Calculation of Angular Deflection Limits of a Mobile Free-Space Optical Link Beam

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Abstract. *This paper describes the effect of optical beam angular deflection on the power received by the receiver of a mobile free-space optical (FSO) link. Permissible fluctuations in the power received were studied on a steady model of the FSO link. It was assumed that these fluctuations were caused by oscillations of the optical beam across the receiver aperture. The formula for beam angular deflection limit was derived for two different types of optical intensity profile. The task was solved for two different types of atmosphere. The first type of atmosphere was considered a homogeneous and lossless environment. In the second type, atmospheric radiation attenuation was included in the calculations. Also, this article includes graphs of dependencies of the angular deflection limits upon the distance between the link stations.*

Keywords

Free space optical link, steady model, angular deflection of the beam, power fluctuations, optical intensity profile, power budget.

1. Introduction

Free space-optical (FSO) links are an alternative solution to radio frequency wireless links. FSO links are naturally resistant to jamming and tapping because of their low optical beam divergence and their small receiver field of view. However, these system parameters are the reason for high link sensitivity to spatial fluctuations in the link stations. [1] The fluctuations cause angular deflections of both the beam and the receiver field of view that deteriorate the power budget. This results in link fade if the link margin is depleted. The mentioned features are important, namely for directional mobile FSO links as they are affected by natural oscillations of the optical beam and the receiver field of view in the required direction. These oscillations are caused by pointing errors of the link stations. [2].

Beam angular deflection limits were calculated. These deflections are important for the system controlling the angular position of the FSO link stations. The dependence of the deflection limit upon the distance between the two link stations was studied on a Gaussian beam and a Top Hat beam.

The radiation attenuation caused by absorption and scattering in the atmosphere was also taken into consideration. The influence of other phenomena, such as atmospheric turbulence and deflections of the receiver field of view were not assessed in this article.

2. The Steady Model of a Free-Space Optical Link

The steady model of a free-space optical link is represented by a link power budget used for calculation of the optical power P_{PD} received by a photodiode. The power budget provides the basic inputs for the statistical model that assesses link reliability taking into account its atmospheric interface. When the span of the required power P_{PD} is defined, the steady model provides:

- Information on the value of the power P_{PD} gained from the steady model and if it lies in the given span.
- The link margin for random effects.

The steady model describes the power budget of an ideal link where additional random factors affecting the power received are not considered.

2.1 The One Channel Arrangement of the FSO Link

The scheme of the atmospheric part for one channel of the FSO link is shown in Fig. 1 [3], where F is the filter, PD is the photodiode, RW is the receiver window, RXA is the receiver optical system, SR is the source of radiation, TXA is the transmitter optical system, TW is the transmitter window, P_{SR} and P_{PD} are the optical power emitted by the source of radiation and detected by the photodiode, respectively, P_r is the power received by station No. 2, P_t is the power transmitted by the station No. 1, R is the distance between the stations, α is the optical attenuation, and Θ is the beam divergence. Circularly symmetric beams were used – thus divergence is the same across all planes.

Knowing both the transmittance of the individual FSO link elements and the losses caused by linkage imperfections, both the radiation attenuation caused at the individual transmission sections and the total attenuation can be calculated. After that, the received power P_{PD} (dBm) can be calculated from the known transmitted power P_{SR} (dBm) and the total attenuation α (dB).

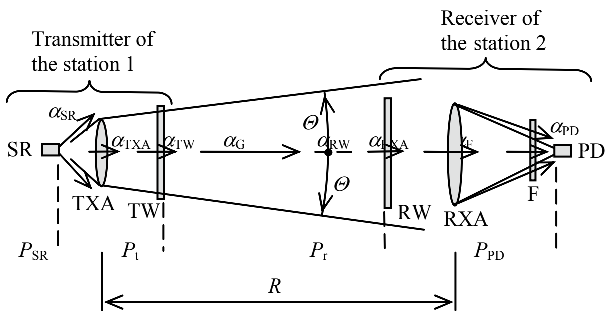

Fig. 1. One channel arrangement of the FSO link [2].

The atmosphere is considered as a homogeneous and lossless transmission environment. So called "ideal pointing of the link" is assumed when the transmitter optical beam axis and the receiver field of view axis are identical.

