\ 0 & \frac{w_{snr}^{82}}{w_f^{82}} & 0 & 0 & \frac{w_{snr}^{85}}{w_f^{85}} & \frac{w_{snr}^{86}}{w_f^{86}} \end{bmatrix}_{8\times 6} \tag{16}$$

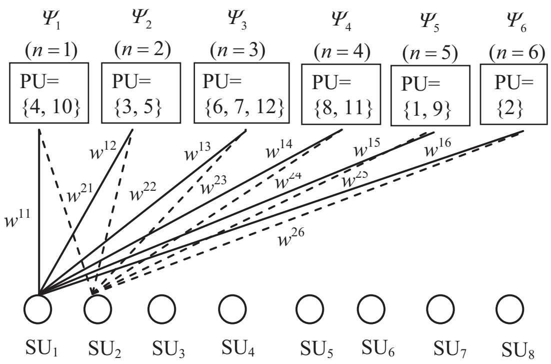


Fig. 2. The weighted bipartite graph of example: for simplicity, only the edges of SU_1 & SU_2 are plotted.

After subchannel allocation at first step, the final interference weight matrix (W_{final}) is updated. For instance, the elements of first row in final interference weight matrix are:

$$\begin{cases} w_f^{11} = (w_{sp}^{4,1} + w_{sp}^{10,1}) + w_{ss}^{51} \\ w_f^{12} = (w_{sp}^{3,1} + w_{sp}^{5,1}) \\ w_f^{13} = (w_{sp}^{6,1} + w_{sp}^{7,1} + w_{sp}^{12,1}) \\ w_f^{14} = (w_{sp}^{8,1} + w_{sp}^{11,1}) + w_{ss}^{21} \\ w_f^{15} = (w_{sp}^{1,1} + w_{sp}^{9,1}) \\ w_f^{16} = (w_{sp}^{2,1}) \end{cases} \tag{17}$$

Also the weight matrix for edges in bipartite graph is updated as follows:

$$W=\begin{bmatrix} \frac{w_{snr}^{11}}{w_f^{11}} & 0 & \frac{w_{snr}^{13}}{w_f^{13}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{w_{snr}^{34}}{w_f^{34}} & \frac{w_{snr}^{35}}{w_f^{35}} & \frac{w_{snr}^{36}}{w_f^{36}} \\ 0 & 0 & 0 & \frac{w_{snr}^{44}}{w_f^{44}} & \frac{w_{snr}^{45}}{w_f^{45}} & \frac{w_{snr}^{46}}{w_f^{46}} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{w_{snr}^{62}}{w_f^{62}} & 0 & 0 & 0 & \frac{w_{snr}^{66}}{w_f^{66}} \\ 0 & \frac{w_{snr}^{72}}{w_f^{72}} & 0 & 0 & \frac{w_{snr}^{75}}{w_f^{75}} & \frac{w_{snr}^{76}}{w_f^{76}} \\ 0 & \frac{w_{snr}^{82}}{w_f^{82}} & 0 & 0 & \frac{w_{snr}^{85}}{w_f^{85}} & \frac{w_{snr}^{86}}{w_f^{86}} \end{bmatrix}_{8\times 6} \quad (18)$$

In (18), in order to ensure that SU_2 and SU_5 will not be selected again by Hungarian method in the next iterations, the rows according to SU_2 and SU_5 are set to zero (denoted by rectangle with solid line). Also, to avoid the intra-cell interference, subchannel $n = 1$ and $n = 4$ become unavailable for SU_6 and SU_1, respectively (denoted by rectangle with dashed line). The weighted bipartite graph in the next iteration is shown in Fig. 3.

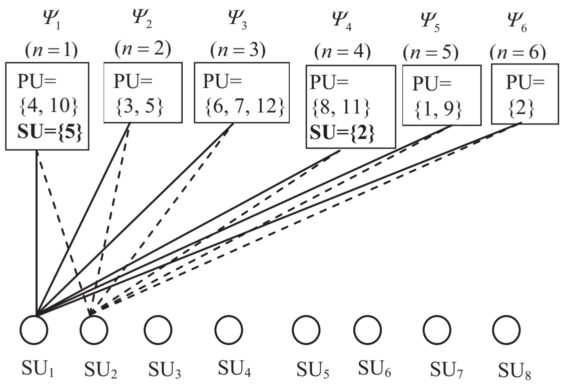


Fig. 3. The weighted bipartite graph of example in second iteration.

