An Adaptive Clutter Suppression Technique for Moving Target Detector in Pulse Doppler Radar

Amritakar MANDAL, Rajesh MISHRA

School of ICT, Gautam Buddha University, Yamuna Expressway, Greater Noida, Uttar Pradesh - 201308, India

amritkar2k@gmail.com, rmishra@gbu.ac.in

Abstract. *An adaptive system performs the processing by using an architecture having time-varying parameters on the received signals which accompanies with clutters. In this paper, an adaptive moving target detector has been designed to meet the challenges of target detection amidst various levels of clutter environments. The approach has been used that is able to overcome the inherent limitations of conventional systems (e.g. Moving Target Indicator, Fast Fourier Transform etc.) having predefined coefficients. In this purpose an optimal design of transversal filter is being proposed along with various weight selection Maps to improve probability of detection in ground based surveillance radar. A modified LMS algorithm based adaptive FIR filter has been implemented utilizing modular CORDIC unit as a main processing element for filtering as well as weight update to suppress clutter of various intensity. Extensive MATLAB simulations have been done using various levels of clutter input to show the effectiveness of adaptive moving target detector (AMTD).*

Keywords

Adaptive MTD, transversal filter, signal-to-clutter ratio, LMS algorithm, WSM, CORDIC.

1. Introduction

In recent years, there has been a significant increase in the requirement for superior performance of radar in all areas of civil as well as military applications [1]. One of the main area of progress in a radar system today is its signal processing. With the advent of Very Large Scale Integration technology and FPGA, implementation of Digital Signal Processing algorithm (DSP) on a single chip has become a reality [2], [3]. Radar signal processor (RSP) manipulates the received radar signal for extraction of desired information of moving target whilst rejecting unwanted clutters. The process of target detection becomes a challenge when the strength and statistical properties of the interfering signals are unknown a priori. The target detection which estimates and detects the target from an ever changing clutter environment is called adaptive moving target detector (AMTD).

So far, a good many numbers of methods have been proposed for adaptive moving target detector (AMTD) and in adaptive implementation of optimum processing (AIOP) in ground based or airborne radar [4-9]. A substantial bulk of work is available in literature about optimum detection of target in non-Gaussian clutter [10], [11]. Imaging technique is also being implemented in some airborne or in UWB radar for target detection and tracking [12], [13]. Over the years, radar scientists have focused on improvement over target detection from various kinds of clutters. Various adaptive processing techniques with clutter map and other criteria also were implemented in the recent past [14-16]. Practically, target always positioned itself amongst the point, area or extended clutter [17]. This background clutter changes with time and as per positions of the antenna. This manifests inevitable requirement of adaptive signal processing technique to apply adaptive threshold as per clutter situation to maintain a constant false alarm rate (CFAR) [17], [18].

In this paper, adaptive moving target detector has been designed keeping in view a known clutter covariance matrix for estimation of filter weights and simultaneously various clutter maps for non-homogeneous clutters. For each coherent processing interval, a bank of digital FIR Doppler filters has been proposed using CORDIC as an integral part of the transversal filter [19], [20]. Filtering as well as weight updating functions can be implemented using multipliers, variable shift registers or commonly used Multiply Accumulator (MAC) units. But in this paper, CORDIC processing element has been used for implementation of these functions efficiently. The advantages of CORDIC compared to MAC based design are many fold. The CORDIC computations are pipelinable at microlevel and are sufficiently robust against internal numerical errors. Since the feedback circuits are extremely sensitive to numerical errors, it would be almost error free to use CORDIC based design. Because of its modular design and easy implementation using VLSI technology makes it a better choice in digital signal processing area [21-23]. In DSP systems, signals are required to be quantized and represented in fixed word-length [24]. In case of MAC based transversal filter design, huge amount of round-off noise increases by decreasing signal to noise ratio (SNR) significantly. To reduce the computation error, a processor designer might simply increase the number of iterations and that will be a huge wastage of processing time and power. To reduce processing time as well as power, pipelined CORDIC architecture has been chosen for design of adaptive FIR filters in conjunction with very robust and popular LMS algorithm [25-27].

The filter banks reject the correlated components of input clutter with using a priori knowledge of the clutter statistics. The efficient clutter rejection is achieved through meticulous design of each filter which rejects the frequency components occupied by the clutter power spectrum resulting in significant increase in signal-to-clutter ratio (SCR). The conventional radar signal processor used to have pre-defined operability. But in modern radar, it should not be pre-defined as it should be operated in a clutter environment which is changing abruptly. The radar receiver gets saturated due to strong clutter and/or hostile noise jamming when the noise received power exceeds detection threshold of the receiver. To identify specific target amongst thick clutter environment and to avoid receiver overloading, an adaptive FIR filtering method has been adopted to lower the false alarm of the receiver significantly and at the same time to increase the probability of detection. For faster convergence of the transversal filter, an optimal operating step size [28] is also verified.

The remainder of this paper proceeds as follows. In Section 2, a mathematical modeling for Doppler filter design has been discussed. In Section 3, architecture of adaptive moving target detector in various clutter environments and related issues has been discussed based on mathematical modeling. A review of CORDIC algorithm which is integral part of the proposed design is incorporated in Section 4. In Section 5, the design of adaptive filter banks using reformulated trigonometric form of LMS algorithm has been explained. Simulated output for the response of filter/filter banks and their analysis has been clearly demonstrated in Section 6. Finally, the concluding remarks are provided in Section 7.

2. A Mathematical Modeling for Doppler Filter Design

The optimal processor is consist of complex weight to facilitate amplitude and phase processing and followed by a modulus extractor. The optimum weight calculation of Doppler filter is based on a probable signal distribution within which Doppler shift of target falls. But target Doppler shift in most of the cases are unknown which leads to design of transversal filter to cover the Doppler frequency band of interest. Therefore, it is obvious that each filter parameter results to optimal filter i.e. each filter is optimum at the centre frequency whereas at other frequencies it is mismatched. Now we will proceed to derive Doppler filter parameters step by step mathematically and thereafter we would like to design the filter/filter banks subsequently. The critical analysis and responses of these filters have been incorporated in the analysis section of this paper.

Let the complex weight of the filter is given by:

$$w^T = [w_0, w_1, \ldots\ldots, w_{n-1}] \tag{1}$$

and the complex signal input vector is given by

$$x^T = [x_0, x_1, \ldots\ldots, x_{n-1}] \tag{2}$$

T is representing transpose operation.

The complex output from the given transversal filter is

$$r = \sum_{i=0}^{n-1} w_i x_i = w^T x = x^T w\,. \tag{3}$$

The output power of the given filter is

$$P_o = E|r|^2 = E(rr^*) = w^T E(xx^{*T})w^*\,. \tag{4}$$

The asterisk operation indicates a complex conjugate operation.

Here input is taken as clutter interference and the corresponding clutter covariance matrix is R_n. The input power can be calculated as:

$$P_n = w^T R_n w^* \tag{5}$$

when $R_n = E(NN^{*T})$

where $N^T = [N_0, N_1, \ldots\ldots, N_{n-1}]$ is the complex input noise vector.

The complex input signal of amplitude A to the n-tap Doppler transversal filter is given by

$$S_i = Ae^{jw_d t}\sum_{i=0}^{n-1}\delta(t-iT) \tag{6}$$

where w_d is the radian Doppler frequency.

