Continuous Wavelet Transform and Hidden Markov Model Based Target Detection

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Abstract. *Standard tracking filters perform target detection process by comparing the sensor output signal with a predefined threshold. However, selecting the detection threshold is of great importance and a wrongly selected threshold causes two major problems. The first problem occurs when the selected threshold is too low which results in increased false alarm rate. The second problem arises when the selected threshold is too high resulting in missed detection. Track-before-detect (TBD) techniques eliminate the need for a detection threshold and provide detecting and tracking targets with lower signal-to-noise ratios than standard methods. Although TBD techniques eliminate the need for detection threshold at sensor's signal processing stage, they often use tuning thresholds at the output of the filtering stage. This paper presents a Continuous Wavelet Transform (CWT) and Hidden Markov Model (HMM) based target detection method for employing with TBD techniques which does not employ any thresholding.*

Keywords

HMM, target detection, track-before-detect, continuous wavelet transform, sea clutter.

1. Introduction

In standard tracking filters, target detection is performed by comparing the sensor output signal with a predefined threshold. However, using a threshold brings out two major problems. The first problem occurs when the threshold is selected very low which increases the false alarm rate. This implies that measurements not originating from targets of interest which exceed the threshold are detected as target. The second problem arises when the selected threshold is very high which increases the track loss rate. This means, targets with low signal-to-noise ratio (SNR) may not be detected. Track-before-detect (TBD) techniques eliminate the need for a detection threshold and help detecting and consequently tracking targets with lower signal-to-noise ratios.

Numerous studies can be found on TBD techniques in the literature, but Dynamic Programming (DP) and particle filter based implementations are the most common and well-known techniques. In [1]-[3], DP based TBD methods were proposed for detecting and tracking low SNR targets. The analysis in [1] showed that the tracking performance of the DP algorithm is poor, even though detection performance is good and track separation phenomenon is one of the factors that deteriorate the tracking performance. An alternative approach is the Particle Filtering (PF), which has been used extensively for TBD [4]-[7]. It is a numerical approximation technique that uses randomly placed samples to solve the non-linear function of the target state, which describes target's kinematic evolution. However it was found that, performance of the PF based detection algorithm depends on correctness of the particle weights which are calculated using system dynamics which were assumed to be known [8].

Moreover, although TBD techniques mentioned above eliminate the need for detection threshold at sensor's signal processing stage, they often use tuning thresholds at the output of the filtering stage. It is the motivation in this work to propose a novel method for TBD that avoids employing any thresholding at any stage.

Detecting surface targets in the presence of sea clutter is an important task for applications like surveillance and navigation. The non-stationary nature of the sea clutter makes the target detection task more difficult for radar applications. Therefore, modeling the sea clutter becomes a critical point. Especially, high resolution radars working with low grazing angles showed that sea clutter contains non-Gaussian nature [9], [10]. There have been some efforts to model the real world sea clutter amplitudes to various statistical distributions. Weibull distribution [11], log-normal distribution [12], K distribution [9], [13] and compound Gaussian distribution [10] are the most well-known distributions among these studies. However, fitting real world sea clutter data to statistical distributions were not successful enough and did not provide adequate help to detect targets in the sea clutter environments [14]. The major reason for this conclusion is that, these algorithms try to model the sea clutter as a stationary process. In reality, sea clutter has a time varying nature and in order to adapt to this nature the methods using statistical distributions need to become more complex.

Continuous Wavelet Transform (CWT) has become a widely used tool in signal processing applications. Unlike

Fourier transform, CWT performs analysis in the time domain and because of this feature it has been a useful tool for applications such as data compression, pattern recognition, radar signal processing, de-noising and image processing. Additionally, the flexibility at selecting a mother wavelet becomes a major advantage, because selecting a proper wavelet for a particular problem increases the processing capability. CWT can be interpreted as 2-D correlator and if characteristics of the signal to be detected are known a-priori, a proper mother wavelet can be created. In that sense, CWT can be considered as a matched filter. Because of this feature, selection of an appropriate mother wavelet CWT becomes an efficient detection tool.