To assess the influence of the pointing errors on the fluctuations in the power received, a reduced steady model was used. This model expresses the relationship between the output power P_t of station 1 and the radiation incident power P_r hitting a receiver window area of the same size as the receiver aperture size, see Fig. 1.

The mathematical reduced steady model of a link only takes into account the dependence of the power received on the geometric attenuation α_G, occurring during the propagation of the optical signal between the link stations.

2.2 The Mathematical Formula of the Reduced Steady Model

If the additional gain of the receiver, resulting from a specific distribution of the beam's optical intensity, is ignored, the reduced mathematical steady model can be expressed by the following formula [4], [5]:

$$P_r = P_t + \alpha_G = P_t + 20 \cdot \log \frac{D_{RXA}}{D_{TXA} + 2\Theta \cdot R} \tag{1}$$

where D_{TXA} (m) is the diameter of the transmitter aperture, D_{RXA} (m) is the diameter of the receiver aperture, P_t is the transmitted optical power, R (m) is the distance between the stations, α_G is the geometrical attenuation, and 2Θ (rad) is the angular width of the beam.

The formula (1) has to satisfy the following condition [5], [6]:

$$S_r < P_t + 20 \cdot \log \frac{D_{RXA}}{D_{TXA} + 2\Theta \cdot R} < P_{rsat} \tag{2}$$

where P_{rsat} (dBm) is the maximal permissible received optical power, and S_r (dBm) is the receiver sensitivity.

The difference between the maximum permissible received optical power P_{rsat} and the receiver sensitivity S_r is the link dynamic range Δ (dB):

$$\Delta = P_{rsat} - S_r \,. \tag{3}$$

The difference between the received optical power P_r and the receiver sensitivity S_r is the link margin M (dB)

$$M = P_r - S_r \,. \tag{4}$$

For stationary FSO links, M is a constant. For mobile FSO links, the link margin depends on the instantaneous distance between the two stations.

2.3 The Graphical Representation of the Reduced Steady Model

In this subsection, a graphical example of the steady model is shown including its use for modification of some relevant link parameters with the following components using formulas (1) and (2):

- transmitted optical power P_t = 14.77 dBm, (30 mW),
- transmitter aperture diameter D_{TXA} = 0.03 m,
- receiver aperture diameter D_{RXA} = 0.20 m,
- receiver sensitivity S_r = – 43 dBm, ($5 \cdot 10^{-5}$ mW),
- link dynamic range Δ = 30 dB,
- optical beam divergence Θ = 0.017 rad, (≈ 1°).

By entering the Δ and S_r into (3), the P_{rsat} = – 13.00 dBm, ($5 \cdot 10^{-2}$ mW).

Fig. 2 shows a graphical representation of the reduced steady link model for given values of the individual components assuming the distances between the stations are within the range of $R \in \langle 100 \text{ m}, 2000 \text{ m}\rangle$. Marked is the link margin M = 9.64 dB for the distance R = 1500 m.

It is clear from the graph that the minimum distance R_{min} is 150 m. It is the shortest link length at which the link can be used. For distances where R < 150 m then $P_r > P_{rsat}$ which does not satisfy the condition (2). However, a situation can occur where the received power, obtained from the steady model, drops down to the receiver sensitivity level. Then $P_r = S_r$ and the distance $R = R_{max0}$ is the link range with zero link margin. For $R > R_{max0}$ the link cannot be used even if the meteorological visibility is at its maximum.

If the link is required to be usable for the entire span of the distances R, the power budget must be adjustable. Assuming two options that either only the beam divergence Θ or only the transmitted optical power P_t can be altered, then the required value of both the divergence Θ and the power P_t can be reached using the condition $P_r(R) = P_{rsat}$, with R = 100 m. If the divergence is altered, and the other parameters remain constant, the limit state is reached for Θ = 25 mrad. If the transmitted power P_t is altered, the limit state occurs for P_t = 11.67 dBm (14.7 mW).