Let the signal vector is

$$S^T = [S_1, S_2, \ldots\ldots, S_k] \tag{7}$$

where $S_k = Ae^{jw_d(k-1)T}$, $k = 1,2,\ldots., n$.

The signal output power for signal covariance matrix M_s is given by:

$$P_s = w^T E(Se^{j\phi}S^{*T}e^{-j\phi})w^* = w^T M_s w^* \tag{8}$$

where $M_s = E(SS^{*T})$.

The normalized (divided by A^2) covariance matrix can be given by:

$$\mathbf{M}_s = \begin{bmatrix} 1 & e^{-jw_dT} & e^{-j2w_dT} & \bullet & \bullet & \bullet & e^{-jw_d(n-1)T} \\ e^{jw_dT} & 1 & e^{jw_dT} & & & & \\ e^{j2w_dT} & e^{jw_dT} & 1 & & & & \\ \bullet & e^{j2w_dT} & e^{jw_dT} & \bullet & & & \\ \bullet & \bullet & e^{j2w_dT} & & \bullet & & \\ e^{jw_d(n-2)T} & \bullet & \bullet & & & \bullet & \\ e^{jw_d(n-1)T} & e^{jw_d(n-2)T} & e^{jw_d(n-3)T} & \bullet & \bullet & \bullet & 1 \end{bmatrix} \tag{9}$$

The improvement factor ($I.F$) of a Doppler processor is defined as the ratio of output S/N to the input S/N.

$$I.F = \frac{(S/N)_o}{(S/N)_i} . \tag{10}$$

In case of normalized improvement factor of a transversal filter, input signal and noise power are equal. Therefore, the normalized improvement factor ($I.F$) becomes:

$$I.F = (S/N)_o = \frac{P_o}{P_n} = \frac{w^T M_s w^*}{w^T R_n w^*} . \tag{11}$$

The above equation shows that signal-to-noise (Clutter) ratio becomes maximum at the output of the transversal filter at maximum value of $I.F$. Ultimately, it confirms improvement in probability of detection. The maximum $I.F$ occurs at the following condition:

$$M_s w^* = \gamma R_n w^* \tag{12}$$

where γ is a scalar quantity. The above equation (12) can be written in a matrix equation by introducing an identity matrix I :

$$(R_n^{-1} M_s - \gamma I) w^* = 0 \tag{13}$$

where R_n^{-1} is the matrix inverse of the clutter covariance matrix and γ is eigenvalues. The nontrivial solution exists for (13) when its determinants become zero.

$$\left| R_n^{-1} M_s - \gamma I \right| = 0 . \tag{14}$$

For an n-pulse Doppler processor, n-number of optimal weights can be found as the eigenvectors that corresponds to maximum eigenvalues of the matrix $R_n^{-1} M_s$. Optimum weights can be determined using statistical detection theory as

$$w_0 = R_n^{-1} S^* . \tag{15}$$

The optimal complex weights are given by:

$$w_i = \sum_{j=1}^{n} \alpha_{ij} \cos(j-1) w_d T - j \sum_{i=1}^{n} \alpha_{ij} \sin(j-1) w_d T \tag{16}$$

α_{ij} is an element of the inverse covariance matrix.

3. Proposed Architecture of Adaptive Moving Target Detector (MTD)

With the advent of digital signal processing techniques, implementation of algorithm on a complex signal processor has become a very easier task. The filters using favorable adaptive algorithm can be realized for target detection and clutter cancellation in a modern radar system. Generally in radar system, the Doppler filtering is achieved through a certain number of FIR filter banks. The filter banks cover frequency response of the radar receiver. Each FIR filter is tuned to a definite Doppler frequency that is within f_d= 0 Hz to f_d= PRF, where f_d is the Doppler frequency and PRF is Pulse Repetition Frequency. The frequency tuning on the desired Doppler frequency is achieved by utilizing coefficients, called "weights". The 'weight' used for specific frequency response in the radar signal processing is complex in nature as the echo signal from target is complex in nature. As a result, the receiver frequency response curve is subdivided into a number of f_d values and the filtering of echo signal which received by the radar is performed by just multiplying it with the coefficient associated to each Doppler filter according to the following formula:

$$(\text{Response})_x = \sum_{n=1}^{N} S_n * (w_n)_{F_x} \tag{17}$$

where: $(\text{Response})_x$ = Output of filter F_x, S_n = Complex echo at sweep n, $(w_n)_{F_x}$ = Complex coefficient for F_x filter.

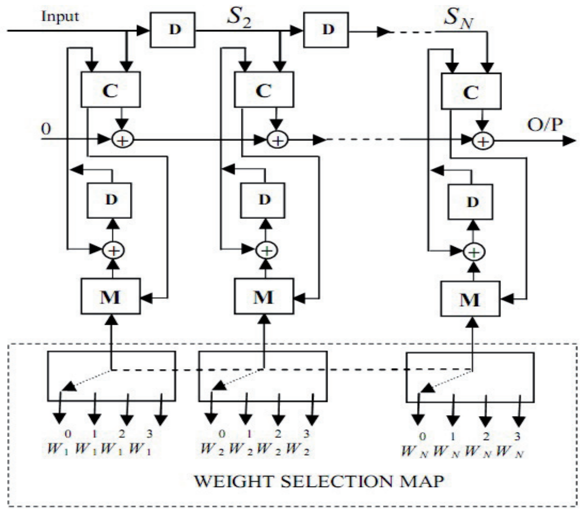

Fig. 1. Digital Filter using Weight Selection MAP in which weights are in the form of equivalent angles.

Each filter therefore will give a significant response only when the filter is tuned at or very close to f_d component of the echo signal. The responses of the filters are generally changed as per different set of complex filter weighting coefficient, w_i, to force many filtering conditions. With reference to strength of the clutter returns, filter banks are divided into four categories (Bank-0 to Bank-3). As per the clutter representation on the scope, the respective bank of filter weights to be selected from Weight Selection Map (WSM). These sets of weights are stored in ROM memory as equivalent angular terms. The WSM maps are categorized as:

a) **Absence of clutter** Bank-0 $(w_1^0 w_2^0 \ldots w_N^0)$

b) **Weak clutter** Bank-1 $(w_1^1 w_2^1 \ldots w_N^1)$

c) **Strong clutter** Bank-2 $(w_1^2 w_2^2 ....w_N^2)$

d) **Very Strong clutter** Bank-3 $(w_1^3 w_2^3 ....w_N^3)$

Each bank of filter is made of n filters. The $n-2$ filters are tuned within the Doppler band and pair of filters at zero velocity (ZVF) is tuned to the right and to the left of the zero Doppler frequency. The $n-2$ filters cover a Doppler band of amplitude which depends on the clutter condition. On an increase of the CNR (Clutter Noise Ratio), the said amplitude decreases in a manner to achieve good rejection of ground clutter. Weight selection map (WSM) is used to select the appropriate bank of filters in each cell.