In recent years, Hidden Markov Model (HMM) based methods have become indispensable in applied mathematics and modern pattern recognition. Hidden Markov Models are especially known for their application in temporal pattern recognition such as speech, sound, handwriting and image recognition. Recently, Markov based procedures and algorithms have also been applied successfully in pattern recognition for radar target detection and classification [15]], [[16]. The underlying assumption of the HMM is that the data samples can be well characterized as a parametric random process and the parameters of the stochastic process can be estimated in a well-defined framework.

This paper presents a CWT-HMM based target detection method for employing with TBD techniques. CWT is a useful tool for extracting the required information from noisy data. In the proposed method CWT is used to detect the singularities originated from target in the received radar echo signal. HMM is a powerful statistical method to characterize the observed data samples of a discrete time series and used to separate target related signals from clutter. In order to evaluate the performance of the proposed method, CSIR database [17] was used to detect surface targets from real radar data. For this purpose clutter and target models have been constructed. Pulse integration was used in order to increase signal-to-noise (SNR) ratio. Target detection is performed over 10 coherent processing intervals (CPI). Then, clutter and target models were trained by Baum-Welch algorithm with sufficient amount of observation data. Finally, detection process was performed on radar observations by using Viterbi algorithm.

In Section 2 brief description of the HMM theory and the three basic HMM problems is presented. Section 3 gives a brief description of the CWT. Section 4 describes how solution of the basic HMM problems and the HMM structure can be applied to target detection is introduced. Section 5 gives a summary of the proposed method.

2. Brief Overview of HMM

An HMM consists of a set of N states, each of which is associated with a set of M possible observations. The parameters of the HMM include:

- An initial matrix of state probabilities given by

$$\pi = [p_1, p_2, \cdots, p_N]^T \tag{1}$$

whose elements, p_i $i \in [1, N]$ describe the position distribution probabilities of the target over the initial state set at the beginning $t = 1$.

- A transition matrix defined as

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \tag{2}$$

whose elements a_{ij}; $i, j \in [1, N]$ are the transition probabilities from state i to state j.

- An observation matrix given by

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1M} \\ b_{21} & b_{22} & \cdots & b_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N1} & b_{N2} & \cdots & b_{NM} \end{bmatrix} \tag{3}$$

whose elements b_{im} are the probabilities of observing symbol $m \in [1, M]$ given that the system is at the state $i \in [1, N]$.

The HMM parameter set is denoted by $\lambda = (A, B, \pi)$. The transition probabilities express which type the model is, i.e., ergodic, left-right or coupled. Three basic problems have to be address with the HMM [18]:

- *Evaluation problem*: What is the probability of the observation O, given the model λ, i.e. $P(O|\lambda) = ?$
- *Decoding problem*: What is the most likely state sequence given the observation O, i.e. $arg_s[\max P(O|\lambda)] = ?$
- *Estimation problem*: How can one estimate the parameters given the training observation sequences, $\lambda^* = arg_\lambda[\max P(O|\lambda)] = ?$

3. Brief Overview of CWT

CWT is defined as the convolution product of time series signal $f(t)$, with the scaled and translated versions of wavelet function ψ and integrating over time.

$$C(\sigma,\tau) = \int_{-\infty}^{\infty} \frac{1}{\sigma} \overline{\psi}(\frac{t-\tau}{\sigma}) f(t)dt \tag{4}$$

where σ and τ are scale and translate parameters respectively, $\overline{\psi}$ is the complex conjugate of the wavelet function ψ. The result of the CWT is the numerous number of wavelet coefficients, C, as a function of scale and translation.

In recent years, CWT has been used to analyze radar signals and become an efficient tool for target detection. The main goal of employing CWT for target detection is the presence of singularities caused by the target itself. As these singularities are not visible, they can be detected by analyzing the CWT coefficients which include detailed level information of the received time series signal [19]-[22]. In this paper, complex valued Morlet wavelet is proposed for analyzing the received radar signal for detection of target. In literature, Morlet wavelet has been utilized in sonar and radar applications due its good response to transient complex exponential sinusoids in noise [23].

