Independent Component Analysis of Complex Valued Signals Based on First-order Statistics

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Abstract. *This paper proposes a novel method based on first-order statistics, aims to solve the problem of the independent component extraction of complex valued signals in instantaneous linear mixtures. Single-step and iterative algorithms are proposed and discussed under the engineering practice. Theoretical performance analysis about asymptotic interference-to-signal ratio (ISR) and probability of correct support estimation (PCE) are accomplished. Simulation examples validate the theoretic analysis, and demonstrate that the single-step algorithm is extremely effective. Moreover, the iterative algorithm is more efficient than complex FastICA under certain circumstances.*

Keywords

Independent component analysis, complex valued signals, first-order statistics.

1. Introduction

Independent component analysis (ICA), a multi-channel signal processing technique, has been heat debated during the past decades. As an important tool for blind source separation, ICA has been widely used in a variety of applications such as biomedical image analysis, face recognition and radar data [1]-[3]. However, most researchers focused on real domain of ICA. Recently, due to the crying need of frequency domain signal processing [4], array signal processing [5] and wireless communication [6], [7], some efforts have been made to explore its extension to complex domain.

The complex-valued sources could be sub-Gaussian or super-Gaussian with circular or noncircular symmetric distributions [8], [9]. The algorithms of complex-valued ICA might be divided into two types: one type of the algorithms is based on second-order statistics [10], [11], which mainly includes algorithm for multiple unknown signals extraction (AMUSE), second-order blind identification (SOBI), strong uncorrelating transform (SUT), and Jacobi angles for simultaneous diagonalization (AJD); another type is based on higher-order statistics[12], which includes complex FastICA [8], forth-order blind identification (FOBI) [13], kurtosis based algorithms [14] and so on. However, for most of the algorithms using second and higher order statistical information, these methods generally have higher complexity, and are not efficient enough especially in power-limited scenarios. Recently, some research has aimed at exploiting prior knowledge about the mixing system or the sources themselves. This may include information on the support of the source of interest, i.e., the time or frequency indices where the desired source is positive or presents significant power [15], [16]. But these source extraction methods essentially require the solution of eigenvalue decomposition (EVD) or generalized EVD problem per iteration. Vicente Zarzoso et al. proposed another algorithm which exploits first-order statistics of the whitened observations, under the circumstance that the positive support of the real-valued sources is known [17]. The positive support denoted the sample indices where the source of interest presents positive values. Compared to second and higher order statistical algorithms, the algorithm behaves more efficient to separate the mixed real-valued signals, particularly when the positive support is entirely seized. Nevertheless, it is a pity that the algorithm and conclusion can't be directly applied to solving the complex case.

In this paper, aiming to solve the problem of the independent component extraction of complex valued signals in instantaneous linear mixtures, we propose an extremely effective method using only the first-order statistics of the signals. In order to extend this method in engineering practice of wireless communications, a brief scheme is also proposed and discussed. Section 2 introduces the data model and assumptions in brief. All source signals are circular symmetric distributed, which is an important signal form in wireless communication systems. Section 3 puts forward a novel method for complex-valued issue based on first-order statistics. Two kinds of single-step segregators are proposed and discussed. Theoretically speaking, prior knowledge of the positive support that is distinct from the definition in [17] must be required by the method; while in engineering practice, achieving the prior knowledge is difficult and even impossible. Therefore, a project is considered to solve this problem, in which a combination between the method and the training-based communication system is made. An iterative algorithm is also provided in order to overcome the inaccuracy rooting in engineering practice. Furthermore, theoretical performance analysis about asymptotic interference-to-signal ratio (*ISR*) and probability of correct support estimation (*PCE*) are accomplished in Section 4. Section 5 carries out several simulations and discussion on the performance, including *ISR*, correct support estimation ratio *r* and performance index (*PI*). The theoretic analysis is validated by simulation results. Comparison of the performance between this method and classic Complex FastICA is made. In Section 6, a concise conclusion is given. What's more, this paper can be regarded as an important complement for Vicente Zarzoso's method in [17].

2. Data Model and Assumptions

The complex-valued ICA model used in this article is

$$\mathbf{z}(t) = \mathbf{Q}\mathbf{s}(t) \tag{1}$$

where $\mathbf{z}(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T$ is the vector of whitened observations, $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_n(t)]^T$ is the vector of sources, and $\mathbf{Q}$ is an unknown unitary mixing matrix connecting sources and whitened observations. Without loss of generality, we will presume that $s_1(t)$ is the source of interest. In our complex-valued ICA model, all source signals are circular symmetric distributed, which is quite realistic in practical problems [8]. Therefore the sources satisfy the following four assumptions:

AS1: The source signals $\mathbf{s}(t)$ are zero mean.

AS2: The source signals $\mathbf{s}(t)$ are statistically independent.

AS3: For all source signals, real and imaginary parts are uncorrelated and their variances are equal.

AS4: All source signals have unit variances, i.e., $E\{s_k(t)s_k^*(t)\} = 1$ for $\forall k \in \{1, 2, \ldots, n\}$.

