Radar HRRP Modeling using Dynamic System for Radar Target Recognition

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Abstract. *High resolution range profile (HRRP) is being known as one of the most powerful tools for radar target recognition. The main problem with range profile for radar target recognition is its sensitivity to aspect angle. To overcome this problem, consecutive samples of HRRP were assumed to be identically independently distributed (IID) in small frames of aspect angles in most of the related works. Here, considering the physical circumstances of maneuver of an aerial target, we have proposed dynamic system which models the short dependency between consecutive samples of HRRP in segments of the whole HRRP sequence. Dynamic system (DS) is used to model the sequence of PCA (principal component analysis) coefficients extracted from the sequence of HRRPs. Considering this we have proposed a model called PCA+DS. We have also proposed a segmentation algorithm which segments the HRRP sequence reliably. Akaike information criterion (AIC) used to evaluate the quality of data modeling showed that our PCA+DS model outperforms factor analysis (FA) model. In addition, target recognition results using simulated data showed that our method based on PCA+DS achieves better recognition rates compared to the method based on FA.*

Keywords

Radar Target Recognition, High Resolution Range Profile (HRRP), Dynamic System, PCA Coefficients, Akaike Information Criterion (AIC).

1. Introduction

The advent of high resolution radars made it possible to extract more information from targets which are used for radar target recognition. High resolution range profile (HRRP) is being known as one of the most powerful tools for radar target recognition [1-4]. A high resolution range profile is a one-dimensional signature of target scatterers along the radar line of sight (LOS) that the signal amplitude in each of its range cells shows the strength of the target return at that range. So, it can specify the location and dominance of target scatterers. It can be obtained through employing a wideband signal (which lead to a high range resolution) such as linear frequency modulated (LFM) pulse by the radar. The main problem with range profile for radar target recognition is its sensitivity to aspect angle [2], [5]. That is, change of aspect angle during the target maneuver causes significant changes in amplitude of signal in range cells of the range profile. So the signal amplitude in a range cell can be regarded as a nonstationary signal. Moving toward range cells (MTRC) and Speckle are two main phenomena cause HRRPs to change due to change of aspect angle during the maneuver of an aerial target [2].

To overcome this problem and simultaneously utilize the information in a frame of consecutive range profiles in recognition process, we need a mathematical model for the statistical relation of the consecutive range profiles. Some solutions are proposed using the Gaussian distribution and its variants in [6] and [7]. In [6] the features extracted from the range profiles are modeled by Gaussian mixture distribution. In [7], the factor analysis (FA) model proposed for modeling sequence of HRRPs assumes independency between consecutive samples of HRRPs in small frames of aspect angles. Note that in all of these works, the consecutive range profiles are assumed to be identically independently distributed (IID) in an aspect frame.

Here we seek for an alternative model ignoring the independency assumption. According to the physical behavior of linear and rotational movement of the target and taking into account the electromagnetic backscattering considerations, Dynamic system (DS) seems to be able to model the statistical behavior of range profile variations during the target maneuver. Dynamic System is a general model that a lot of models and processes such as auto regressive (AR), moving average (MA), and ARMA can be regarded as its special cases. In a DS model, nonstationary behavior of the observations (as it is the case in the sequence of HRRPs) can be modeled through the existence of sequence of continuous hidden states (like HMM; of course, there, states are from a discrete finite set).

In our previous work [8] we used dynamic system to model the sequence of feature vectors extracted from the sequence of HRRPs. Features were the location of main scatterers (range cells with the largest amplitudes) extracted from each sample of HRRP using RELAX algorithm introduced in [9] and also used in [6]. But, in this paper we proceed with another approach.