4. Application of HMM to Target Detection

In the proposed method, the HMM is used to detect targets in the presence of clutter. Let each radar measurements be represented by a sequence of measurement vector or observations O, defined as

$$O = o_1, o_2, ..., o_T . \tag{5}$$

The target detection problem can then be regarded as that of computing

$$\arg\max_i \{P(w_i|O)\} \tag{6}$$

where w_i is the i^{th} detection. In this study, detections are divided into two main classes as clutter and target. If needed, these classes can be divided into subclasses. For example, clutter class can be divided into sea and land clutter subclasses and target class can be divided into constant velocity and coordinated turn subclasses. The probability given in (6) is not computable directly; however, Bayes' Rule yields

$$P(w_i|O) = \frac{P(O|w_i)P(w_i)}{P(O)} . \tag{7}$$

Thus, for a given set of prior probabilities $P(w_i)$, the most probable detection depends only on the likelihood $P(O|w_i)$. If the dimensionality of the observation sequence O considered, the direct estimation of the joint conditional probability $P(o_1, o_2, ...|w_i)$ from examples of radar measurements is not practical. However, if a parametric model of radar measurement production such as a Markov model is assumed, then estimation from data is possible since the problem of estimating the class conditional observation densities $P(O|w_i)$ is replaced by the much simpler problem of estimating the Markov model parameters [24].

In HMM based target detection, it is assumed that the sequences of observed radar measurement vectors corresponding to each detection are generated by a Markov model as shown in Fig. 1. A Markov model is a finite state machine which changes state once every time unit (in this application time unit is a radar scan) and each time t that a state j is entered, a radar measurement vector is generated from the probability density $b_j(o_t)$. Furthermore, the transition from state i to state j is also probabilistic and is represented by the discrete probability a_{ij}. The joint probability that O is generated by the model M moving through the state sequence X is calculated as the product of the transition probabilities and the output probabilities. So, for the state sequence X given in Fig. 1

$$P(O, X|M) = a_{12}b_2(o_1)a_{22}b_2(o_2)a_{23}b_3(o_3)... \tag{8}$$

In practice, only the observation sequence O is known and the underlying state sequence X is hidden.

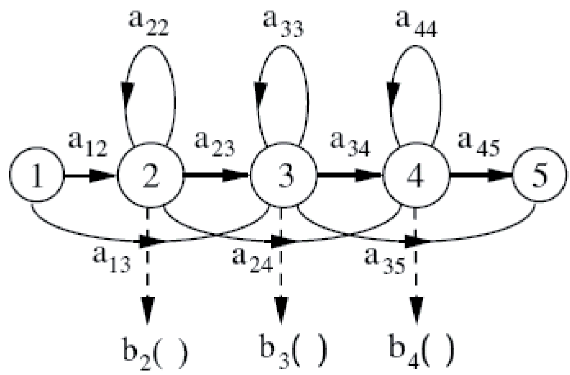

Fig. 1. The Markov generation model.

Given that X is unknown; the required likelihood is computed by taking the sum over all possible state sequences (Bayesian approach) $X = x(1), x(2), x(3), ..., x(T)$, that is

$$P(O|M) = \sum_X a_{x(0)x(1)} \prod_{t=1}^{T} b_{x(t)}(o_t) a_{x(t)x(t+1)} \tag{9}$$

where $x(0)$ is constrained to be the model entry state and $x(T + 1)$ is constrained to be the model exit state.

As an alternative to (9), the likelihood can be approximated by considering the most likely state sequence that is (Viterbi approach)

$$\hat{P}(O|M) = \max_X \left\{ a_{x(0)x(1)} \prod_{t=1}^{T} b_{x(t)}(o_t) a_{x(t)x(t+1)} \right\} . \tag{10}$$

Given a set of models, M_i, corresponding to detections w_i, equation (6) is solved by using (7) and assuming that

$$P(O|w_i) = P(O|M_i) . \tag{11}$$

Given sufficient number of training examples of each detection, a HMM can be constructed which implicitly models all of the many sources of variability inherent in real radar measurements. Fig. 2 summarizes the use of HMMs for target detection. Firstly, a HMM is trained for each detection using a number of examples of that detection. In this case, just two detection models: "clutter" and "target" models are used. Secondly, to detect some unknown radar measurements, the likelihood of each model generating that measurement is calculated and the most likely model identifies the detection.