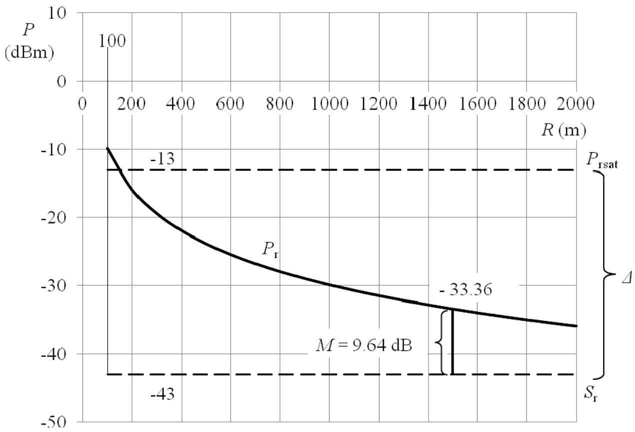

Fig. 2. Graph of the reduced steady model [4].

The link margin M is an important result of the link power budget and plays a crucial role in link reliability [4]. It specifies the maximum permissible value of the real link's additional power losses caused by the atmosphere, background radiation, pointing errors etc. If the link is to be sufficiently resistant against these influences, the link margin must be as high as possible for all required link lengths. Theoretically, the maximum possible link margin is equal to the dynamic range: $M_{max} = \Delta$.

It is clear that for links with constant parameters, the margin decreases with increasing distance between the stations. Thus, the link length affects the link sensitivity to undesirable phenomena, showing deterioration of the transmission characteristics and degradation of the link availability. In such a situation, when beam divergence is high and the link is relatively insensitive to pointing errors, it is necessary to ensure compliance with the condition (2) and provide the receiver with a big dynamic range Δ. Only that can assure a sufficient link margin.

3. Effect of Beam Deflection on the FSO Received Power

As the real mobile FSO link suffers from pointing errors, the link parameters and the parameters of the control tracking system have to be chosen with regards to the permissible fluctuations in power received.

Received power fluctuations are caused by changes in the linear and angular mutual positions of the link stations. These changes are a natural, permanent and unavoidable part of a link operation environment, generally affecting all platforms and depending on the nature of movement of the specific platform.

The fluctuations in received power depend on the type of platform and its features, and on the quality and accuracy of the control system that provides automatic tracking of opposing link stations.

The pointing errors are:

- Angular deflection of the receiver field of view from its ideal position.
- Angular deflection of the optical beam from its ideal position.

The angular deflection of the receiver causes random shifts of the radiation footprint from the sensitive detector surface. As a result, some power losses can occur if the detector surface is not large enough.

However, mentioned errors are not the focus of this article. This article only focuses on losses caused by a shift of the optical beam axis from the center of the receiver aperture surface.

The angular deflection of the optical beam δ_{ba} (rad) causes random linear beam deflections δ_{bl} (m) at the receiver aperture surface, see Fig. 3. The main requirement for these deviations is to fulfill the following inequation:

$$\delta_{bl}\left(R\right) < w\left(R\right) - \frac{D_{RXA}}{2} \tag{5}$$

where $w(R)$ is the beam width at the distance R.

The following equations apply to a beam with a small divergence and tiny pointing deflections:

$$w\left(D\right) = \Theta \cdot R\,, \tag{6}$$

$$\delta_{bl}\left(D\right) = \delta_{ba} \cdot R\,. \tag{7}$$

Fulfilling the formula (5) ensures that the optical beam does not miss the receiver aperture totally. In general, even if (5) is fulfilled, the link margin could still become zero (M = 0 dB) for certain deflection limit $\delta_{bl} = \delta_{blm} = \delta_{bam} \cdot R$, where δ_{blm}, δ_{bam} are the beam linear and angular deflection limits, respectively. However, for deviations $\delta_{bl}(R) > w(R) - D_{RXA} / 2$ the received power could still be sufficient.

To assess the effect of pointing errors on fluctuations in the received power P_r, it is necessary to know the dependence of the optical intensity on the radial distance from the beam axis. For example in the direction of the x-axis, see Fig. 3. Then this dependency can be generally expressed by a relative optical intensity $I'(x, R)$ (–):

$$I'\left(x, R\right) = \frac{I\left(x, R\right)}{I_{0R}\left(R\right)} \tag{8}$$

where $I(x, R)$ ($W \cdot m^{-2}$) is the optical intensity, $I_{0R}(R)$ is the optical intensity on the beam axis at the distance R from the transmitter, and x is the radial distance in the direction of the x-axis.

As the optical beams are circularly symmetrical optical beams, the x coordinate can be substituted by the y

coordinate. The function $I'(x)$ influences both the fluctuation amplitude of the received power P_r and the angular deflection limit of the beam δ_{bam}.