3. The Proposed Algorithm

In this section, before concentrating on the proposed algorithm, we introduce the definition of first-order statistics first. First-order statistics can be mainly described by mathematical expectation and conditional expectation of the signals.

Assume that the probability density function of a continuous random variable $\mathbf{x}$ is $p(\mathbf{x})$, and the conditional probability density function of $\mathbf{x}$ given continuous random variable $\mathbf{y}$ is $p(\mathbf{x}|\mathbf{y})$. If $\int_{-\infty}^{\infty} \mathbf{x} p(\mathbf{x}) d\mathbf{x}$ and $\int_{-\infty}^{\infty} \mathbf{x} p(\mathbf{x} \mid \mathbf{y}) d\mathbf{x}$ converge absolutely at constants, then the former is the mathematical expectation of $\mathbf{x}$, and the latter is the conditional expectation of $\mathbf{x}$ [18]. In this work, we utilize the conditional expectation as the first-order statistics. Since the probability density function is generally unknown, the conditional expectation is appropriately obtained by averaging the samples under given condition.

Now we present the algorithm for complex-valued signals under the data model in Section 2. The algorithm for one source of interest is

Segregator 1 $$\mathbf{w} = E\{\mathbf{z} \mid R_1 > 0\} \tag{2}$$

where R_1 is the real part of s_1. The segregator is gained by the conditional mean, which amounts to averaging the observations over the samples where R_1 is positive-valued. Then, the source of interest can be estimated as:

$$y = \mathbf{w}^H \mathbf{z} = \alpha s_1 \text{ with } \alpha = E\{R_1 \mid R_1 > 0\} > 0. \tag{3}$$

The reason why the estimated source signal equals αs_1 can be illustrated as follows:

According to model (1), we have

$$\mathbf{w} = E\{\mathbf{z} \mid R_1 > 0\} = E\{\mathbf{Q}\mathbf{s} \mid R_1 > 0\} = \mathbf{Q}\mathbf{g},$$

where

$$\mathbf{g} = E\{\mathbf{s} \mid R_1 > 0\}. \tag{4}$$

Considering AS1, AS2 and the un-correlation assumption in AS3, we have:

$$g_1 = E\{s_1 \mid R_1 > 0\} = E\{R_1 \mid R_1 > 0\} + jE\{I_1 \mid R_1 > 0\} = \alpha,$$
$$g_k = E\{s_k \mid R_1 > 0\} = 0$$

where I_1 denotes the imaginary part of s_1.

Hence, $\mathbf{g} = \alpha \mathbf{e}_1$, where $\mathbf{e}_1 = [1, 0, 0, \ldots, 0]^T$.

Consequently, $y = \mathbf{w}^H \mathbf{z} = \mathbf{g}^H \mathbf{Q}^H \mathbf{Q}\mathbf{s} = \mathbf{g}^H \mathbf{s} = \alpha s_1$.

Therefore, s_1 is estimated as y, and s_k for $\forall k \in \{2, \ldots, n\}$ can be recovered in the same way. Similarly, we can conceive another algorithm based on the positive support of imaginary part, the segregator of which turns into $\mathbf{w} = -jE\{\mathbf{z} \mid I_1 > 0\}$. Furthermore, joining these two algorithms together will fabricate an algorithm with better performance, which can be proved in Section 4. The segregator is expressed as follows:

Segregator 2 $$\mathbf{w} = [E\{\mathbf{z} \mid R_1 > 0\} - jE\{\mathbf{z} \mid I_1 > 0\}]/2. \tag{5}$$

Undoubtedly, the Segregator 2 needs more prior knowledge of the sources. Both of the segregators, simple yet effective, can independently accomplish the source separation task. To choose which kind of the segregator depends on how much prior knowledge is provided.

In engineering practice, achieving the prior knowledge is difficult and even impossible. Therefore, we consider the following project to win through:

Procedure 1. Transmit the pilot sequence of each source one by one, i.e., *n* pilot sequences take turns occupying the channel. The receiver synchronously gets the positive support of each pilot sequence during the interval $n\Delta t$, where Δt is the interval of single pilot sequence.

Procedure 2. Retransfer the pilot sequences during the interval Δt, i.e., n pilot sequences occupying the channel simultaneously. Calculate the segregator using Segregator 1 or Segregator 2 after the mixtures of pilot sequences are received.

Procedure 3. Transmit the communication source signals simultaneously. Use the segregator gained from pilot sequences to separate the mixed signals directly.

Thus, if the wireless communication system follows the rule above, mutually independent source signals will be able to transmit simultaneously in the same frequency band in the stationary or slowly varying non-stationary environment. It can be seen from the project that the separation performance relies on the accuracy of the segregator obtained from pilot sequences. Whereas the estimation of the pilot's positive support wouldn't be totally accurate in practice. To overcome the limitation, we think of adding the following iterative algorithm before Procedure 3:

Iterative Segregator

Step 1. Calculate the estimate of each pilot sequence.