The amplitude of the filter output is calculated by modulus extractor without any loss. WSM is a dynamic map with updating periodicity defined through antenna scans. The loading of the WSM map occurs analyzing the clutter residues present on filters F_1 and F_{n-2}, that is the filter after F_{0^+} and the last filter before F_{0^-}. Not to lose the radar operativity during the map updating, a specific set of weights is selected so that the frequency response of the filters utilized to load WSM (these filters are not usable for detection) is covered by the adjacent filters; in other words, if we consider for example an 8-filter bank, filters F_1 and F_6 are used to load the WSM map but the response curve of filters $F_2 - F_5$ are widened in order to cover the frequencies of F_1 and F_6 as well.

4. A Review of CORDIC Algorithm in Complex Signal Processing

The main idea of CORDIC computation is to decompose the desired rotation angle into the weighted sum of a set of predefined elementary rotation angles through each of them can be accomplished with simple shift-add operation for a desired rotational angle θ, it can be represented for M iterations of an input vector $(x,y)^T$ setting initial conditions $x_0 = x$, $y_0 = y$ and $z_0 = \theta$ as

$$z_f = \theta - \sum_{i=0}^{M-1} \delta_i \alpha_i \ . \tag{18}$$

If $z_f = 0$ holds, then $\theta = \sum_{i=0}^{M-1} \delta_i \alpha_i$ i.e. the total accumulated rotation angle is equal to θ. $\delta_i, 0 \le i \le M-1$, denotes a sequence of ±1s that determines the direction of each elementary rotation.

Almost every signal used in DSP module is complex in nature. So all these signal will follow the Euler's theorem resulting one sided spectrum with direction of rotation (positive or negative frequency) and with known real (cosine) and imaginary (sine) components.

$$\cos \omega_c t + j \sin \omega_c t = e^{j\omega_c t} \ . \tag{19}$$

Let a signal vector $\hat{\mathbf{v}}$ with angle θ is passed through CORDIC processor. The outcome from CORDIC can be shown as follows.

$$\begin{aligned}
\hat{\mathbf{v}} &= ve^{j\theta} = v.\exp(j(\sum_{i=0}^{M-1} \sigma_i.\alpha_i)) = v.(\prod_{i=0}^{M-1} e^{j\delta_i \alpha_i}) \\
e^{j\delta_i \alpha_i} &= \cos(\delta_i.\alpha_i) + j\sin(\delta_i.\alpha_i) \\
&= \cos(\delta_i.\alpha_i).(1 + j\tan(\delta_i.\alpha_i)) \\
&= \cos(\delta_i.\alpha_i).(1 + j\delta_i.2^{-i}) \\
&= \cos(\alpha_i).(1 + j\delta_i.2^{-i})
\end{aligned} \tag{20}$$

Therefore,

$$\begin{aligned}
\hat{\mathbf{v}} &= v.(\prod_{i=0}^{M-1} \cos(\alpha_i).(1 + j\delta_i.2^{-i})) \\
&= v.(\prod_{i=0}^{M-1} \cos(\alpha_i)).(\prod_{i=0}^{M-1} (1 + j\delta_i.2^{-i})) \\
\hat{\mathbf{v}} &= v.K_i.(\prod_{i=0}^{M-1} (1 + j\delta_i.2^{-i}))
\end{aligned} \tag{21}$$

where $K_i = \cos\left(\arctan 2^{-i}\right) = \sqrt{(1+2^{-2i})}$ is known as gain factor for each iteration.

In iterative terms, the signal can be represented with known number of iterations, the equation can be given by:

$$v_{i+1} = v_i.K_i.(1 + j\delta_i.2^{-i}) \tag{22}$$

The complex signal can be represented as:

$$\begin{aligned}
x_{i+1} + jy_{i+1} &= K_i.(x_i + jy_i).(1 + j\delta_i.2^{-i}) \\
&= K_i[(x_i - y_i.\delta_i.2^{-i}) + j(y_i + x_i.\delta_i.2^{-i})] \ .
\end{aligned} \tag{23}$$

The simplified iterative CORDIC algorithm can be shown as follows.

$$\begin{aligned}
x_{i+1} &= K_i\left(x_i - y_i \delta_i 2^{-i}\right), \\
y_{i+1} &= K_i\left(y_i + x_i \delta_i 2^{-i}\right)
\end{aligned} \tag{24}$$

The elementary functions sine and cosine can be computed using the rotation mode of the CORDIC algorithm if the initial vector starts at $(|K|,0)$ with unit length. The final outputs of the CORDIC for the given input values $x_0 = 1, y_0 = 0$ and $z_0 = \theta$ are as follows:

$$x_f = K\cos\theta \ , \ y_f = K\sin\theta \text{ and } z_f = 0 \ . \tag{25}$$

Since the scale factor is constant for a given number of rotations, $x_0 = 1/K$ can be set to get purely $\sin\theta$ and $\cos\theta$ values.

In this CORDIC architecture, a number of identical rotational modules have been incorporated and each module is responsible for one elementary rotation. Because of identical CORDIC iterations, it is convenient to map them into pipelined architecture. The purpose of pipelined implementation is to design a minimum critical path. Therefore, this kind of architecture provides better throughput and lesser latency compared to other designs. It is associated with a number of stages of CORDIC Units where each of the pipelined stages consists of a basic CORDIC engine. The CORDIC engines are cascaded through intermediate latches as shown in Fig. 2. The shift operations are hardwired using permanent oblique bus connections to perform multiplications by 2^{-i} reducing a large silicon area as required by barrel shifters. The critical-path of the pipelined CORDIC is the time required by the Add/Substract operations in each of the stages.

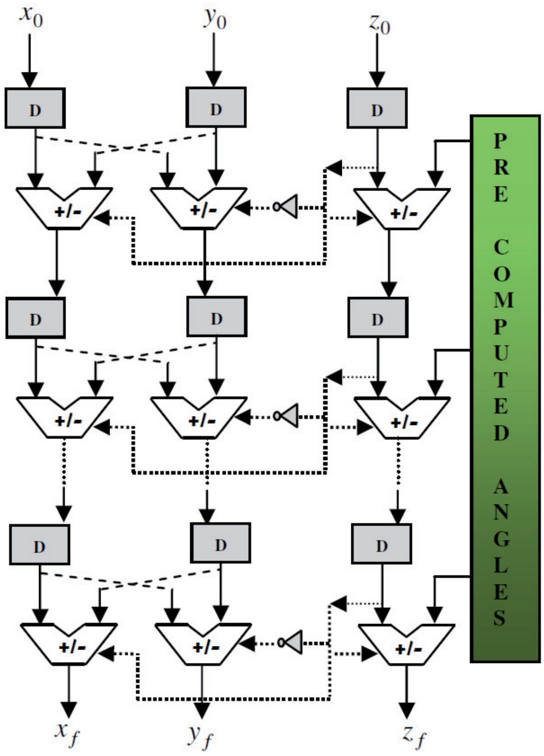

Fig. 2. CORDIC Processing element.