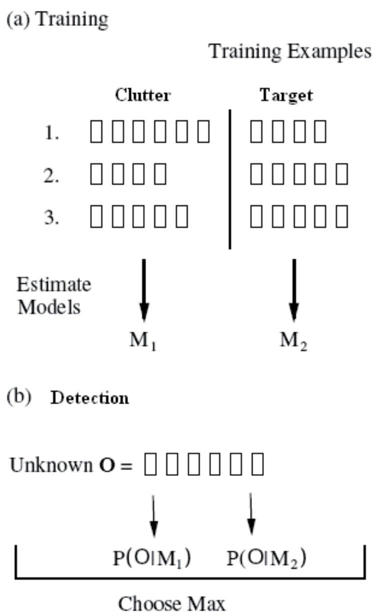

Fig. 2. Using HMMs for target detection.

4.1 CWT-HMM Structure for Detection

The measurements, feature set and HMM structure determine the overall performance of the detection system designed. Efficient way of detecting the target in the clutter environment is to correlate the target radar echo signal with itself and maximize the SNR at the receiver. In that sense, CWT can be considered as correlator or a matched filter bank. If signal to be detected can be known a priori, this signal can be used for modeling a mother wavelet. Using CWT, received radar signal spectrum can be separated into different frequency bands. The goal is, using different filter resolutions for adapting the variability in the received radar signal frequency components. In this paper, complex Morlet mother wavelet was used for CWT analysis. The ability of complex Morlet wavelet for detection of transient sinusoidals in the noisy signal efficiently, gives an opportunity for detection of target signal in the sea clutter environment. Fig. 3 shows an example of received radar signal which includes target and sea clutter measurements whereas Fig. 4 gives percentage of energy for each wavelet coefficient with respect to wavelet scale. As seen from the figure, maximum energy values correspond to the points where the correlation between received radar signals and mother wavelet is maximized. Considering that the mother wavelet is designed according to the target signal to be detected, the maximum energy points are the potential evidence for target presence. However, it has been observed that, determining a single wavelet scale which optimizes the target detection is a difficult task, because the presence of a target in the frequency bands is affected by target's physical characteristics, maneuvers, speed and etc. Fig. 5 shows wavelet scale change during 10 CPI of target maneuver. As it can be seen from the figure, wavelet scales which are in accordance with the target presence change over time. Therefore, a range of wavelet scale was selected for processing. In order to make sure about the target presence at these points, the range and bearing changes corresponding to these points need to be taken into the consideration. The proposed method uses range and bearing values related with the maximum energy points of CWT coefficients along with their delta, acceleration and third differential coefficients as a feature vector. Delta, acceleration and third differential coefficients provide the ability to detect the target maneuver in an efficient way [25] and are calculated through,

$$d_t = \frac{\sum_{\theta=1}^{\Theta} \theta (c_{t+\theta} - c_{t-\theta})}{2\sum_{\theta=1}^{\Theta} \theta^2} \tag{12}$$

where d_t is a delta coefficient at time t and Θ is the window size. The same formula is applied to the delta coefficients to obtain the acceleration coefficients.

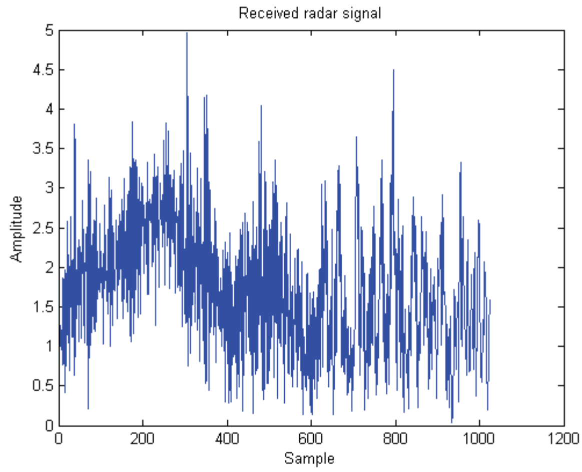

Fig. 3. Received radar signal.