3.4 Complexity Analysis

The computational complexity of the proposed algorithm is the sum of complexities of the aforementioned three steps. The complexity of our proposed algorithm is dominated by the complexity of Hungarian algorithm which is used in Step 3. In the worst case, K iterations of the Hungarian algorithm are required for subchannel allocation among all SUs. In each iteration, the complexity of applying Hungarian algorithm on the $K\times N$ weight matrix W is $O(\max\{K,N\}^3)$. As mentioned in Section 2.1 page 2, K and N denote the total numbers of SUs and subchannels, respectively. Usually, the number of users is larger than the number of shared subchannels, therefore the worst-case computational complexity of Hungarian algorithm in each iteration is given by $O(K^3)$ and finally the worst case complexity of the solving subchannel allocation is $O(K^4)$ which is significantly lower than the computational complexity which is needed to directly resolve this type of combinatorial problems. The optimal solution to the subchannel allocation in an OFDMA network requires an exhaustive search to find the optimal subchannel assignment to the K SUs, and the complexity of this exhaustive search exponentially grows as $O(K^N)$. Therefore, these exhaustive-search resolutions are computationally intractable for practical OFDM-based systems when a very large number of subchannels are utilized.

4. Simulation Results

In this section we demonstrate the performance results of the proposed resource allocation algorithm. Simulations have been carried out for an OFDMA-based cellular CR system. The basic OFDMA setups are summarized in Tab. 4. We consider a network with multiple cells in 9 km × 9 km area. Here we assume 19 cells, with the number of PUs and SUs in each cell being 14 and 20, respectively. The PUs and SUs are randomly placed on a hexagonal cell of radius $R = 1$ km where the PBS is located at the center of the cell and the SBS is located randomly at a circle of radius $R\_SBS = 0.5$ km. Fig. 4 shows the cell topology used in the simulation. All base stations of primary and secondary network have 40 dBm transmission power budget and the noise power on each subchannel is set to -119 dBm. For simplicity, we assume all the PUs have the same tolerable interference limit. The intra-cell interference weights W_A and W_N, are chosen as $W_A = 10^5$ and $W_N = 50$ in order to guarantee that the intra-cell interference is avoided. For primary subchannel allocation, we use the simplest FRF (frequency reuse factor) of 3 such that once a subchannel is allocated to a cell, the nearby cells are not allowed to use that subchannel. Also, no two PUs occupy the same subchannel in each cell. The channel gain between the base station and the receiver (primary or secondary) is modeled as a combination of path-loss and fading. Without loss of generality, shadowing is excluded in this paper. The path-loss in dB is given by $\varphi = 130.62 + 37.6 \times \log_{10}(d)$, where d is the transmitter-receiver distance in kilometers.

System Bandwidth	10 MHz
FFT Size	1024
Sampling Frequency	11.2 MHz
Carrier Frequency	2.5 GHz
Number of Subchannels (N)	42
Number of subcarriers Per Subchannel	12

Tab. 4. OFDM parameters.

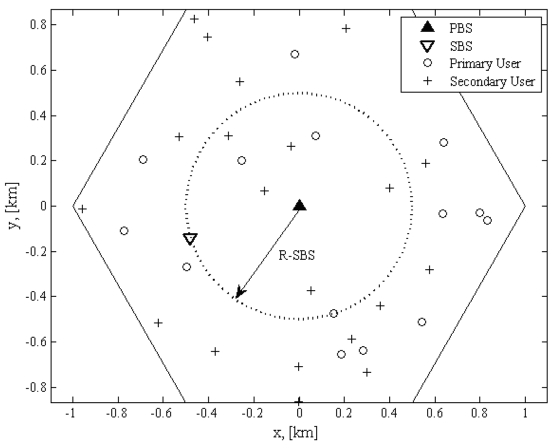


Fig. 4. Sample placement of 14 PUs and 20 SUs.