The pre-computed angles of the i-th iteration angle α_i required at each CORDIC engine can be stored at a ROM memory location, are known. Therefore, the need of multiplexing and sign detection is avoided to reduce critical path. The latency of computation thus depends primarily on the adder used. Since no sign detection is needed to force $z_f = 0$, the carry save adders are well suited in this architecture. The use of these adders reduces the stage delay significantly. The delay can be adjusted by using proper bit-length in the shift register. With the pipelining architecture, the propagation delay of the multiplier is the total delay of a single adder. So ultimately the throughput of the architecture is increased to a many fold as the throughput is given by: "1/delay due to a single adder". It implies that speed up factor becomes more than M and latency of the design is M times of the delay of a single adder. It is obvious that if we increase the number of iterations then the latency of the design also will increase significantly. If an iterative implementation of the CORDIC were used, the processor would take several clock cycles to give output for a given input. But in the pipelined architecture, it converts iterations into pipeline phases. Therefore, an output is obtained at every clock cycle after pipeline stage propagation. Each pipeline stage takes exactly one clock cycle to pass one output. (Pipelined stages outputs are shown in Fig. 3.)

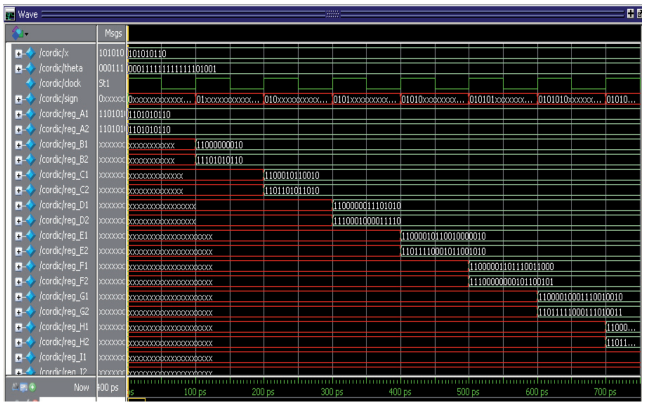

Fig. 3. Simulated output at pipelined stages.

Theoretically, CORDIC realization has infinite number of iterations and that leads to accurate result. But practically CORDIC realization uses finite number of iterations resulting in approximation error. This kind of error arises due to approximations in angle as well as finite word length. The total approximation error will follow an inequality for a particular bit length. We can get a consolidated truncation error due to finite wordlength using scale factor K and number of finite iterations M.

$$K * \sqrt{2} * 2^{-b}\left(1+\sum_{j=0}^{M-1}\left(\prod_{i=j}^{M-1}\sqrt{(1+2^{-2i})}\right)\right). \tag{26}$$

The scaling operation also introduces some error which amounts to maximum of 2^{-b}. If we use 12 bit in the fractional part of wordlength, then total quantization error becomes:

$$\leq \frac{1}{2^{M-1}} * |v^*| + K * \sqrt{2} * 2^{-b}\left(1+\sum_{j=0}^{M-1}\prod_{i=j}^{M-1}\sqrt{(1+2^{-2i})}\right) + 2^{-b} \leq 2^{-12} \tag{27}$$

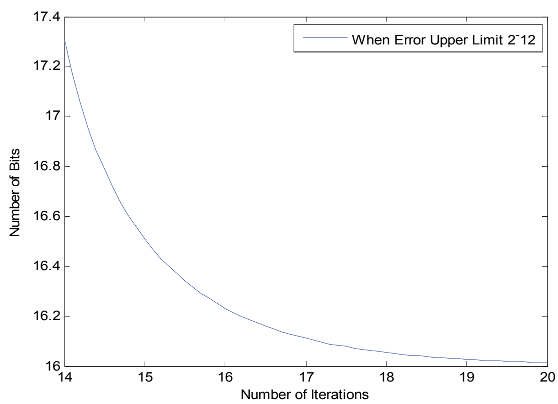

Fig. 4. Optimization of required number of bits for CORDIC Processor.

The above inequality is simulated in MATLAB to find out fractional bits of the internal word length of the CORDIC as shown in Fig. 4.

5. Reformulation of LMS Algorithm for CORDIC Based Design

The LMS algorithm is one of the simplest well known adaptive algorithms. The evolution of reformulated Trigonometric LMS algorithm for adaptive FIR filter design has been explained briefly. Let the linear discrete time FIR filter has input signal of $x(n)$ and the desired output sequence of $d(n)$ with zero mean values. Again, $e(n)$ is the error between the desired output and the estimated output as shown in Fig. 5.

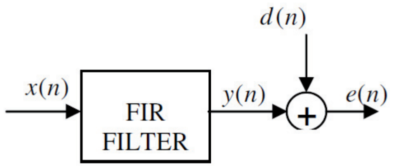

Fig. 5. Basic FIR filter.

Let w be the filter weights coefficient vector for an N tap filter, then,

$$\mathbf{w} = \left[w_0, w_1, \ldots\ldots, w_{N-1}\right]^T \tag{28}$$

and $\mathbf{x}(n) = \left[x(n), x(n-1), \ldots\ldots, x(n-N+1)\right]^T$.

Since, the correlation between any two samples at k distance apart is constant i.e. independent of n.

$$E[x(n)x(n-k)] = r(k) \tag{29}$$

Let $\mathbf{R}$ be a symmetric $N \times N$ Toeplitz matrix.

$$\mathbf{R} = \begin{bmatrix} r(0) & r(1) & r(2) & \bullet\bullet\bullet & r(N-1) \\ r(1) & r(0) & r(1) & \bullet\bullet\bullet & \bullet \\ r(2) & r(1) & r(0) & \bullet\bullet\bullet & \bullet \\ \bullet & r(2) & r(1) & \bullet\bullet\bullet & \bullet \\ \bullet & \bullet & r(2) & \bullet\bullet\bullet & \bullet \\ r(N-2) & \bullet & \bullet & \bullet\bullet\bullet & \bullet \\ r(N-1) & r(N-2) & r(N-3) & \bullet\bullet\bullet & r(0) \end{bmatrix} \tag{30}$$

and
$$\mathbf{R} = E[\mathbf{x}(n)\mathbf{x}^T(n)]. \tag{31}$$

The mean square error (MSE) can be written as:

$$\text{MSE} = \varepsilon = E[e^2(n)] = E[e(n)e^T(n)] \tag{32}$$

where,
$$e(n) = d(n) - y(n) \tag{33}$$

$$\begin{aligned}\varepsilon &= E[(d(n)-y(n))^T \cdot (d(n)-y(n))] \\ &= E[d^2(n)] - 2E[\mathbf{w}^T \cdot \mathbf{x}(n)d(n)] + E[\mathbf{w}^T \cdot \mathbf{x}(n) \cdot \mathbf{x}^T(n) \cdot \mathbf{w}(n)].\end{aligned} \tag{34}$$

The estimated output, $y(n) = \mathbf{x}^T(n) \cdot \mathbf{w}(n) = \mathbf{w}^T(n) \cdot \mathbf{x}(n)$, can be substituted in the above equation to get the following:

$$\varepsilon = \sigma_d^2 - 2\mathbf{w}^T E[\mathbf{x}(n)d(n)] + \mathbf{w}^T E[\mathbf{x}(n)\mathbf{x}^T(n)]\mathbf{w}. \tag{35}$$

Let $\mathbf{p} = E[\mathbf{x}(n)d(n)]$ which is independent of n due to stationarity.