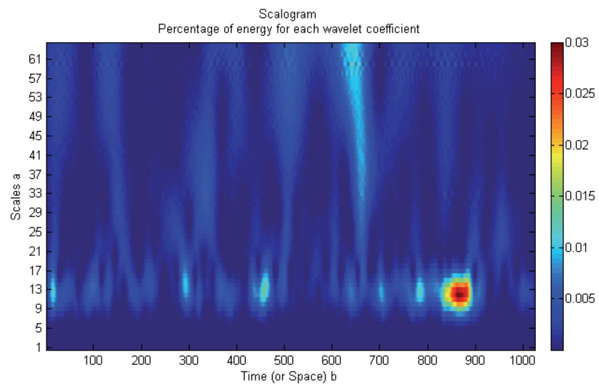

Fig. 4. Percentage of energy for each wavelet coefficient.

4.2 The Database

The database was created by the Council for Scientific and Industrial Research (CSIR) in South Africa, which is one of the leading scientific, and technology research, development and implementation organizations in Africa. One of the key research topics that need to be addressed in developing the database is the detection and tracking of small boats at sea and in the littoral which is important for weapons trafficking, smuggling, poaching, piracy, illegal

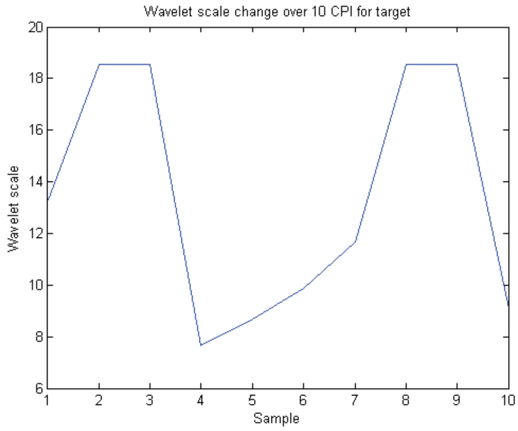

Fig. 5. Wavelet scale change according to target maneuver.

immigration and terrorism. The radar deployed on the measurement trials is an experimental, monopulse, pulsed Doppler X-band radar with an instrumented range of 60 km [17]. During the measurement trials different types of surface vessels were used whose lengths vary between 4.2 m – 10 m. and both sea clutter and boat reflectivity data was recorded with environmental conditions (wind speed, wave height). Boats used during the trials were instrumented with GPS which provides recording the radar data with position information (range and bearing) wrt. to time. In order to test the proposed method, 4.2 m length boat was selected. The environmental conditions of the selected data are defined as wind speed and wave height which are 6.03 Kts and 2.66 m respectively.

4.3 HMM Topology

Each clutter and target detections were modeled by a 12 state HMM. HMM state number was chosen experimentally as a tradeoff between detection performance and computational complexity [24]. State 1 is the entry and state 12 is the exit states of the HMM. States 2 to 10 are the emitting states which correspond to measurements obtained from sequential CPIs. Clutter and target models were trained separately. Feature set consists of range and bearing values along with their delta, acceleration and third differential coefficients which were collected from each CPI. The HMMs are estimated from the training database in an offline training stage. The CSIR database contains raw radar measurements for sea clutter only and target with sea clutter separately. For each HMM 75% of related measurements were used for training and remaining measurements were used for test purposes. The measurements which were used for training were not used for testing. HTK toolkit was used to perform HMM training and testing processes [24].

4.4 Target Detection Process

When target is in the radar illuminated area, there is numerous number of measurements at the radar receiver. The efficient way of increasing SNR at the radar receiver is to add up the received signals reflected from the target. Therefore, coherent pulse integration is used in order to preserve phase information and 0.5 sec. CPI time was selected. Target detection is performed over 10 consecutive CPI. The detection process is as follows:

1. CWT is applied to signals inside the CPI for every range-bearing cell. As the frequency band which the presence of target resides may change according to its maneuver or speed, different frequency bands needed to be considered. For this reason, a range of wavelet scales between 1-64 is selected. In addition, to increase the resolution of the frequency bands, the selected range was divided into 100 equal sub-bands.
2. Potential target range-bearing cells are identified according to the maximum energy level of CWT coefficients.
3. Step 2 is repeated 10 times and resulting range-bearing cell changes were accumulated into a buffer.
4. HMM was used for accumulated range-bearing changes in order to detect a target trajectory.