The power profile considered for the multipath channel corresponds to the ITU Channel B models for Vehicular and Pedestrian environments described in [24]. While in the proposed subchannel allocation algorithm, the averaged interference channel gains (such as path-loss) are used instead of instantaneous channel gains between CR transmitters and primary receivers, for better protection of PRN, we set the interference constraint for each PU 5 dB less than I_{th}.

In order to better evaluate the proposed subchannel allocation, we compare it with two other spectrum sharing methods as benchmarks. The first one is a blind subchannel allocation, in which no interference-aware mechanism is employed i.e. in each cell the SBS independently performs its own subchannel allocation without intra-cell interference between their SUs while doesn't consider mutual interference introduced to the PUs. To get insight of the upper bound on subchannel allocation scheme, we propose a subchannel allocation algorithm based on perfect CSI (SA-PCSI) as second benchmark. It is assumed that the SA-PCSI algorithm can track the instantaneous changes of the channels between CR transmitters and primary receivers. In this method, for each pair of SU_k (located in cell c) and subchannel n, we define the efficiency indicator as follows:

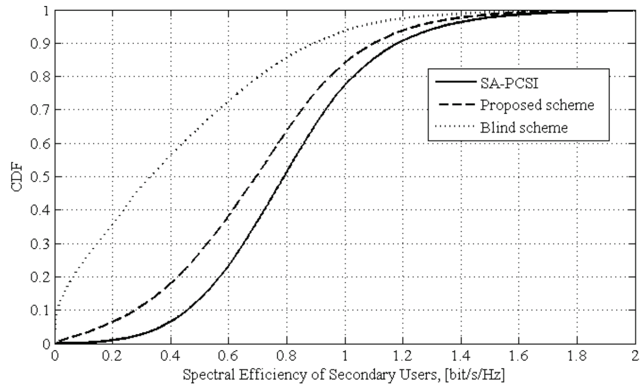
$$\omega_{k,n} = \begin{cases} \dfrac{\lambda_{k,n}}{I_{\max}} & if\ n \notin n_c \\ 0 & if\ n \in n_c \end{cases} \tag{19}$$

where $\lambda_{k,n}$ represents the SNR for SU_k on subchannel n and I_{max} is defined as:

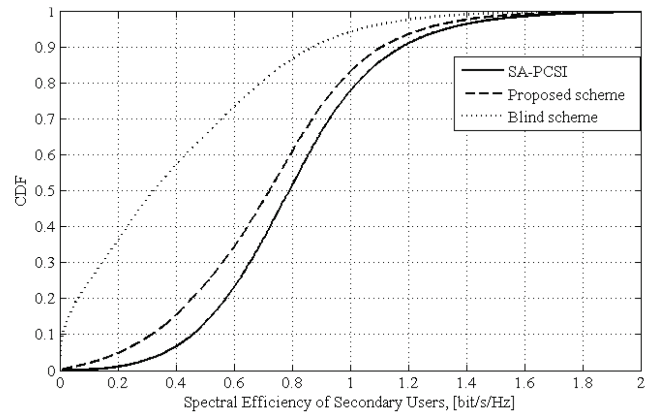
$$\begin{aligned} I_{\max} &= \max\{I(m) \quad \forall m \in \Psi_n\} \\ I(m)_{[dB]} &= P_S + g^{SP,i}_{m,n} \end{aligned} \tag{20}$$

and index i refers to serving SBS of SU_k. $I_{\max}$ represents the maximum interference that imposes on PRN by assignment of subchannel n to SU_k. The algorithm proceeds as follows: For every pair of subchannel n and SU_k, the algorithm searches for the pair $(\tilde{n},\tilde{k})$ with the maximum $\omega_{k,n}$, and sets $\alpha_{k,n}$ = 1 if the interference constraints of all PUs which receive signal from their serving PBS on subchannel n, are guaranteed. Then, the algorithm searches for the subchannel-SU pair with the second highest $\omega_{k,n}$ and checks whether the interference of that pair is less than or equal to the remaining interference threshold. If this is the case, the algorithm proceeds as aforementioned. If the induced interference for the subchannel-SU pair is greater than the remaining interference threshold, a new corresponding efficiency $\omega_{k,n}$ is calculated. The algorithm then proceeds by searching for the pair with the highest efficiency among pairs that are not yet assigned.