Step 2. Obtain the positive support of the estimate.

Step 3. Recalculate the modified segregator 1 $\mathbf{w} = E\{\mathbf{z} | \tilde{R} > 0\}$ or the modified segregator 2 $\mathbf{w} = [E\{\mathbf{z} | \tilde{R} > 0\} - jE\{\mathbf{z} | \tilde{I} > 0\}]/2$, and then return to Step 1 to estimate the plot sequence. Repeat these steps until the estimate converges. $\tilde{R}$ and $\tilde{I}$ denote the real and imaginary part of the estimated pilot sequence respectively.

4. Performance Analysis

In this section, the theoretical analysis on source extraction performance of the segregators (i.e. Segregator 1, Segregator 2 and Iterative Segregator regardless of the engineering project) is carried out. Both the performance of interference-to-signal ratio (*ISR*) and the probability of correct support estimation (*PCE*) will be deduced based on all assumptions of the source. In order to simplify the issue, two further assumptions are made:

AS5: The source signals consist of i.i.d. samples.

AS6: Both the real and imaginary parts of the source signals are symmetric distributed.

For the observation lengths are finite in practice, we assume that the observations are composed of T samples, the index of which is expressed as $\mathrm{S} = \{0, 1, \ldots, T-1\}$. The set S can be divided into two exclusive sets S_{r1} and $\overline{\mathrm{S}}_{r1}$ (or S_{i1} and $\overline{\mathrm{S}}_{i1}$) where S_{r1} (or S_{i1}) is the positive support of source's real parts (or imaginary parts) and $\overline{\mathrm{S}}_{r1}$ (or $\overline{\mathrm{S}}_{i1}$) is the complement. The index set estimated as the positive support of the source's real parts (or imaginary parts) is denoted F_r (or F_i), the cardinality of which is $N \approx T/2$ according to AS6. Here we suppose the correct support estimation ratios for real and imaginary parts are equal. Therefore, the set F_r (or F_i) is the union of set F_{r1} (or F_{i1}) composed of N_1 indices correctly identified, i.e., for which actually $R_1(t) > 0$ (or $I_1(t) > 0$), and its complement $\overline{\mathrm{F}}_{r1}$ (or $\overline{\mathrm{F}}_{i1}$) of $N - N_1$ indices in F_r (or F_i) where $R_1(t) > 0$ (or $I_1(t) > 0$). In short, we can express these relations as follows:

$$\mathrm{S} = \mathrm{S}_{r1} \cup \overline{\mathrm{S}}_{r1}, \quad \mathrm{F}_r = \mathrm{F}_{r1} \cup \overline{\mathrm{F}}_{r1}, \quad \mathrm{F}_{r1} = \mathrm{F}_r \cap \mathrm{S}_{r1}, \quad \overline{\mathrm{F}}_{r1} = \mathrm{F}_r \cap \overline{\mathrm{S}}_{r1}$$

and

$$\mathrm{S} = \mathrm{S}_{i1} \cup \overline{\mathrm{S}}_{i1}, \quad \mathrm{F}_i = \mathrm{F}_{i1} \cup \overline{\mathrm{F}}_{i1}, \quad \mathrm{F}_{i1} = \mathrm{F}_i \cap \mathrm{S}_{i1}, \quad \overline{\mathrm{F}}_{i1} = \mathrm{F}_i \cap \overline{\mathrm{S}}_{i1}.$$

Being interested in average *ISR* per interfering source, we refer to [17] and reach an amendatory definition

$$ISR = \frac{E\{\tilde{b}\tilde{b}^*\}}{(n-1)E\{\tilde{y}_1\tilde{y}_1^*\}} = \frac{\sum_{k=2}^{n} E\{\tilde{g}_k \tilde{g}_k^*\}}{(n-1)E\{\tilde{g}_1 \tilde{g}_1^*\}} \tag{6}$$

where the component of the source of the interest $\tilde{y}_1$ and the interfering component $\tilde{b}$ make up of the estimated signal, i.e., $\tilde{y} = \tilde{\mathbf{g}}^H \mathbf{s} = \tilde{y}_1 + \tilde{b}$. Given $\tilde{\mathbf{g}} = [\tilde{g}_1, \tilde{g}_2, \ldots, \tilde{g}_n]^T$, we have $\tilde{y}_1 = \tilde{g}_1 s_1$ and $\tilde{b} = \sum_{k=2}^{n} \tilde{g}_k s_k$.

Then, two theoretical expressions of *ISR* are gained corresponding to Segregator 1 and 2 respectively:

Expression 1 $\quad ISR = \dfrac{1}{T(2r-1)^2 \alpha^2}$,

Expression 2 $\quad ISR = \dfrac{1}{2T(2r-1)^2 \alpha^2}$

where $r = N_1/N$ represents the correct support estimation ratio, and α is summarized in Tab. 1 for some common probability distributions. These expressions indicate that *ISR* is in inverse proportion to sample size T, and extremely increased when r tends to 0.5.