Detection process starts with forming the observation sequence, that is, by collecting 10 consecutive CPI data. There is no assumption as to which part of the target trajectory is sampled in the observation sequence and the aim of the CWT-HMM detection is to perceive the target trajectory through observation sequences. Detection was performed by applying the Viterbi algorithm on accumulated CPIs to find the most likely model, i.e., whether the measurement has been originated from a target or from clutter, then at the end of the detection process measurements in each CPI is marked as clutter or target.

4.5 Experimental Results

Experiments showed that, in real world conditions the reflected radar signal power from the low observable targets can be below the sea clutter signal level. To visualize this statement, Fig. 6 gives an example detection process which considers that the maximum signal power at the radar receiver is considered as a target. As it can be seen, such a process fails in detecting the target when its signal power is below the clutter level. By looking at Fig. 6, it can also easily be predicted that, conventional algorithms which uses thresholding at the radar receiver results in a high false alarm rate under such conditions. The main idea behind the utilization of CWT is to detect the transient sinusoids from the received radar signal which were caused by the target itself. This approach makes the proposed algorithm independent from the target signal level. The usage of the HMM gives the ability to recognize the target trajectory in the observation data. In this work, the observation data is the range cell of the target related transient sinusoids detected by selecting the coefficients which has maximum energy level after CWT. The main idea behind the TBD approach is to detect the target trajectory, rather than detecting the signal on every time instant. Therefore, detection process is performed on consecutive time intervals.

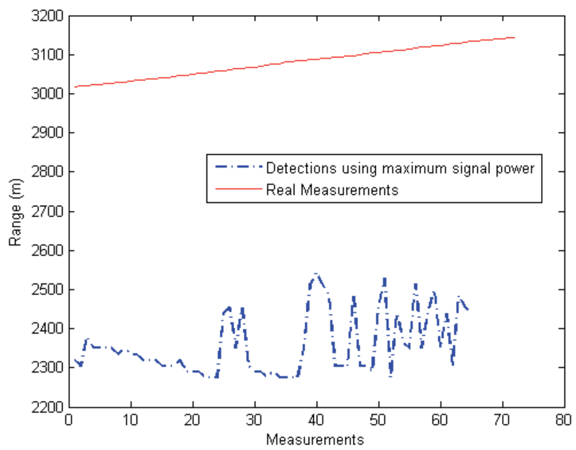

Fig. 6. Detection process which considers that the maximum signal power at the radar receiver is considered as a target.

The obtained results have revealed that the proposed method has a potential to detect surface targets in the presence of sea clutter without using any thresholding. Fig. 7 shows sequences of clutter and target originated measurements in an observation sequence respectively. As it can be seen from Fig. 7, clutter is a noise like signal and has no apparent relation between range samples, whereas measurements from a target, which moves according to a trajectory, display strong relation. Fig. 5 shows the wavelet scale change during the maneuver shown in Fig. 7, the maneuver of a target leads to change in the frequency band where the transient signal resides. As the target platform accelerates or maneuvers the presence of target in the frequency bands change. For example, as shown in Fig. 5 and Fig. 7, during measurement samples between 3-6 target platform changes its speed and course which affects the frequency of the target presence in the received radar echo. Therefore, wavelet scale range 1-64 was divided into 100 parts in order to increase the time-frequency resolution.

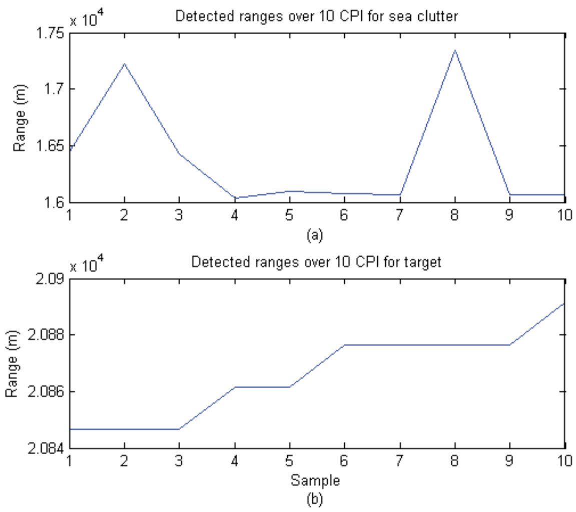

Fig. 7. Example of a (a) clutter (b) target marked frames.