Then,
$$\varepsilon = \sigma_d^2 - 2\mathbf{w}^T\mathbf{p} + \mathbf{w}^T\mathbf{R}\mathbf{w}. \tag{36}$$

The above expression is a quadratic expression in **w**. It is obvious that the above expression will have minima. It is obvious that the weights resulting from the minima will be the weights of an optimal FIR filter. Let us consider

$$\nabla_w \varepsilon = \begin{bmatrix} \dfrac{\partial \varepsilon}{\partial w_0} \\ \dfrac{\partial \varepsilon}{\partial w_1} \\ \bullet \\ \bullet \\ \dfrac{\partial \varepsilon}{\partial w_{N-1}} \end{bmatrix} \tag{37}$$

Solving the equation, $\nabla_w \varepsilon = 0$, we get the optimal weights of an FIR filter.

The expression for ε has three terms. The gradient of the first term with respect to w is zero, i.e.

$$\nabla_w \sigma_d^2 = 0. \tag{38}$$

Let us evaluate the gradient of the second term:

$$\nabla_w[-2\mathbf{w}^T.\mathbf{p}] = -2\begin{bmatrix} \dfrac{\partial[\mathbf{w}^T.\mathbf{p}]}{\partial w_0} \\ \dfrac{\partial[\mathbf{w}^T.\mathbf{p}]}{\partial w_1} \\ \bullet \\ \bullet \\ \dfrac{\partial[\mathbf{w}^T.\mathbf{p}]}{\partial w_{N-1}} \end{bmatrix} \tag{39}$$

It is obvious that $\mathbf{p}$ is a $N \times 1$ matrix. The term $\mathbf{w}^T\mathbf{p}$ can be expanded to get the following simple equations: $\mathbf{w}^T \cdot \mathbf{p} = w_0p_0 + w_1p_1 + \cdots\cdots + w_{N-1}p_{N-1}$

Therefore,
$$\nabla_w[-2\mathbf{w}^T.\mathbf{p}] = -2\begin{bmatrix} p_0 \\ p_1 \\ \bullet \\ \bullet \\ p_{N-1} \end{bmatrix} = -2\mathbf{p}. \tag{40}$$

Let us evaluate the gradient of the third term:

$$\nabla_w[\mathbf{w}^T\mathbf{R}\mathbf{w}] = \nabla_w[\sum_{i-0}^{N-1} w_i(\mathbf{R}\mathbf{w})_i] \tag{41}$$

where $(\mathbf{R}\mathbf{w})_i$ is the i-th element of the column matrix $\mathbf{R}\mathbf{w}$. Evaluating gradient with respect to one particular weight, say w_k.

$$\frac{\partial}{\partial w_k}[\sum_{i-0}^{N-1} w_i(\mathbf{R}\mathbf{w})_i] = \frac{\partial}{\partial w_k}[\sum_{\substack{i-0 \\ i \neq k}}^{N-1} w_i(\mathbf{R}\mathbf{w})_i + w_k(\mathbf{R}\mathbf{w})_k]\cdot \tag{42}$$

Substituting $(\mathbf{Rw})_i = \sum_{j=0}^{N-1} R_{ij} w_j$ in the above expression:

$$\frac{\partial}{\partial w_k}[\sum_{i-0}^{N-1} w_i(\mathbf{Rw})_i] = \frac{\partial}{\partial w_k}[\sum_{\substack{i-0\\i\neq k}}^{N-1} w_i \sum_{j=0}^{N-1} R_{ij} w_j + w_k \sum_{j=0}^{N-1} R_{kj} w_j]$$

$$= \frac{\partial}{\partial w_k}[\sum_{\substack{i-0\\i\neq k}}^{N-1} w_i \sum_{\substack{j=0\\J\neq K}}^{N-1} R_{ij} w_j + \sum_{\substack{i=0\\i\neq k}}^{N-1} w_i R_{ik} w_k + w_k \sum_{\substack{j=0\\j\neq k}}^{N-1} R_{kj} w_j + w_k R_{kk} w_k]$$

$$= \sum_{\substack{i=0\\i\neq k}}^{N-1} w_i R_{ik} + \sum_{\substack{j=0\\j\neq k}}^{N-1} R_{kj} w_j + 2 w_k R_{kk}$$

$$= \sum_{i=0}^{N-1} w_i R_{ik} + \sum_{j=0}^{N-1} R_{kj} w_j \tag{43}$$

Since **R** is a symmetric matrix, $R_{ij} = R_{ji}$,

$$\frac{\partial}{\partial w_k}[\sum_{i-0}^{N-1} w_i(\mathbf{Rw})_i] = 2\sum_{i=0}^{N-1} R_{ik} w_i \,. \tag{44}$$

From the above equation,

$$\nabla_w[\mathbf{w}^T\mathbf{Rw}] = 2\mathbf{Rw}\,. \tag{45}$$

From (38) to (45), we get

$$\nabla_w \varepsilon = -2\mathbf{p} + 2\mathbf{Rw}\,. \tag{46}$$

The minimum mean square error will give the optimum filter weight. Using the steepest descent search algorithm, we will get the optimum filter weight from the equation $\nabla_w \varepsilon = 0$

$$\mathbf{w}_{opt} = \mathbf{R}^{-1}\mathbf{p}\,. \tag{47}$$

But it is not always easy to find out optimal weight by direct solution of the above equation. Finding a solution of $\mathbf{R}^{-1}$is not an easy task for ever varying environment. Instead of direct computing w_{opt}, we usually follow an iterative method to find out w_{opt}. This iterative method is called steepest descent search procedure. The expression for the steepest descent search procedure in the i-th iteration is:

$$\mathbf{w}(i+1) = \mathbf{w}(i) - \frac{\mu}{2} \times \nabla_w \varepsilon \;\Big|_{w=w(i)} \tag{48}$$

where
$$\nabla_w \varepsilon = -2\mathbf{p} + 2\mathbf{Rw} \tag{49}$$

The term in (48), $\frac{\mu}{2} \times \nabla_w \varepsilon \;\Big|_{w=w(i)}$, is a correction or update term. It is very important to choose an optimum value of μ for optimal convergence. The too much smaller value of it will decrease the rate of convergence. On the other hand, very high value of μ may produce oscillation or system response may divulge out and never converges.

Now, we change the iterative process in time format or the update should occur with change in clock cycles for practical implementation of the adaptive system. We replace i term by n term to compute weight update in real-time situation. Then we get the updated equation as:

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{\mu}{2} \times [-2(\mathbf{p} - \mathbf{Rw})]\,. \tag{50}$$

Replacing **R** and **p** by $x(n)x^T(n)$ and $x(n)d(n)$ respectively, we get the LMS algorithm in the form of:

$$\begin{aligned} w(n+1) &= w(n) + \mu[x(n)d(n) - x(n)x^T(n)w(n)]\,. \\ &= w(n) + \mu x(n)\,[d(n) - x^T(n)w(n)] \\ &= w(n) + \mu x(n)\,[d(n) - x(n)w^T(n)] \\ &= w(n) + \mu x(n)\,[d(n) - y(n)] \\ &= w(n) + \mu x(n)e(n) \end{aligned} \tag{51}$$

where μ is called the step size. The above LMS algorithm cannot be used directly in practice as signals are delayed by D-amounts in the filter taps. Therefore, delayed LMS is can be extracted from the original LMS algorithm by introducing delay. Let the gradient is evaluated at some previous iteration, say L iterations, before present iteration. Then the update equation of delayed LMS algorithm can be expressed as:

$$w(n+1) = w(n) + \mu x(n-L)e(n-L)\,. \tag{52}$$

In our design, CORDIC has been used and naturally it demands weight equation in terms of angular form. We can follow in the following way to get the equivalent weights in terms of angle updates. Let the tap weight w_k be

$$w_k = A_k \sin\theta_k \tag{53}$$

so that that each w_k satisfies , $-A_k \le w_k \le +A_k$, and maps uniquely to a θ_k in the interval $\left[-\frac{\pi}{2}, +\frac{\pi}{2}\right]$. The function $\sin\theta_k$ is a monotonically increasing, continuous function of θ_k , as $-\frac{\pi}{2} \le \theta_k \le +\frac{\pi}{2}$, which means $\frac{\partial \varepsilon}{\partial \theta_k}$ has the same sign as that of $\frac{\partial \varepsilon}{\partial w_k}$ everywhere within the hypercube.