Proof. See Appendix A.

In Section 3, we considered Iterative Segregator which can probably be applied to practical wireless communication. Here we use *PCE* to evaluate the performance of the iterative algorithm. For iterative algorithm based on modified Segregator 1 (abbr. Iterative Segregator 1), the *PCE* is defined as $r' = P(R_1 > 0 \mid \tilde{R} > 0)$; while for the iterative algorithm using modified Segregator 2 (abbr. Iterative Segregator 2), its *PCE* is

$$r' = \frac{num(R_1 > 0 \mid \tilde{R} > 0) + num(I_1 > 0 \mid \tilde{I} > 0)}{num(\tilde{R} > 0) + num(\tilde{I} > 0)} \approx [P(R_1 > 0 \mid \tilde{R} > 0) + P(I_1 > 0 \mid \tilde{I} > 0)]/2$$

where $\tilde{R} = \tilde{R}_1 + \tilde{b}_r$ and $\tilde{I} = \tilde{I}_1 + \tilde{b}_i$ are the real and imaginary parts of the estimated signal, $num(\cdot)$ denotes the counter. The closed-form expressions are deduced respectively

Expression 3 $$r' = \frac{1}{2} + \int_0^{+\infty} p_{\tilde{\alpha}R_1}(\tau)\, erf\left(\frac{\sqrt{2}T\tau}{n-1}\right) d\tau$$

Expression 4 $$r' = \frac{1}{2} + \int_0^{+\infty} p_{\tilde{\alpha}R_1}(\tau)\, erf\left(\frac{2\sqrt{2}T\tau}{n-1}\right) d\tau$$

where $\tilde{\alpha} = (2r-1)\alpha$, $p_{\tilde{\alpha}R_1}(\tau)$ is the probability density function of $\tilde{\alpha}R_1$, and $erf(\cdot)$ is the error function. In the course of iteration, the correct support estimation ratio will be updated according to Expression 3 and 4.

Proof. See Appendix B.

In the proposed iterative segregator, the complexity of the algorithm mainly depends on step 1, which means that the algorithm needs $O(nT)$ products per iteration. Complex FastICA [8], one of the most computationally attractive Complex ICA methods proposed to date, needs the same quantity level products per iteration, and AJD approach [19], which converges slower than Complex FastICA, needs $O(n^2T)$ products per iteration.

Bernoulli	Sinusoid	Uniform	Laplacian	Gaussian
$\frac{\sqrt{2}}{2}$	$\frac{2}{\pi}$	$\frac{\sqrt{6}}{4}$	$\frac{1}{2}$	$\sqrt{\frac{1}{\pi}}$

Tab. 1. Conditional mean $\alpha = E\{R_1 \mid R_1 > 0\} = E\{I_1 \mid I_1 > 0\}$ for some normalized distributions.

5. Simulation Results and Discussion

Here, several sets of simulation results are provided to demonstrate the performance of the proposed algorithm. Generally speaking, experiments on both single-step and iterative algorithms have been carried out.

Simulation 1. Performance of the single-step algorithm

In this simulation, we suppose that the prior knowledge is totally known, and use Segregator 1 and 2 to separate unitary mixtures of n independent sources with the same distribution. The unitary mixing matrix $\mathbf{Q}$ in data model of (1) is created randomly.

Fig. 1 illustrates the *ISR* performance between Segregator 1 and 2 when $n = 20$ for different sample size. The 4QAM sources, whose real and imaginary parts are both Bernoulli distributed, are adopted in this simulation. Results are averaged over 100 Monte Carlo runs. We can see that the theoretical performance analysis by Expression 1 and 2 well approximates the simulation results, and the performance of Segregator 2 is about 3 dB better than Segregator 1 just as Expression 1 and 2 indicate.

Fig. 2 shows the *ISR* performance for different values of the correct support estimation ratio r when the sample size $T = 1000$. The simulation results are well approximated by the theoretical performance analysis.

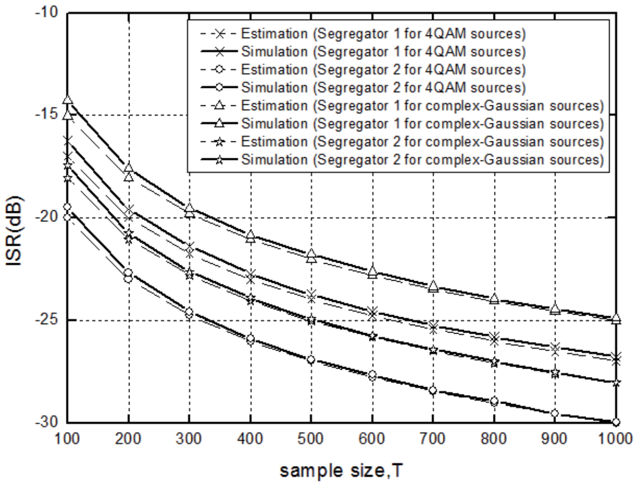

Fig. 1. *ISR* versus sample size for Segregator 1 and 2 when $n = 20$, $r = 1$.