The average performance of the given scenarios is obtained through over 200 topologies by randomly arranging the location of Pus and SUs. Fig. 5 and Fig. 6 illustrate the cumulative distribution function (CDF) of the spectral efficiency and SINR of SUs for three different methods, respectively. It is clear in these plots that a significant performance advantage can still be realized by using our proposed method, even if the subchannel allocation at the secondary network is based on imperfect network information. It should be noted that the good performance of the SA-PCSI algorithm is at the cost of having perfect CSI and needs a huge amount of information to be transferred between network entities. Comparing with the blind method, the improvement of spectral efficiency and SINR by the proposed method is obvious.



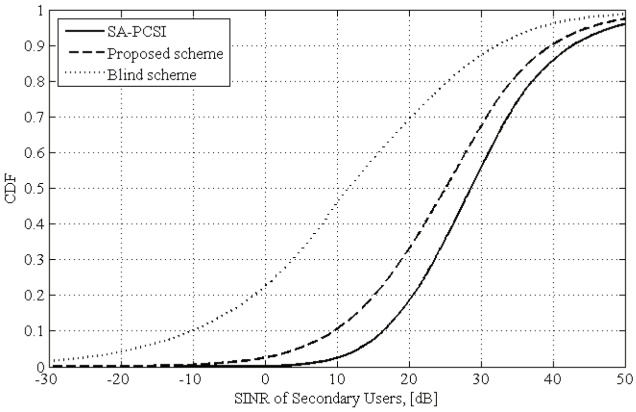
(a) I_{th}= -110 dBm



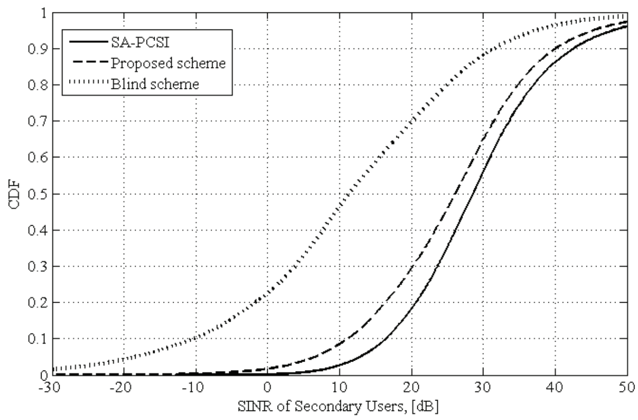
(b) I_{th}= -100 dBm

Fig. 5. Spectral Efficiency of SUs for different scenarios, a) I_{th} = -110 dBm, b) I_{th} = -100 dBm.

Another important issue in CRNs is the interference avoidance with PN. In Fig. 7, we plot the CDF of total interference introduced by SUs to PRN.



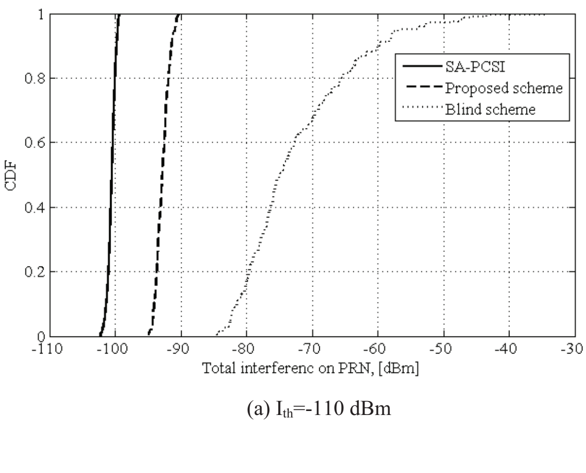
(a) I_{th} = -110 dBm



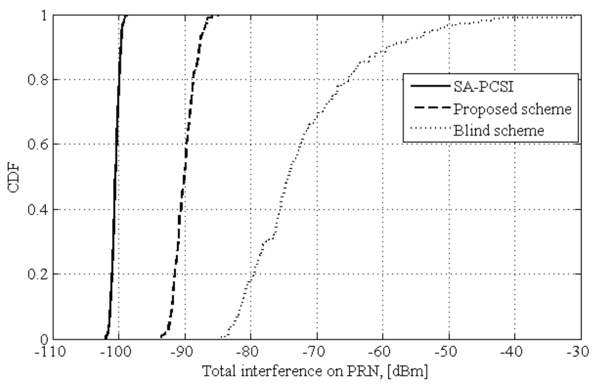
(b) I_{th} = -100 dBm

Fig. 6. SINR distribution of SUs for different scenarios, a) I_{th} = -110 dBm, b) I_{th} = -100 dBm.