$$\frac{\partial \varepsilon}{\partial \theta_k} = \frac{\partial \varepsilon}{\partial w_k} \,.\, A_k \cos\theta_k \tag{54}$$

Therefore we can write an equivalent equation as (49),

$$\nabla_\theta \varepsilon = -2\mathbf{p} + 2\mathbf{Rw}\,. \tag{55}$$

The angle update equation can be written as following:

$$\theta(i+1) = \theta(i) - \frac{\mu}{2} \times \nabla_\theta \varepsilon \;\Big|_{\theta=\theta(i)} \tag{56}$$

Then recursively updated LMS algorithm can be uniquely mapped into the trigonometric form of the following algorithm as :

$$\theta(n+1) = \theta(n) + \mu\Delta(n)x(n)e(n) \tag{57}$$

$$e(n) = d(n) - \sum_{k=0}^{N-1} A_k \sin\theta_k(n)\, x(n-k).$$

The above form of LMS algorithm is updated in terms of angles and we know that CORDIC based realization is possible as sine and cosine terms used in filtering as well as update of coefficient can be computed by using a CORDIC processor for implementation of FIR filter.

5.1 Design of Pipelined Adaptive FIR Filter

In this paper, a bank of filters has been used within a required bandwidth of Doppler shift to detect target signal and all the filters are designed in such a way that clutters are cancelled adaptively. After deciding response frequency for each filter, we are going to introduce adaptive criteria within the filters. For that a reformulated LMS algorithm in association with CORDIC processing unit has been chosen for better implementation and stability. The main advantage of CORDIC compared to MAC based design is the former is sufficiently robust against internal numerical errors. Since the feedback circuits are extremely sensitive to finite wordlength errors, it would be almost error free to use CORDIC based design. We have shown (Fig. 4) that the upper limit of total quantization error for 12 bit CORDIC is in the magnitude of 2^{-12} which is negligible.

We have seen that in circular rotation mode, CORDIC unit gives sinθ and cosθ as an output against the input signal bit stream. In the adaptive FIR filter architecture, the modified LMS algorithm has been implemented using pipelined multiplier to reduce delay in filtering and weight update (Fig. 7). It is seen in the previous section that the convergence rate depends on micro-rotations used in the design. The combination of CORDIC processor and reformulated LMS algorithm expedites the convergence resulting faster adaptation in reality. This makes it a better choice in real time signal processing as is required in modern radar. A MATLAB simulated result in Fig. 6 shows that convergence rate is faster in CORDIC and LMS combination than the simple LMS algorithm based design.

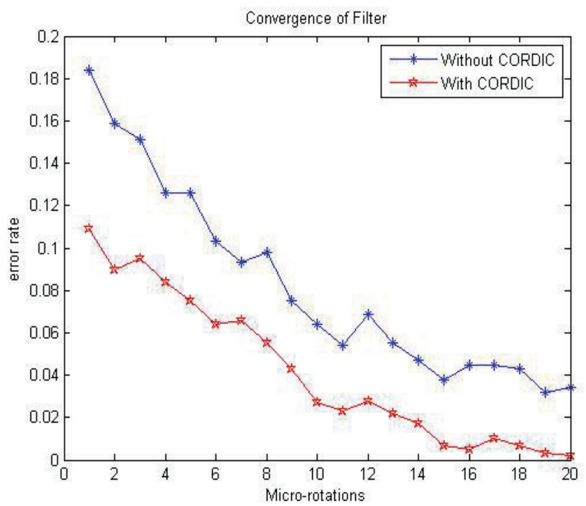

Fig. 6. Convergence of filter output with/without CORDIC.

The performance of many computational problems is dominated by speed at which a multiplication is performed. The multiplier has two parts. One is for generating partial product and another for adding the partial product. With the pipelining technique, the propagation delay of the multiplier becomes the total delay of a single adder. That is why overall speed of signal processing is increased. The responses of the filters can be changed using a different set of weighting coefficients in Weight Selection Map.

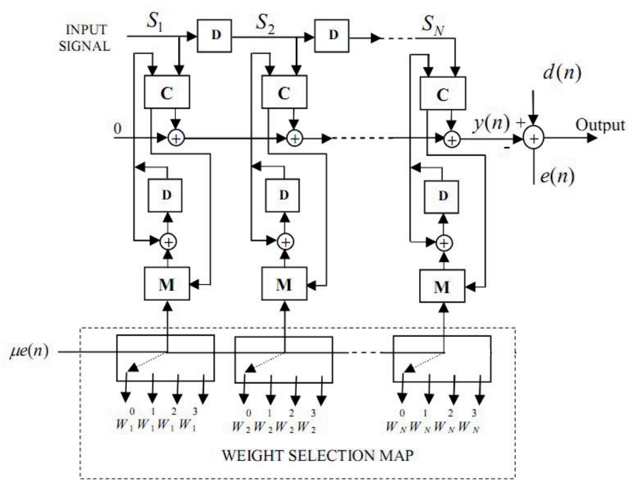

Fig. 7. Adaptive FIR filter using modified LMS algorithm.

One reset clock is activated to initialize the system. Initially, the error at the output of the final stage of the filter is maximum. An automatic gate generator can be incorporated for the selection of weight bank as per the intensity of clutter. Once weight bank is selected, the error is fed back to the adaptive filter in association with the selected weights. The internal clock is adjusted in such a way that every sweep of signal is processed within single system clock. That means, the internal clock frequency is much higher than the system clock. The architecture shown in Fig. 8 comprises a single CORDIC element and a pipelined multiplier (PM) which is driven by a primary clock and it is N times faster than the input. Three dedicated FIFOs (F_1, F_2, F_3) have been utilized to maintain required sequence of sampled input data, to keep intermediate results and tap weights. The n number of input data samples are loaded and circulated in F_1 at primary clock rate. At every $(N+1)$-th clock cycle, a new set of input data samples is loaded by flipping a MUX. The sinusoidal output produced by CORDIC unit is fed to the accumulator which is shared by two registers namely R_1 and R_2. The accumulator takes N primary clock cycles to update output terms. Both the registers share the same input. But R_2 updates at sample frequency of f_s and at the same time partially accumulated products are continuously circulated within the accumulator register R_1 at a clock frequency of Nf_s. One 'Clr' control has been introduced to clear the accumulator to start computation for next output. The Cosine output of CORDIC is simultaneously used in weight update in association with pipelined multiplier (PM) unit to compute the product terms used in (57). A FIFO, F_2, of length $N+1$ has been introduced for accurate synchronization of error term $e(.)$ and product term $\Delta(.)x(.)$ at the input of PM. The internal architecture has been shown in Fig. 8.