Since dimensionality of HRRP vectors is high (about 100 or more), modeling the sequence of HRRP vectors using dynamic system makes a model complicated and with a lot of parameters to be estimated, resulting in time-consuming computations for model learning. So, Dynamic system is used to model the sequence of PCA (Principal Component Analysis) coefficients extracted from the sequence of HRRPs. Considering this, we have proposed a model called PCA+DS which models consecutive samples of HRRP in a segment. The whole HRRP sequence obtained during a complete maneuver of the target is split into a few segments and modeling is done for each segment. To segment HRRP sequence reliably, we have proposed a segmentation method (Section 3.2). To evaluate the quality and fitness of the proposed model on simulated data, the Akaike information criterion (AIC), introduced in [10], is used. It is shown to be well performed in multivariate model selection problems with limited observation data [11]. Radar Data are simulated using BSS (Backscattering Simulation) software based on simplest components analysis method [12]. The recognition experiments based on simulated data show that our recognition method based on PCA+DS outperforms the method based on factor analysis (FA) which has been proved in [7] that can appropriately model HRRP statistical characteristics and achieve good recognition results.

The remainder of this paper is organized as follows. In Section 2 dynamic system and the expectation maximization (EM) based method for its parameter estimation is discussed and PCA+DS is presented for modeling. In Section 3 the whole recognition procedure from learning phase to test phase will be discussed. Section 4 contains experimental results including model quality test using AIC and recognition results. Finally, Section 5 concludes this paper.

2. Dynamic System for HRRP Sequence Modeling

The dynamic system model used here can be summarized in state and measurement update equations as below:

$$x_{t+1} = Fx_t + w_t, \tag{1}$$

$$y_t = Hx_t + v_t \tag{2}$$

where $x_t \in R^n$ is the hidden state, $y_t \in R^q$ is the observation, w_t and v_t are the model and measurement noise respectively and F and H are two n-by-n and q-by-n matrices respectively. Model and measurement noise are assumed to be white Gaussian and uncorrelated.

$$w_t \sim N\left(\mu_w, Q\right), \tag{3}$$

$$v_t \sim N\left(\mu_v, R\right). \tag{4}$$

The initial state (x_0) is also assumed to be Gaussian with mean μ_0 and Σ_0. The parameters of the model can be summarized in the parameter set λ which should be estimated according to the observations.

$$\lambda = \left(F, H, \mu_w, Q, \mu_v, R, \mu_0, \Sigma_0\right). \tag{5}$$

As noted before, a lot of models and processes (such as AR) can be regarded as special cases of DS model. The sequence of hidden states in DS can be interpreted as a trajectory along which observations are generated, and thereby Dynamic system will be able to model the nonstationarity in observations as it is the case in the sequence of HRRPs (Fig. 1). Fig. 1 shows variations of two of elements of HRRP vectors along one segment.

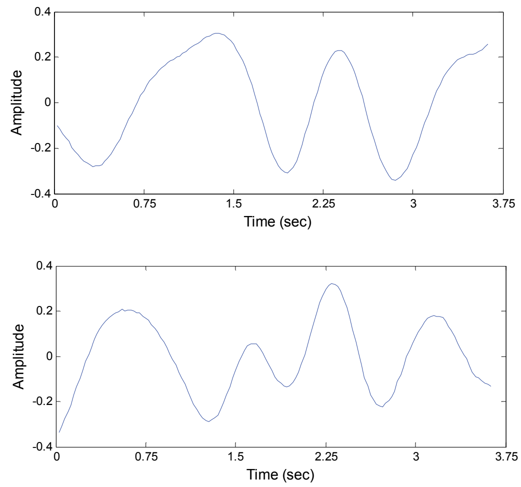

Fig. 1. Variations of two of elements of HRRP vectors along one segment.

2.1 Parameter Estimation

To estimate the parameters, an Expectation Maximization (EM) based technique is used which is first introduced in [13]. Using the EM algorithm for estimating the parameters of the dynamic system model involves computing the conditional expectations of the sufficient statistics for the hidden state during the E-step, using these to re-estimate the parameters during the M-step, and iterating until convergence. If Y = [y_0, y_1, …., y_N] is the segment of observations for training the dynamic system, only the following statistics are needed to be computed in E-step [13]

$$E\{x_t | Y\} = \hat{x}_{t|N}, \tag{6}$$

$$E\{x_t x_t^T | Y\} = \hat{x}_{t|N}\hat{x}_{t|N}^T + \Sigma_{t|N}, \tag{7}$$

$$E\left\{x_t x_{t-1}^T \middle| Y\right\} = \hat{x}_{t|N}\hat{x}_{t-1|N}^T + \Sigma_{t,t-1|N} \tag{8}$$

where:

$$\Sigma_{t|N} = E\left\{\left(x_t - \hat{x}_{t|N}\right)\left(x_t - \hat{x}_{t|N}\right)^T\right\}, \tag{9}$$

$$\Sigma_{t,t-1|N} = E\left\{\left(x_t - \hat{x}_{t|N}\right)\left(x_{t-1} - \hat{x}_{(t-1|N)}\right)^T\right\}. \tag{10}$$

These statistics are calculated using the fixed-interval smoothing form of the Kalman filter, including forward and backward recursions as shown below, augmented with cross-covariance recursions to get second-order statistics.