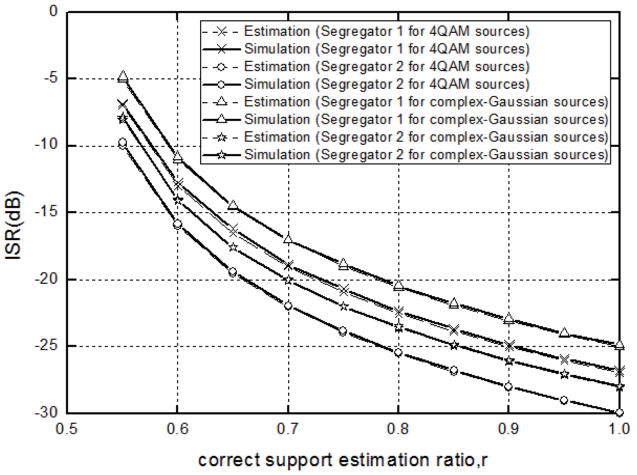

Fig. 2. *ISR* versus correct support estimation ratio for Segregator 1 and 2 when $n = 20$ and $T = 1000$.

Simulation 2. Performance of the iterative algorithm

Here, the convergence speed of iterative segregator when $n = 10$ is considered. The 4QAM sources composed of $T = 1000$ samples are adopted in this simulation. The mixtures are also unitary for the data model of (1) is applied.

In Fig. 3, we compare the convergence speed of correct support estimation ratio r between Iterative Segregator 1 and 2. The different initial values of the correct support estimation ratio are considered and labeled as r_1. Likewise, results are obtained over 100 Monte Carlo runs. One can observe that, after several iterations the correct support estimation ratio r converges at 1.0. When $r_1 = 0.6$, Iterative Segregator 2 (Fig. 3, right) converges more swiftly than Iterative Segregator 1 (Fig. 3, left). And the theoretical curves in Fig. 3-right, are more precise than the left one.

In Fig. 4, the performance of *ISR* is provided between Iterative Segregator 1 (Fig. 4, left) and Iterative Segregator 2

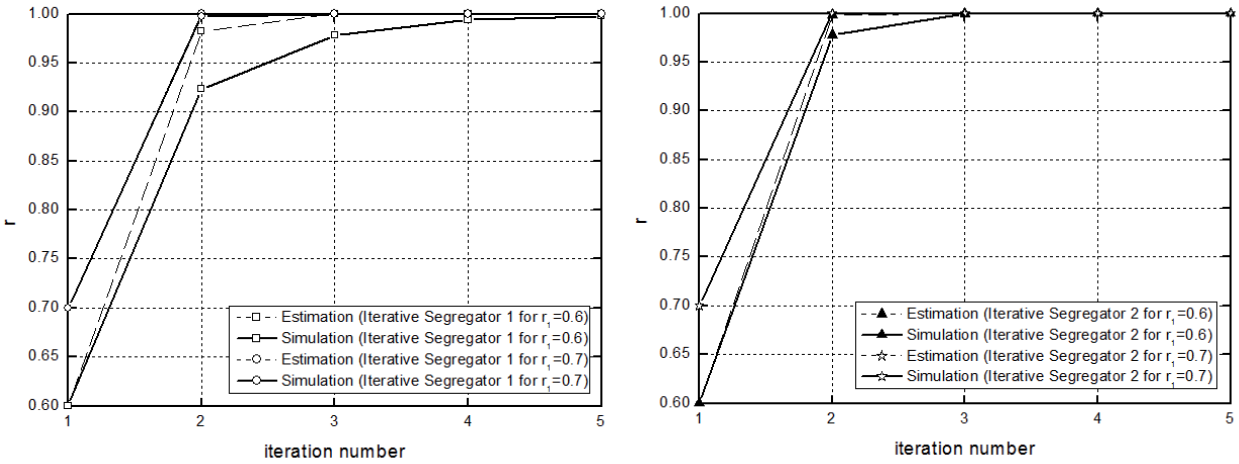

Fig. 3. r of Iterative Segregator 1 and 2 for different initial correct support estimation ratios when n = 10 4QAM sources and T = 1000 samples are adopted.