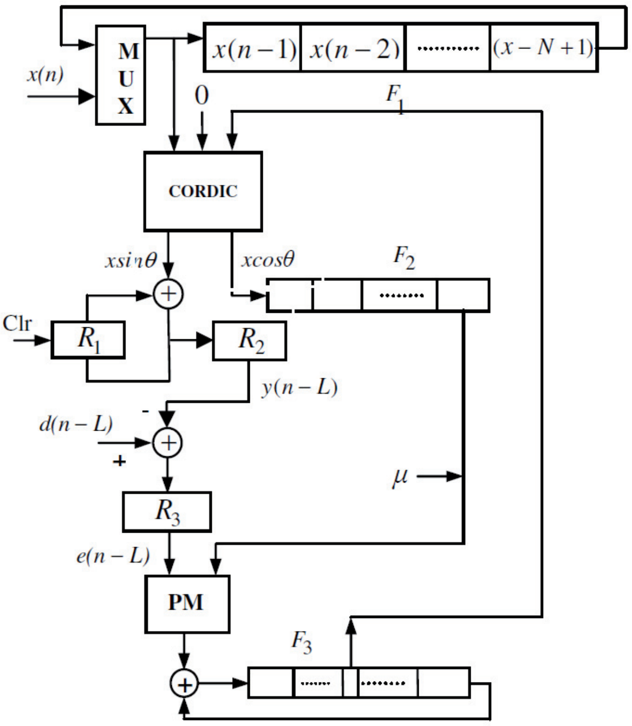

Fig. 8. Internal design of FIR adaptive filter.

Though the CORDIC element computes the sinusoidal and cosine terms concurrently, it takes N+1 extra clock cycles to compute error term. That is why after every N clock cycle, one don't care state has been incorporated at the CORDIC pipe to synchronize output. In the filtering section, feedback error signal continuously updates incoming signals with previous reference signals and at the same time static returns or extremely slow moving target returns are cancelled.

6. Simulation, Analysis and Discussion

To introduce frequency agility in modern surveillance radar, FIR filter based MTD design is an ultimate solution. Design of MTD using IIR filter is impractical as number of pulses varies in each CPI. The designer of MTD has to be well aware of the side lobe requirements for suppression of specific clutter in order to achieve good performance. Many researchers have pointed out that static clutter normally contribute 50-60 dB in the received signals. Keeping in view the requirements for a ground based surveillance radar, we have designed a bank of 8-tap filter to achieve the following pre requisites:

(a) A response of -70 dB in clutter rejection notch.

(b) A response of -50 dB for Chaff rejection.

(c) Two filters (F_0^+, F_0^-) are to respond at zero Doppler velocity and other six to reject fixed clutter.

The banks of filters are designed in such a fashion that combined response of all the filters covers the PRF of the radar. In our radar, PRF is 1000 Hz. An adaptive filtration is required since reception from the nearby area as well as from very slow moving object is strong enough. Therefore, after determination of required response of each filter, a feedback path is incorporated in each filter. As per the value of Doppler shift of the return echo, respective filter will adaptively process the signal to identify target by suppressing clutter to a significant amount. In the filter banks, a predefined variation of response as per the selection of 'Weight Selection MAP' has been introduced so that responses of the filters are aligned according to the strength of clutters

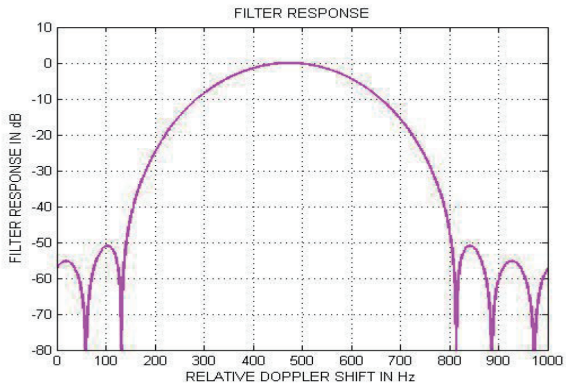

Fig. 9. Relative Doppler frequency response of filter at PRF/2.

Fig. 9. shows that targets are detected when they fall in the middle of the filter response curve. Whereas, the steep notch at nearby zero velocity regions provides very good clutter rejection. That means the filter rejects to all those targets whose Doppler shifts are very near to zero Doppler.

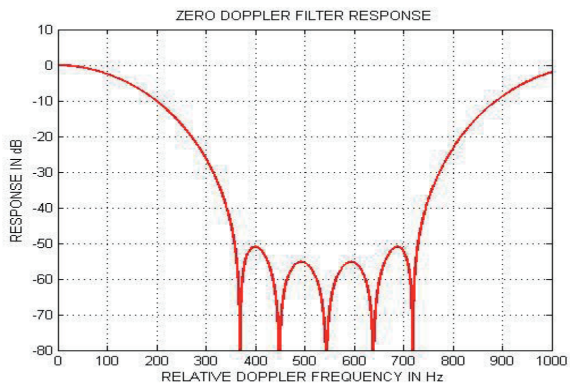

Fig. 10. Response of Zero Velocity Filters (F_{0^+}, F_{0^-})

The F_0^+ filter has been designed to respond at near zero Doppler frequency shift of the target. F_0^- is nothing but the mirror image of F_0^+ as shown in Fig. 10. The mirror image of the original fitter has filter coefficient just complex conjugate of the original. Since vegetation, slow moving objects etc. will be filtered out avoiding to represent themselves on the radar monitor.

A composite response of the filter bank at Nil clutter has been shown in Fig. 11. The output response curve shows that all filter peaks are evenly distributed. During design of these filters, a special attenuation has been given for trading straddling loss at crossover frequencies. Since the MTD here uses DFT, it allows the centre frequency of the filter to move as per clutter strength. When the clutter is very strong, the envelope of the clutter spectrum may touch two or more filter response curve. The response curve of the clutter filter will be automatically widened enough and centre frequency of other filter will be moved away to keep themselves free from clutter. We have considered four cases of response optimization. At Nil clutter condition, the composite response will be affected with the system noise. The responses are shown in Fig. 11, 12, 13 and 14 as per strength of clutters.

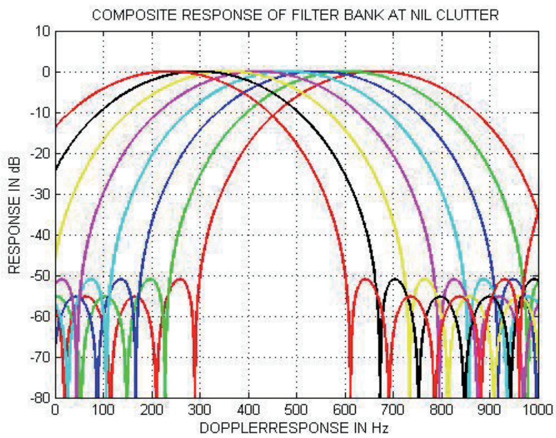

Fig. 11. Composite response of filter banks at Nil clutter.