Also Fig. 8 shows the CDF of interference introduced by SUs to each PU for different interference threshold. Since in blind algorithm, mutual interference introduced to the PUs is not considered, it has higher interference in comparison with other algorithms. Also as expected, due to perfect CSI between CR transmitters and primary receivers, the SA-PCSI scheme has the lowest interference to PRN compared to the other methods.



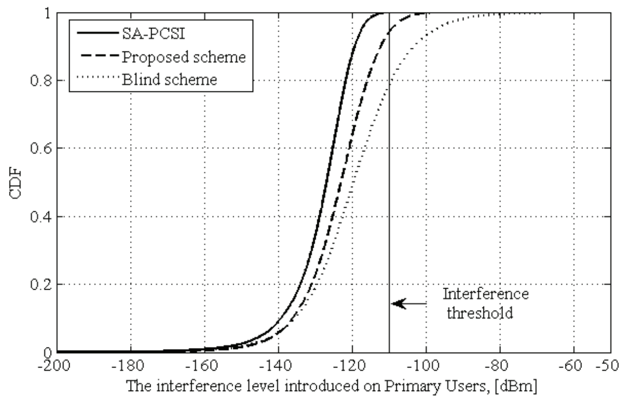
(a) I_{th}=-110 dBm



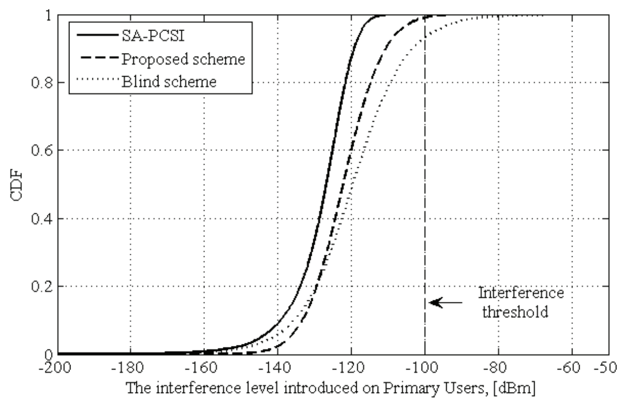
(b) I_{th} = -100 dBm

Fig. 7. Total interference introduced to PRN for different scenarios, a) I_{th} = -110 dBm, b) I_{th} = -100 dBm.

It should be noted that although in our proposed subchannel allocation some SUs violate the interference constraints of some PUs, but due to fast fading of channel, these violation don't have harmful effect on data rate of that PUs. It is clear that the time varying nature of interference channel gain and subsequent violations of secondary network depend on the Doppler spread of the interference channel.



(a) I_{th} = -110 dBm



(b) I_{th} = -100 dBm

Fig. 8. The interference level on PUs for different scenarios, a) I_{th} = -110 dBm, b) I_{th} = -100 dBm.

5. Conclusion

We have studied the problem of resource allocation for an OFDMA based downlink underlay cognitive radio network comprising multiple secondary users and multiple primary users. First, we formulated a generalized optimization problem which is based on maximization of total data throughput of secondary users under interference leakage constraints to primary users. In order to solve this problem, we have proposed a graph approach in which the resource allocation problem is translated as a maximum weighted bipartite matching problem and have solved it by Hungarian algorithm. Based on multi-user diversity gains, our proposed technique assigns subchannels adaptively while considering the interference caused to the PUs. Also one of the key advantages of the proposed method is that the algorithm assigns subchannels based on imperfect CSI in CRN. In the proposed subchannel allocation method, we assume that the CR base station has perfect CSI only between the SUs and itself and the procedure of subchannel assignment in CRN is done based on path-loss components without requirement of tracking the instantaneous changes of the channels between CR transmitters and primary receivers. Lastly, the provided numerical results demonstrate the potential benefits of our proposed approach.

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