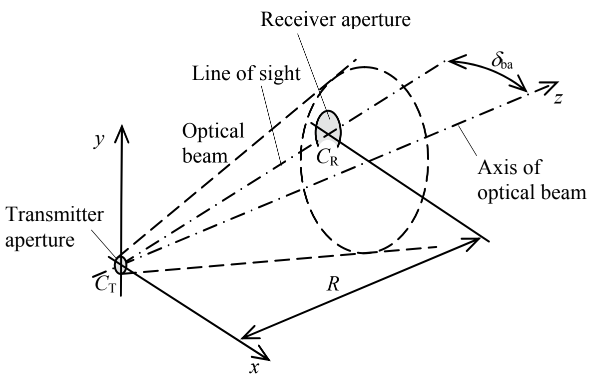

Fig. 3. The coordinate system of the optical beam.

Assuming that there are two optical intensity distributions, the Gaussian beam type and the Top Hat beam type. Then, the definition of the beam edge for both types is a distance from the beam axis $x = w(R) = \Theta \cdot R$, where the optical intensity falls down to the level of $I_{0R}(R)/e^2$. This results in: $\Theta_G = \Theta_{TH} = \Theta$, where Θ_G, Θ_{TH} is the divergence of the Gaussian beam and the Top Hat beam, respectively. It is necessary to define beam edges in the same way for the analysis of the received power sensitivity to the spatial fluctuations of the individual beams.

Taking into consideration the dependency of optical intensity on divergence and distance, the distribution of the relative intensity of the Gaussian beam can be expressed as follows:

$$I'_G(\Theta, R, x) = e^{-2 \cdot \left(\frac{x}{\Theta \cdot R}\right)^2}. \tag{9}$$

The relative intensity of the Top Hat beam can be derived from the Fermi-Dirac and Super-Gaussian approximations [7], or by using the following formula:

$$I'_{TH}(\Theta, R, x) = e^{-2 \cdot \left(\frac{|x| - K_2 \cdot |x|^c}{K_3 \cdot \Theta \cdot R}\right)^2} \tag{10}$$

where c, K_2, K_3 are coefficients.

If the condition $\Theta_G = \Theta_{TH} = \Theta$ is to be fulfilled, one of the parameters K_2 or K_3 has to be dependent on the remaining parameters in (10).

Assuming that the coefficient K_3 is dependent, then the following equation derives from the definition of the beam edge:

$$K_3 \cdot \Theta \cdot R = |x| - K_2 \cdot |x|^c. \tag{11}$$

Entering $|x| = \Theta \cdot R$ into (11) results in

$$K_3 \cdot \Theta \cdot R = \Theta \cdot R - K_2 \cdot (\Theta \cdot R)^c$$

and from that:

$$K_3 = 1 - K_2 \cdot (\Theta \cdot R)^{c-1}. \tag{12}$$

When K_3 from (12) is substituted into (10), then the relative optical intensity of the Top Hat beam becomes

$$I'_{TH}(\Theta, R, x) = e^{-2 \cdot \left\{\frac{|x| - K_2 \cdot |x|^c}{[1 - K_2 \cdot (\Theta \cdot R)^{c-1}] \cdot \Theta \cdot R}\right\}^2}. \tag{13}$$

For example, for $K_2 = 2.4$ and $c \in \langle 1.5, 5 \rangle$, different intensity profiles can be selected for the Top Hat beam. For $c = 1$, the profile is Gaussian. For $c = 1.5$, the edge beam profiles can be formed by changing K_2 within a range of 1.2 to 2.4. Nevertheless, these profiles are only slightly similar to real beams of this type.

The behavior of functions (9) and (13) are graphically depicted in Fig. 4. The graphs were set for the following variables and parameters: $\Theta = 0.017$ rad, $R = 100$ m, $K_2 = 2.4$, $c = 1.5$.