Assuming $\mu_v = 0$, we have:

Forward recursion:

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t e_t, \tag{11}$$

$$\hat{x}_{t+1|t} = F\hat{x}_{t|t} + \mu_w, \tag{12}$$

$$e_t = y_t - H\hat{x}_{t|t-1}, \tag{13}$$

$$K_t = \Sigma_{t|t-1} H^T \Sigma_{e_t}^{-1}, \tag{14}$$

$$\Sigma_{e_t} = H\Sigma_{t|t-1}H^T + R, \tag{15}$$

$$\Sigma_{t|t} = \Sigma_{t|t-1} - K_t \Sigma_{e_t} K_t^T, \tag{16}$$

$$\Sigma_{t,t-1|t} = (I - K_t H) F \Sigma_{t-1|t-1}, \tag{17}$$

$$\Sigma_{t+1|t} = F\Sigma_{t|t}F^T + Q. \tag{18}$$

Backward recursion:

$$\hat{x}_{t-1|N} = \hat{x}_{t-1|t-1} + A_t[\hat{x}_{t|N} - \hat{x}_{t|t-1}], \tag{19}$$

$$\Sigma_{t-1|N} = \Sigma_{t-1|t-1} + A_t\left[\Sigma_{t|N} - \Sigma_{t|t-1}\right]A_t^T, \tag{20}$$

$$A_t = \Sigma_{t-1|t-1} F_{t-1}^T \Sigma_{t|t-1}^{-1}, \tag{21}$$

$$\Sigma_{t,t-1|N} = \Sigma_{t,t-1|t} + \left[\Sigma_{t|N} - \Sigma_{t|t}\right]\Sigma_{t|t}^{-1}\Sigma_{t,t-1|t}. \tag{22}$$

After calculation of (6), (7), and (8) in E-step, we must re-estimate the model parameters in M-step. Let us, define the following operators:

$$< o >_1 = \frac{1}{N+1}\Sigma_{t=0}^{N} o, \tag{23}$$

$$< o >_2 = \frac{1}{N}\Sigma_{t=1}^{N} o. \tag{24}$$

Then, in M-step estimates of the model parameters are obtained through (23) to (26):

$$\begin{aligned}\left[\hat{F} \quad \hat{\mu}_w\right] = \langle[E\{x_{t+1}x_t^T|Y\} \quad E\{x_{t+1}|Y\}]\rangle_2 \cdot \\ \left(\left\langle\begin{bmatrix} E\{x_t x_t^T|Y\} & E\{x_t|Y\} \\ E\{x_t^T|Y\} & 1\end{bmatrix}\right\rangle_2\right)^{-1},\end{aligned} \tag{25}$$

$$\begin{aligned}\hat{Q} = \langle E\{x_{t+1}x_{t+1}^T|Y\}\rangle_2 \\ -\langle[\mathrm{E}\{x_{t+1}x_t^T|Y\} \quad \mathrm{E}\{x_{t+1}|Y\}]\rangle_2 \cdot \left[\hat{F} \quad \hat{\mu}_w\right]^T,\end{aligned} \tag{26}$$

$$\hat{H} = \langle y_t E\{x_t^T|Y_{a_i}\}\rangle_1 (\langle E\{x_t x_t^T|Y\}\rangle_1)^{-1}, \tag{27}$$

$$\hat{R} = \langle y_t y_t^T\rangle_1 - \hat{H}\langle E\{x_t|Y\}y_t^T\rangle_1. \tag{28}$$

2.2 PCA+DS Model

As noted before, Dynamic system model is used to model the sequence of PCA coefficients obtained from sequence of HRRPs. PCA is a linear transformation through which a set of vectors of possibly correlated variables are converted to a new set of vectors with uncorrelated variables called principal components (PCs). The number of principal components can be lower than the number of original variables because the variance of principal components follows a descending order, i.e. the first component has the largest variance and the last one has the smallest variance. So, a number of components with smaller variances can be omitted.