Fig. 8 compares the detection performance with the real target measurements. The proposed method identifies the range cells in which the target resides. As the mother wavelet was modeled to optimize the detection performance, the energy level of CWT coefficients maximizes at the points where target reflected signals reside. Moreover, with the usage of the HMM, the proposed method detects the trajectory of the CWT coefficients and separates clutter from target measurements. Since there is no means to indicate where the target is within the identified cell, as a common practice, the target is assumed to be located in the center of that particular cell. Therefore, CWT-HMM method assumes the target is located in the center of the cell.

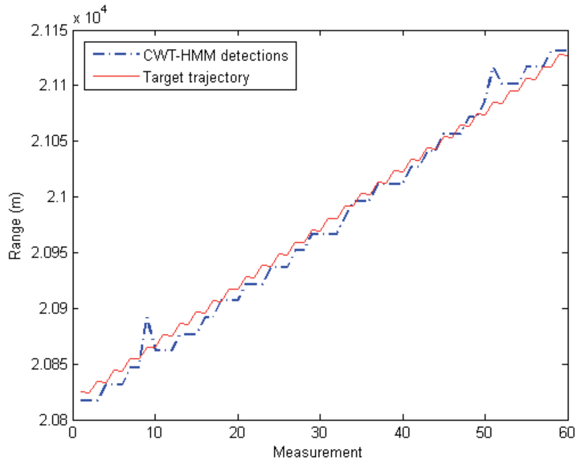

Fig. 8. Detection performance of CWT-HMM method.

Fig. 9 shows range differences between real measurements and CWT-HMM detections. Considering that each range cell is 15 m wide, this assumption leads to a maximum ±7.5 m deviation from the real target location which can be acceptable. On the other hand, it is also seen from Fig. 9 that some range differences are outside of the ±7.5 m uncertainty interval which indicates false detections. Since the processing of CWT can be considered as 2-D cross-correlator, performance of the proposed algorithm is highly dependent on the selection of the mother wavelets with its parameters. Especially for targets with maneuvers, the characteristics of radar echo signal changes wrt. to the reflection points of the target physical surface. This condi-

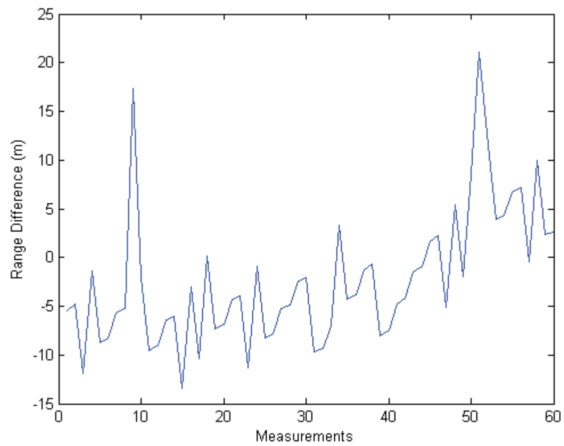

Fig. 9. Range difference between real measurements and CWT-HMM detections.

tion is an important aspect which degrades the performance of conventional detection algorithms which use thresholding. Due to the reflected radar echo amplitude varies during the target maneuver, definition of an appropriate threshold value becomes a difficult process. For this purpose, proposed algorithm uses wavelet scales between 1-64 during CWT in order to capture the radar echo signal during target maneuver in an efficient way.

5. Conclusion

In this paper a CWT-HMM based target detection method for TBD techniques was presented. The main idea in this work is to propose a novel detection method for TBD which does not use any of the thresholding methods. The proposed method was tested with real radar measurements. Simulations showed that proposed method has a potential to detect targets in a sea clutter environments. It has also been showed selection of a proper mother wavelet optimizes the detection performance, the energy level of CWT coefficients maximizes at the points where target reflected signals reside. Moreover, with the usage of the HMM, the proposed method detects the trajectory of the CWT coefficients and separates clutter from target measurements.

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