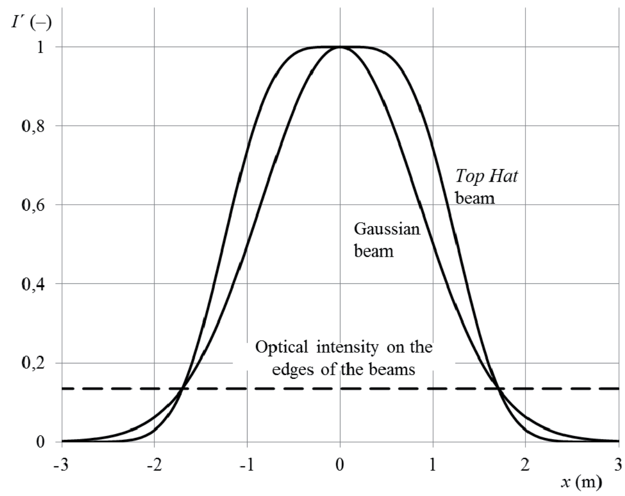

Fig. 4. Dependency of relative optical intensity on the radial distance x.

3.1 The Deflection Limits of the Gaussian Beam

The optical intensity of the Gaussian beam can be described as [4]:

$$I_G(\Theta, R, r) = 0.741 \cdot \frac{P_t}{(\Theta \cdot R)^2} \cdot e^{-2 \cdot \frac{r^2}{(\Theta \cdot R)^2}} \tag{14}$$

where r (m) is the absolute distance from the beam axis along the receiver aperture plane, $r \in \langle 0, \infty \rangle$.

Substituting I_{0RG} for $0.741 \cdot P_t/(\Theta \cdot R)^2$ gives:

$$I_G(\Theta, R, r) = I_{0RG} \cdot e^{-2 \cdot \frac{r^2}{(\Theta \cdot R)^2}}. \tag{15}$$

The quantity I_{0RG} represents the optical intensity on the axis of the Gaussian beam at the distance R from the transmitter.

As a quantitative indicator of the effect of pointing errors on the link reliability, beam angular deflection limit δ_{bam} can be used. The deflection is then derived from the formula expressing the power propagating through the receiver aperture center which is located out of the optical beam axis.

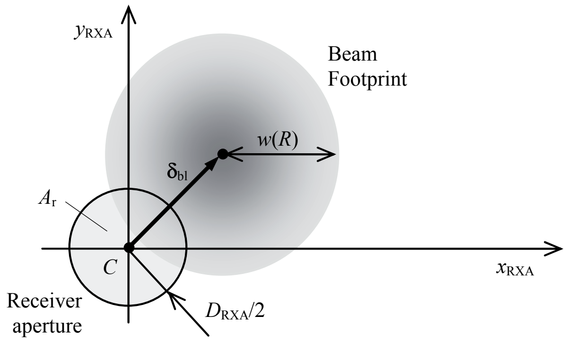

Fig. 5. Misalignment of the beam footprint [8].

The shift of the beam footprint from the receiver aperture in the receiver coordinate system $Cx_{RXA}y_{RXA}$, where C is the center of the receiver aperture, is depicted in Fig. 5 [8]. If the area of the receiver aperture A_r (m^2) is comparable with the area of the beam footprint, the received power $P_r(\delta_{bl}, \Theta, R)$ should be calculated from the following general equation [8], [9]

$$P_r(\delta_{bl}, \Theta, R) = \iint_{A_r} I(\mathbf{r} - \boldsymbol{\delta}_{bl}, \Theta, R) \cdot \mathrm{d}A \tag{16}$$

where $I(\mathbf{r} - \boldsymbol{\delta}_{bl}, \Theta, R)$ is the optical intensity in the receiver coordinate system, and $\mathbf{r}$ is the radial vector from the beam center.

If $D_{RXA} << 2\Theta \cdot R$, then the optical intensity at the receiver aperture can be considered constant and its value depends on the distance between the receiver aperture center C and the beam axis. Then, the received power can be expressed as a dependency on the angular beam deflection.

For the Gaussian beam, the following designation was introduced: $\delta_{bl} = \delta_{Gl,}$ $\delta_{ba} = \delta_{Ga}$. When r is replaced by δ_{bl} in (14) and δ_{bl} is replaced by (7), then the power incident on the receiver aperture with the area of $A_r = \pi \cdot D_{RXA} / 4$ will be

$$P_r(\delta_{Ga}, \Theta, R) = I(\delta_{Ga}, \Theta, R) \cdot A_r \, ,$$

$$P_r(\delta_{Ga}, \Theta, R) = 0{,}185 \cdot \pi \cdot D_{RXA}^2 \cdot \frac{P_t}{(\Theta \cdot R)^2} \cdot \mathrm{e}^{-2 \cdot \left(\frac{\delta_{Ga}}{\Theta}\right)^2} \tag{17}$$