Here we propose a model called PCA+DS. Assume $Z_s = [z_0, z_1, ..., z_N]$, $z_t \in R^d$ is the sequence of HRRPs in a segment of observations from the whole maneuver of the target, and $Y_s = [y_0, y_1, ..., y_N]$, $y_t \in R^q$ is the corresponding sequence of PCA coefficients, where $q < d$. Then, we have:

$$z_t = Ay_t + \mu + \varepsilon_t \tag{29}$$

where the columns of $A \in R^{d\times q}$ are bases of the PCA subspace (q eigenvectors of covariance matrix of Z_s with the largest eigenvalues). ε_t is a portion of z_t which cannot be represented in this subspace and is modeled as a noise with zero mean and covariance matrix $\Psi \in R^{d\times d}$ which is obtained by taking average over the available data. μ is the mean of Z_s. PCA coefficients are modeled by dynamic systems through (1) and (2).

So, the final PCA+DS model for observations can be described through (30):

$$\begin{cases} x_{t+1} = Fx_t + w_t \\ y_t = Hx_t + v_t \\ z_t = Ay_t + \mu + \varepsilon_t \end{cases} \tag{30}$$

where the estimates of parameters of the two first equations of (30) are obtained using the method described in Section 2.1.

3. Target Recognition Scheme

Target Recognition Scheme consists of train (or learning) and test (or recognition) phases. In the train phase as noted before, since notable changes of aspect angle cause significant changes in the statistical behavior of range profiles, the maneuver of the target is split into a few segments and a PCA+DS model is trained for each segment. So, a reliable segmentation scheme is needed. We have presented a segmentation method which will be explained in Section 3.2. Note that only one maneuver is used for model learning which must cover all aspect angles in the test data. Before the segmentation was done, a pre-processing is required to be applied to the data which is the topic of the following section.

3.1 Pre-Processing

It is required to do a pre-processing on HRRP sequence. This pre-processing is done so as to eliminate the effect of jet engine modulation and to make the data smoother. Jet modulation is due to the effect of jet propellers and jet cavity which influences a few range cells of range profiles and causes them to change more rapidly than other range cells during the maneuver depending on target velocity and pose. So, not only it doesn't help for recognition but it also degrades the recognition performance. To overcome this problem, we have used a low-pass filter which is applied separately to each range cell along the segment. This filter is a FIR filter with Gaussian weightings. It eliminates the effect of jet modulation in range cells affected by it, and makes the variations smoother in other range cells, as can be seen in Fig. 3. The value of -3dB bandwidth of this filter depends on the time between two consecutive HRRP samples. Originally this time is equal to radar pulse repetition interval (PRI). But due to very close similarity between consecutive HRRPs, the sequence is usually downsampled. In our experiments with a repetition frequency of 40 Hz for HRRP samples (i.e. 25 msec between two consecutive samples) the -3dB bandwidth of the filter has been chosen to be 0.02π.

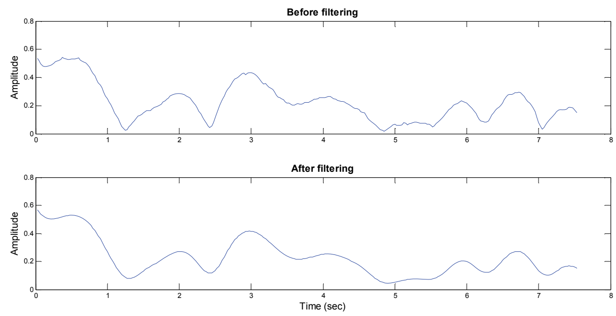

Fig. 2. Variations of one element of HRRPs during the maneuver before and after the filter is applied.