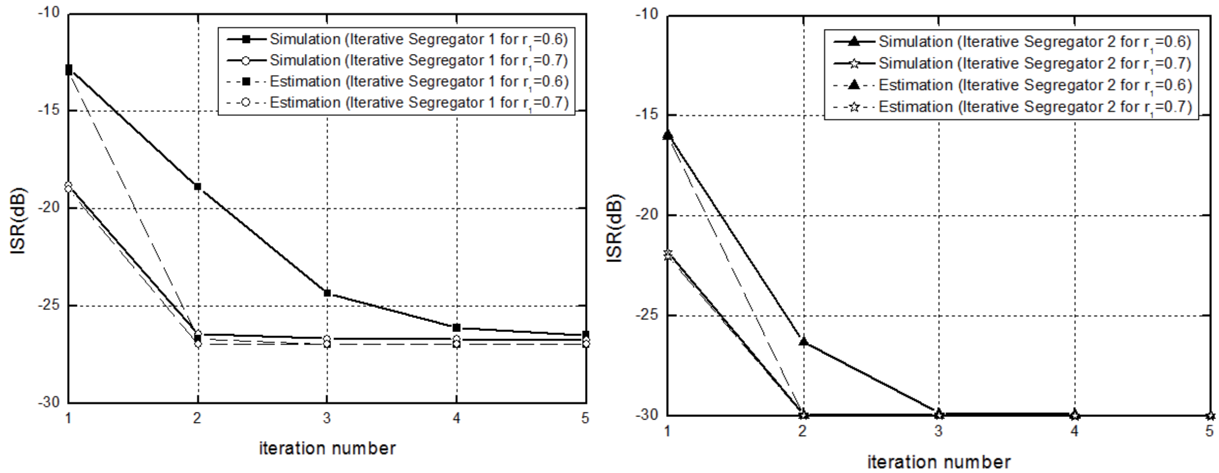

Fig. 4. *ISR* of Iterative Segregator 1 and 2 for different initial correct support estimation ratios when n = 10 4QAM sources and T = 1000 samples are adopted.

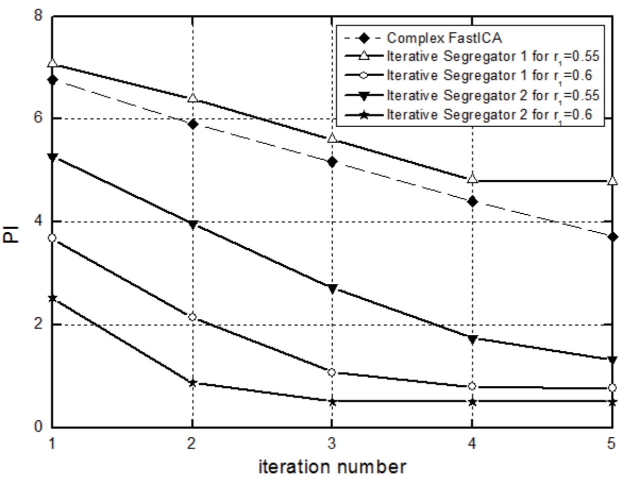

Fig. 5. Performance index (*PI*) among Complex FastICA, Iterative Segregator 1 and 2 when n = 10 4QAM sources and T = 1000 samples are adopted.

(Fig. 4, right). As iterations increase, the correct support estimation ratio r is improved. So it is not difficult for us to understand the amelioration of the *ISR* performance. The estimated *ISR* from Expression 1 and 2 well approximates the simulated one if r_1 is not close to 0.5.

In Fig. 5, the comparison of performance index (*PI*) [20] is made between Complex FastICA and Iterative Segregators, where the classical expression of *PI* is

$$PI = \frac{1}{n}\left[\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\frac{|g_{ij}|}{\max_{j}|g_{ij}|}-1\right)+\sum_{j=1}^{n}\left(\sum_{i=1}^{n}\frac{|g_{ij}|}{\max_{i}|g_{ij}|}-1\right)\right]$$

with $g_{ij}=[\mathbf{WQ}]_{ij}$, and $\mathbf{W}$ denotes the estimated mixing matrix. The *PI* performance of Iterative Segregator 1 for $r_1 = 0.55$ is slightly worse than Complex FastICA, but it will be greatly improved if we adopt the Iterative Segregator 2 or enhance a little accuracy of prior knowledge. Meanwhile, considering the complexity per iteration discussed in Section 4, the proposed algorithm is more efficient than Complex FastICA to some extent.

The analysis can be summed up by a few empirical rules to indicate the advantages and disadvantages of our method, as well as the requirements for application:

- For single-step algorithms, large samples and high accuracy of prior knowledge of support information lead to excellent separation performance. And the ISR performance of Segregator 2 is generally 3dB better than Segregator 1, which demonstrates the theoretical performance analysis.
- For engineering practice such as wireless communications, although prior knowledge seems to be available over the pilot sequences, it is difficult to achieve accurate prior knowledge. But further iterations (i.e. Iterative Segregators) can improve the accuracy and obtain the same performance as the single-step one when r converges at 1.0.
- Since the proposed method belongs to semi-blind scenarios and $r_1 = 0.5$ indicates that no prior knowledge is available, the r_1 should be not too close to 0.5 for our method. The method is extremely cost-effective, if a little more prior knowledge is adopted.

6. Conclusion

In this paper, a novel method for complex-valued ICA based on the conditional first-order statistics of the whitened observations is proposed. The performance of the method relies on sample size, the segregator type, and the accuracy of prior knowledge, but further iterations can improve the accuracy. Simulation results support the theoretic analysis, and demonstrate that compared to classic Complex FastICA, the proposed iterative algorithm is more efficient if a little more prior knowledge is adopted. The practical project considered in Section 3 indicates that the method can be compatibly applied to wireless communication system. Future research includes the improvement of the method on fully blind scenarios, the extension to non-circular symmetric distributed sources and the investigation on the method in non-stationary environment.