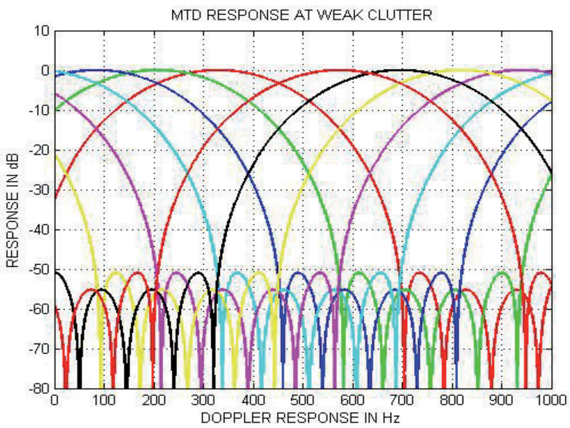

Fig. 12. Composite response of filter banks at weak clutter.

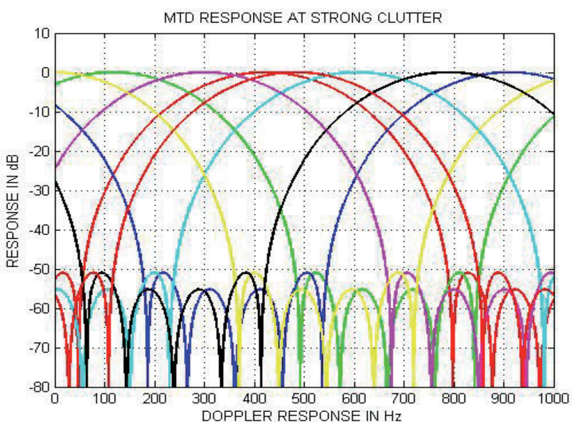

Fig. 13. Composite response of filter banks at strong clutter.

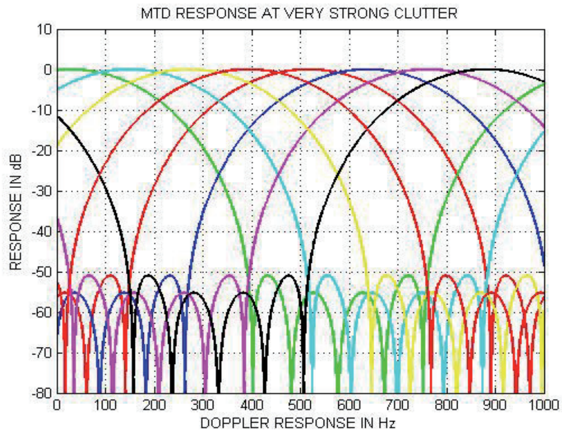

Fig. 14. Composite response of filter banks at very strong clutter.

Now we change our focus towards adaptive filter which uses reformulated LMS algorithm for its adaptation. It is well known that pipelined CORDIC itself has got good convergence property which has been efficiently used in this application. Exact convergence analysis of proposed architecture using trigonometric LMS algorithm is not possible due to presence of nonlinearity of the filter as well as angle update equation. Using the angle iteration error in the given LMS algorithm, approximate convergence studies have been carried out. The rate of convergence has already been shown in Fig. 6. A 9-tap adaptive filter with centre placed at the 5-th tap has been taken for simulation studies. The step size which is the guiding force for fast or slow conversion of the system is appropriately taken care. A MATLAB simulated outcome in terms of learning curves of the design at $\mu = 0.0045$ and $\mu = 0.002$ have been shown in Fig. 15 and Fig. 16 respectively. At $\mu = 0.0045$, the architecture gives its optimum output. At larger value of step size stability of the system reduces rapidly. Again at less than $\mu = 0.002$, the convergence rate becomes very low.

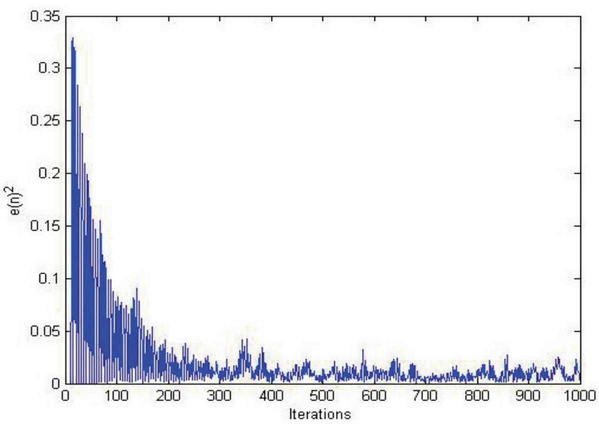

Fig. 15. Learning curve of the design at $\mu = 0.0045$.

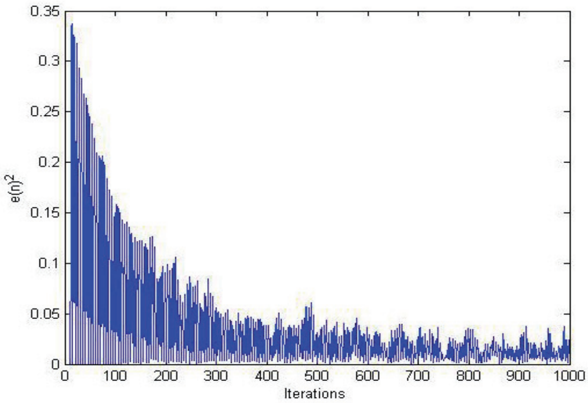

Fig. 16. Learning curve of the design at $\mu = 0.002$.

7. Conclusions

In this paper, we present a simple and efficient technique for adaptive moving target detector using a transversal filter. The implementation based on reformulated LMS algorithm using CORDIC processing element to enhance adaptive filtering capability during radar target detection in an ever changing environment. The proposed design is highly effective even in very strong clutter conditions. Since it is very difficult to extract target embedded with non-homogeneous clutters, the optimum filter has been modified by introducing so called LMS algorithm after its reformulation. The CORDIC has been used as a central processing element for the filter architecture. Due to the inherent convergence property of CORDIC along with pipelined architecture, the system converges faster. The MATLAB simulation results for adaptive MTD shows that the proposed design works satisfactorily in various clutter environments.

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About Authors ...

Amritakar MANDAL was born in Krishnanagar, West Bengal, India. He is currently working toward the Ph.D degree in Electronics and Communication Engineering in the School of Information and Communication Technology, Gautam Buddha University, India. He has vast experience in electronic warfare and surveillance radar systems. His research interests include radar signal processing and high speed VLSI design for communication systems.

Rajesh MISHRA is currently working as an Assistant Professor in the School of Information and Communication Technology, Gautam Buddha University Greater Noida (India). He received his BE (Electronics Eng.), M. Tech., and Ph. D. degree (Reliability Eng.) from the Reliability Engineering Centre, IIT Kharagpur (India) in year 2000, 2004, and 2009 respectively. He has worked in the area of reliability engineering, layout design for capacitated networks, and network optimization. His current interests and activities are in telecommunication circuit design and bio-inspired VLSI vision processing systems. He has published papers in several international journals such as IJPE, IEEE, RESS, QTQM etc. He has been involved as a convener of various faculty development programmes, workshops and international conferences.