3.2 Segmentation Method

To divide the complete maneuver of each target into appropriate segments some constraints must be chosen. The constraint we use here is that the ratio of energy of HRRP vectors in the PCA subspace to their total energy must be greater than a threshold. According to this raw idea, we have proposed an algorithm whose flowchart can be seen in Fig. 3, where $Z=[z_0,z_1,...,z_T]$, $z_t\in R^d$ is the whole sequence of HRRPs during the maneuver, K is the minimum number of observations in a segment that $K>q$, where q is the number of principal components, and N is the current number of observations in the current segment.

We start with the first K observations in the first segment. Then, PCA bases and coefficients are computed for this segment. Then, with receiving the next observation z_{K+1}, the ratio of its energy in the PCA subspace to its total energy is computed and is put to R. If $R > TH$, this observation will be added to the segment and we continue with this updated segment and the next observation. Otherwise, a new segment will be initialized with K observations starting from z_{K+1}. These steps are continued until the last observation.

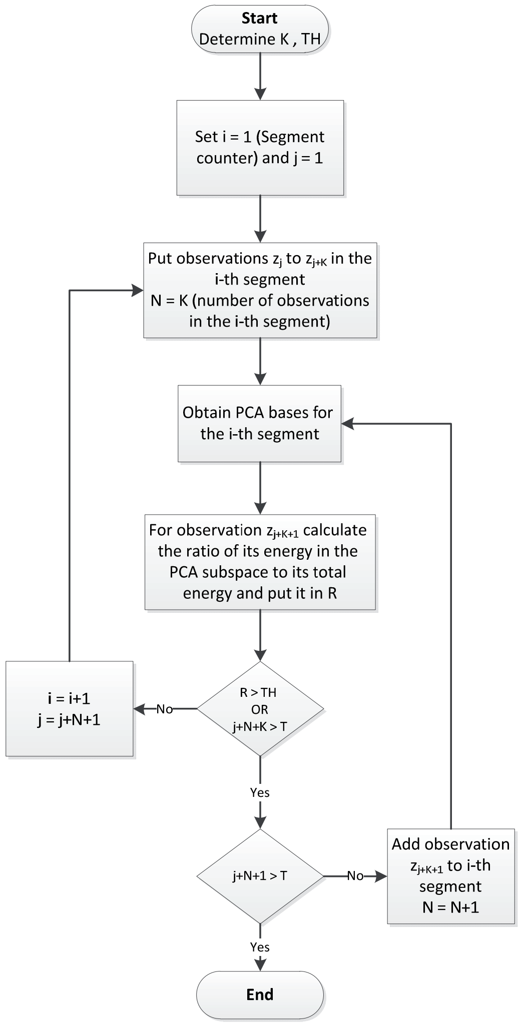

Fig. 3. Segmentation method flowchart.

3.3 Test Phase

In test phase, the input to the classifier is a segment of HRRPs during the target maneuver. Decision is made based on maximum a posteriori (MAP) criterion. That is, the recognition result is the target which maximizes (31):

$$\mathrm{P}(T_i|Z)\propto \mathrm{p}(Z|T_i)\ .\ P_i,\quad i=1,2,...,M \tag{31}$$

where T_i denotes the i-th target and $Z=[z_1, z_2, \dots, z_N]$ is the sequence of HRRPs in the segment. $\mathrm{P}(T_i|Z)$ denotes the posterior probability of target T_i given segment observation Z, $\mathrm{p}(Z|T_i)$ is the probability density function of Z conditioned on target T_i, and P_i is the prior probability of i-th target. If P_i's are assumed to be equal (ML classifier case), the target which maximizes likelihood ($p(Z|T_i)$) or its logarithm (log-likelihood) is chosen. To compute the log-likelihood of the observations in PCA+DS model, note that if we combine the two last equations of (30), a new dynamic system is constructed. We know for a dynamic system model described by (1) and (2) the log-likelihood of the observed sequence Y is obtained by the innovations representations, as

$$\log p(Y|\theta)=\sum_{t=1}^{N}\left\{log\left|\Sigma_{e_t}\right|+e_t^T\Sigma_{e_t}^{-1}e_t\right\}+constant \tag{32}$$

where prediction errors e_t ,and their covariances Σ_{e_t} can be computed through Kalman filter using (11) to (15).