Appendix A: Proof of *ISR* Expressions

According to (6), computing the *ISR* needs the calculation of $E\{\tilde{g}_k\tilde{g}_k^*\}$ for $1 \le k \le n$. Here, we consider Expression 1 based on Segregator 1 in the first instance. In practice, we should first remove the sample mean from every sample of the observations so as to ensure the rationality of AS1. Therefore the global transformation (4) can be expressed as the following sample version:

$$\begin{aligned}\tilde{\mathbf{g}} &= \frac{1}{N}\sum_{t\in \mathrm{F}_r}\mathbf{s}(t) - \frac{1}{T}\sum_{t\in \mathrm{S}}\mathbf{s}(t) = \frac{1}{T}\sum_{t\in \mathrm{F}_r}\mathbf{s}(t) - \frac{1}{T}\sum_{t\in \overline{\mathrm{F}}_r}\mathbf{s}(t) \\ &= \frac{1}{T}\sum_{t\in \mathrm{F}_r}\mathbf{R}(t) - \frac{1}{T}\sum_{t\in \overline{\mathrm{F}}_r}\mathbf{R}(t) + \frac{j}{T}\sum_{t\in \mathrm{F}_r}\mathbf{I}(t) - \frac{j}{T}\sum_{t\in \overline{\mathrm{F}}_r}\mathbf{I}(t)\end{aligned} \tag{A-1}$$

where $\tilde{\mathbf{g}} = [\tilde{g}_1, \tilde{g}_2, ..., \tilde{g}_n]^T$. For $2 \le k \le n$, under the assumption AS3 and AS5, we obtain

$$\begin{aligned}E\{\tilde{g}_k\tilde{g}_k^*\} &= \frac{1}{T^2}\sum_{t\in F_r}E\{R_k^2\} + \frac{1}{T^2}\sum_{t\in \overline{F}_r}E\{R_k^2\} \\ &+ \frac{1}{T^2}\sum_{t\in F_r}E\{I_k I_k^*\} + \frac{1}{T^2}\sum_{t\in \overline{F}_r}E\{I_k I_k^*\}.\end{aligned} \tag{A-2}$$

Considering the independence assumption AS2 and unit-power assumption AS4, we have $E\{s_k(t)\} = 0$ and $\mathrm{var}\{R_k(t)\} = \mathrm{var}\{I_k(t)\} = 1/2$, so that

$$E\{\tilde{g}_k\tilde{g}_k^*\} = \frac{1}{T}. \tag{A-3}$$

For $k = 1$, under the symmetry assumption AS6, we have

$$\begin{aligned}E\{R_1(t)\} &= \begin{cases}\alpha & t \in \mathrm{S}_{r1} \\ -\alpha & t \in \overline{\mathrm{S}}_{r1}\end{cases} \\ \mathrm{var}\{R_1(t)\} &= \frac{1}{2} - \alpha^2, \quad t \in \mathrm{S}_{r1}\ or \quad t \in \overline{\mathrm{S}}_{r1}\end{aligned} \tag{A-4}$$

Now, N_1 indices of F_r belong to S_{r1} and the remaining $N - N_1$ to $\overline{\mathrm{S}}_{r1}$; by symmetry, set $\overline{\mathrm{F}}_r$ contains $N - N_1$ indices in S_{r1} and N_1 in $\overline{\mathrm{S}}_{r1}$. Hence, we obtain

$$\begin{aligned}E\{\tilde{g}_1\} &= (2r-1)\alpha, \\ \mathrm{var}\{\tilde{g}_1\} &= \frac{1}{2T} + \frac{1}{T}\left(\frac{1}{2} - \alpha^2\right).\end{aligned} \tag{A-5}$$

For sufficient sample size, $E\{\tilde{g}_1\tilde{g}_1^*\}$ is dominated by $E\{\tilde{g}_1\}E\{\tilde{g}_1\}^*$, as a result, $E\{\tilde{g}_1\tilde{g}_1^*\} = (2r-1)^2\alpha^2$. Bring this expression and (A-3) into (6), we obtain

$$ISR = \frac{1}{T(2r-1)^2\alpha^2}. \tag{A-6}$$

To acquire Expression 2 based on Segregator 2, another global transformation can be given

$$\begin{aligned}\tilde{\mathbf{g}} &= \frac{1}{T}\sum_{t\in \mathrm{F}_r}\mathbf{s}(t) - \frac{1}{T}\sum_{t\in \mathrm{S}}\mathbf{s}(t) - \frac{j}{T}\sum_{t\in \mathrm{F}_i}\mathbf{s}(t) + \frac{j}{T}\sum_{t\in \mathrm{S}}\mathbf{s}(t) \\ &= -\frac{1}{T}\sum_{t\in \overline{\mathrm{F}}_r}\mathbf{R}(t) - \frac{j}{T}\sum_{t\in \overline{\mathrm{F}}_r}\mathbf{I}(t) - \frac{j}{T}\sum_{t\in \overline{\mathrm{F}}_i}\mathbf{R}(t) + \frac{1}{T}\sum_{t\in \overline{\mathrm{F}}_i}\mathbf{I}(t)\end{aligned} \tag{A-7}$$