Segment No.	F-15		MIG-21		Tornado	
	FA	*PCA+DS*	*FA*	*PCS+DS*	*FA*	*PCA+DS*
1	-1.0490×10^6	-1.5141×10^6	-1.4438×10^6	-2.0022×10^6	-1.0559×10^6	-1.5257×10^6
2	-7.3227×10^5	-1.1066×10^6	-7.4308×10^5	-1.1124×10^6	-1.0829×10^6	-1.5306×10^6
3	-5.5414×10^5	-8.5640×10^5	-4.6054×10^5	-7.0197×10^5	-4.3600×10^5	-6.8795×10^5
4	-4.0600×10^5	-6.1965×10^5	-4.5729×10^5	-6.6533×10^5	-3.5237×10^5	-5.4203×10^5
5	-3.9771×10^5	-6.0846×10^5	-2.6803×10^5	-3.9478×10^5	-5.4029×10^5	-7.7222×10^5
6	-6.0196×10^5	-8.5402×10^5	-4.1444×10^5	-5.8971×10^5	-3.7756×10^5	-5.4879×10^5
7	-4.0730×10^5	-6.2255×10^5	-8.9106×10^5	-1.2763×10^6	-3.6234×10^5	-5.3635×10^5
8	-4.8577×10^5	-7.4939×10^5	-4.9740×10^5	-7.2853×10^5	-7.4489×10^5	-1.0743×10^6
9	-6.6939×10^5	-1.0170×10^6	-7.4911×10^5	-1.0899×10^6	-8.2546×10^5	-1.0743×10^6
10	-1.3190×10^6	-1.8166×10^6	-1.3640×10^6	-1.8315×10^6	-7.0996×10^5	-1.0680×10^6
11	-5.9569×10^5	-9.5843×10^5	-1.1265×10^6	-1.6037×10^6	-1.0971×10^6	-1.5757×10^6

Tab. 1. AIC values for some segments of the maneuver of F-15, MIG-21, and Tornado for FA and PCA+DS models.

Note that since a target is far from radar and its pose is unknown, the aspect angles will be unknown too. So the likelihood should be computed for all models trained for different aspect frames of target T_i and one with the highest value is considered as final likelihood to be used in test process. Of course, if the aspect angles can somehow be estimated well enough, there is no need to do so and the likelihood is computed only for models trained for corresponding aspect frames.

4. Experimental Results

To simulate range profiles, BSS (Backscattering Simulation) software [12] was used. Simulation is based on the simplest components analysis method. In this method, the surface of the target is divided into several geometrical components and some bright points or lines are determined for each of them, and finally the effects of them are superposed with taking into account the effect of shadowing. Details can be found in [12]. Based on comparison made with real data, there has been shown that the proposed method can appropriately simulate range profiles.

The radar considered here for simulation is a tracking radar with high range resolution. Its bandwidth is 1 GHz in X-band, equivalent to a resolution of 15 cm in range domain which is sufficient enough to separate target scatterers. Polarization is horizontal. The signal used by the radar is an LFM pulse with a width of 100 μs. Radar PRF is 1 kHz. But, the HRRP sequence is downsampled to a repetition frequency of 40 Hz. The frame length of the HRRPs is 40 m. Each HRRP vector consists of 130 range elements. In addition, the -3dB bandwidth of the pre-processing filter has been chosen to be 0.02π.

4.1 Model Quality Test

Here the Akaike information criterion (AIC) is used to evaluate the quality and fitness of the models. It can be used as a tool for comparison between different models. If the observed HRRP sequence in a segment is denoted by Z and the number of independent parameters in the model M is denoted by p, we have:

$$\mathrm{AIC} = -2\log\big(P(Y|M)\big)+2p \tag{33}$$

where $\log\big(P(Y|M)\big)$ is the likelihood of observed sequence for model M. The smaller the AIC, the better the model will be. Forcing Q and R to be diagonal, the number of all parameters of PCA+DS will be $q\times(n+1)+2n^2+3n+d\times(q+1)+d^2$, in which d, q, and n are dimensionality of z, y, and x, respectively. But, there are some dependencies between parameters. These dependencies are due to orthonormality of PCA bases in matrix A and symmetry of matrix Ψ. Thus, a value of $q\times(q-1)/2+q+d\times(d-1)/2$ must be subtracted from the number of parameters. So, the number of independent parameters in PCA+DS will be:

$$p=q\times(n+1)+2n^2+3n+d\times(q+1)+d^2 \\ -\left(q\times\frac{q-1}{2}+q+d\times\frac{d-1}{2}\right). \tag{34}$$

Here we want to compare PCA+DS model with FA model (for further information about FA see [7], [14]). So, HRRP sequence for three different targets including F-15, MIG-21, and Tornado is simulated during a maneuver. Theses sequences then were split into some segments using the algorithm described in Section 3.2. The AIC values for some segments can be found in Tab. 1. 6 PCA coefficients are used for PCA+DS and 6 factors for FA. Hidden state vector dimensionality in PCA+DS is equal to 4.

As can be seen from Tab. 1, PCA+DS outperforms FA in modeling sequence of HRRPs. In Tab. 1 both number of principal components in PCA+DS and number of factors in FA have been set to 6. With these conditions, computational burden for PCA+DS model is more than computational burden for FA model. So, we have increased the number of factors even up to 20 in FA. The corresponding -AIC values is shown in Fig. 4 for 3 different segments for F-15. The -AIC values for PCA+DS model with 6 principal components is denoted by straight lines along the horizontal axis. It is still seen that PCA+DS outperforms FA.

4.2 Target Recognition Results

A radar target recognition scenario has been considered with three jet fighters including F-15, MIG-21, and Tornado which approximately have similar shapes and dimensionalities. To make recognition more difficult an identical maneuver has been used for training all three

	FA			PCA+DS		
	F-15	MIG-21	Tornado	F-15	MIG-21	Tornado
F-15	93.5	2.6	6.4	91.6	1.3	0
MIG-21	1.8	93.6	8.3	4.7	93.6	0.9
Tornado	4.7	3.8	85.3	3.7	5.1	99.1
Total recognition rate (%)	90.5			95.0		

Tab. 2. Confusion Matrix and Average Recognition Rates for PCA+DS and FA.

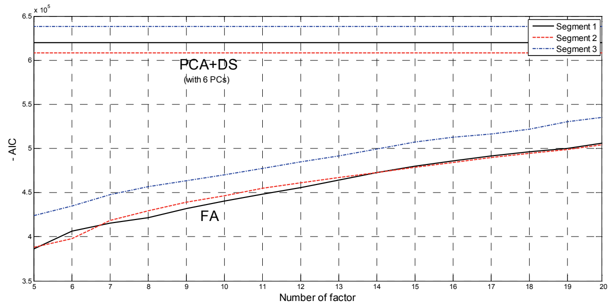

Fig. 4. Values of –AIC vs. number of factors in FA for 3 different segments. Corresponding –AIC values for PCA + DS with 6 PCs has been showed by straight lines.

fighters. Then, using segmentation method described in Section 3.2, the training maneuver were divided into 15, 13, and 15 segments for F-15, MIG-21, and Tornado, respectively. For test data some maneuvers have been used which are completely different from the training maneuver. Each test maneuver is split into some segments which are inputs to the target recognition system. Recognition results for PCA+DS method was compared to FA method. 6 PCA coefficients were considered for PCA+DS and 6 factors for FA. Hidden state vector dimensionality in PCA+DS is equal to 4. Tab. 2 shows confusion matrix and recognition rates for PCA+DS and FA. Total recognition rate for PCA+DS is equal to 95% compared to FA total recognition rate of 90.5%. It should be noted that total recognition rates have been not computed by averaging over correct recognition rates of each target and they have been obtained by involving all segments (for all three fighters) used in the test phase.

5. Conclusion

To model dependency between consecutive samples of HRRP, dynamic system (DS) was proposed. Using PCA coefficients extracted from HRRP sequence, PCA+DS model was presented for HRRP sequence modeling. The whole HRRP sequence is divided into a few segments and modeling is done independently for each segment. So, a segmentation algorithm was proposed. Using AIC as a criterion for modeling quality, superiority of PCA+DS was shown over FA. In addition, recognition results using the method based on PCA+DS was shown that can achieve better results compared to method based on FA.

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