Under the assumptions AS1-AS6, similarly we have

$$\begin{aligned}E\{\tilde{g}_k\tilde{g}_k^*\} &= \frac{1}{2T}, \\ E\{\tilde{g}_1\tilde{g}_1^*\} &= (2r-1)^2\alpha^2.\end{aligned} \tag{A-8}$$

Bring (A-8) into (6), we obtain

$$ISR = \frac{1}{2T(2r-1)^2\alpha^2}. \tag{A-9}$$

Appendix B: Proof of *PCE* Expressions

To deduce the closed form of $r' = P(R_1 > 0 \mid \tilde{R} > 0)$, we note that $P(R_1 > 0 \mid \tilde{R} > 0)$ can be reduced to $P(\tilde{R} > 0 \mid R_1 > 0)$ according to Bayesian theorem and symmetry assumption. To calculate $P(\tilde{R} > 0 \mid R_1 > 0)$ needs the integration of $p_{\tilde{R}|R_1>0}(u)$ over $[0, +\infty)$, where $p_{\tilde{R}|R_1>0}(u)$ is the probability density function of $\tilde{R}$ given $R_1 > 0$. Otherwise, considering $\tilde{y} = \tilde{\mathbf{g}}^H \mathbf{s} = \tilde{y}_1 + \tilde{b}$ and (A-5), $\tilde{y}_1 \approx E\{\tilde{g}_1\} s_1 = \tilde{\alpha} s_1$ for sufficient sample size. Then we have $\tilde{R}_1 = \tilde{\alpha} R_1$, with $\tilde{\alpha} = (2r-1)\alpha$.

Thereby,

$$p_{\tilde{R}|R_1>0}(u) = \int_{-\infty}^{+\infty} p_{\tilde{\alpha}R_1|R_1>0}(\tau) p_{\tilde{b}_r}(u-\tau) d\tau$$
$$= 2\int_0^{+\infty} p_{\tilde{\alpha}R_1}(\tau) p_{\tilde{b}_r}(u-\tau) d\tau$$

where the symmetry assumption exert an influence. The interference term $\tilde{b}_r$ performs as a zero-mean Gaussian random variable with variance $\sigma^2 = E\{b_r^2\} = (n-1)/2T$, for the reason that $E\{\tilde{b}\tilde{b}^*\} = \sum_{k=2}^{n} E\{\tilde{g}_k \tilde{g}_k^*\} = (n-1)/T$ and $E\{\tilde{b}\tilde{b}^*\} = E\{\tilde{b}_r^2 + \tilde{b}_i^2\} = E\{\tilde{b}_r^2\} + E\{\tilde{b}_i^2\} = 2E\{\tilde{b}_r^2\}$, where we use equation (A-3) and assumption AS3.

The discussion above leads to

$$r' = P(\tilde{R} > 0 \mid R_1 > 0) = \int_0^{+\infty} p_{\tilde{R}|R_1>0}(u) du$$
$$= 2\int_0^{+\infty}\int_0^{+\infty} p_{\tilde{\alpha}R_1}(\tau) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(u-\tau)^2}{2\sigma^2}\right) d\tau\, du$$
$$= \frac{1}{2} + \int_0^{+\infty} p_{\tilde{\alpha}R_1}(\tau) erf\left(\frac{\sqrt{2T}\tau}{n-1}\right) d\tau$$

where the fourth equality uses the error function $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-v^2} dv$ and follows from the change of variable $v = (u-\tau)/(\sqrt{2}\sigma)$.

Otherwise, the deduction of $r' = [P(R_1 > 0 \mid \tilde{R} > 0) + P(I_1 > 0 \mid \tilde{I} > 0)]/2$, which represents the performance of iterative algorithm based on Segregator 2, resembles the former. Since $P(R_1 > 0 \mid \tilde{R} > 0) \approx P(I_1 > 0 \mid \tilde{I} > 0)$, the probability is reduced to $r' = P(R_1 > 0 \mid \tilde{R} > 0)$. Whereas the interference term $\tilde{b}_r$ performs distinctively with the variance $\sigma^2 = E\{b_r^2\} = (n-1)/4T$, for $E\{\tilde{b}\tilde{b}^*\} = \sum_{k=2}^{n} E\{\tilde{g}_k \tilde{g}_k^*\} = (n-1)/2T$, where equation (A-8) is exploited. Therefore, the closed-form expression turns into

$$r' = \frac{1}{2} + \int_0^{+\infty} p_{\tilde{\alpha}R_1}(\tau) erf\left(\frac{2\sqrt{2T}\tau}{n-1}\right) d\tau .$$

The proof is completed.

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