The condition for beam angular deflection limit δ_{Gam} is

$$0.185 \cdot \pi \cdot D_{RXA}^2 \cdot \frac{P_t}{(\Theta \cdot R)^2} \cdot \mathrm{e}^{-2 \cdot \left(\frac{\delta_{Gam}}{\Theta}\right)^2} - S_r = 0 \, . \tag{18}$$

Then from (18), the analytical equation for δ_{Gam} will be

$$\delta_{Gam} = \sqrt{-0.5 \cdot \Theta^2 \cdot \ln \frac{S_r \cdot (\Theta \cdot R)^2}{0.185 \cdot \pi \cdot D_{RXA}^2 \cdot P_t}} \, . \tag{19}$$

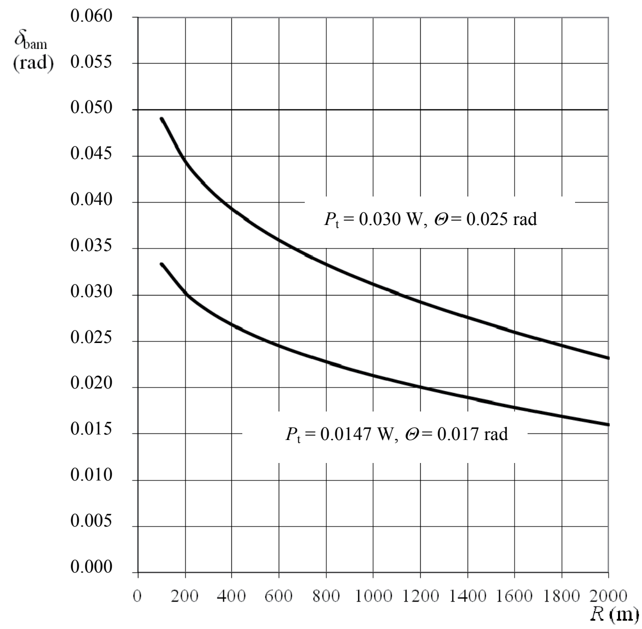

Fig. 6. Behavior of Gaussian beam angular deflection limit.

Fig. 6 shows graphs of the Gaussian beam angular deflection limit δ_{Gam} for the two following configurations of the mobile FSO parameters, using the link parameters mentioned in Subsection 2.3:

- P_t = 11.67 dBm (14.7 mW), Θ = 0.017 rad,
- P_t = 14.77 dBm (30.0 mW), Θ = 0.025 rad.

As expected, Fig. 6 confirms that, as far as the sensitivity to pointing errors is concerned, it is more favorable to have a link with a higher divergence, in our case 0,025 rad. For the design of the link it is important that the value of δ_{Gam} decreases with increasing distance R. Thus, the tracking system should be designed as an adaptive system or should be designed for the worst case scenario when the distance R is at its maximum, i.e. 2000 m. The corresponding beam angular deflection limits δ_{Gam} are 23.0 mrad and 16.0 mrad, for Θ = 25.0 mrad and Θ = 17.0 mrad, respectively.

3.2 The Deflection Limits of the Top Hat Beam

Assuming that the optical intensity distribution of the Top Hat beam is analogically defined as in (15), then

$$I_{\mathrm{TH}}(r,\Theta,R)=I_{0\mathrm{RTH}}\cdot \mathrm{e}^{-2\cdot\left\{\frac{r-K_2\cdot r^{\mathrm{c}}}{[1-K_2\cdot(\Theta\cdot R)^{\mathrm{c}-1}]\cdot\Theta\cdot R}\right\}^2} . \quad (20)$$

Assuming that $I_{0\mathrm{RTH}} = I_{0\mathrm{RG}}$, then the angular deflection limit of the Top Hat beam can be expressed from the following equation for relative intensities as:

$$I'_{\mathrm{G}}(\delta_{\mathrm{Gam}},\Theta,R)=I'_{\mathrm{TH}}(\delta_{\mathrm{THam}},\Theta,R) \quad (21)$$

where δ_{THam} is the angular deflection limit of the Top Hat beam. If x and $|x|$ are replaced by $\delta_{\mathrm{Glm}} = \delta_{\mathrm{Gam}} \cdot R$ and $\delta_{\mathrm{THlm}} = \delta_{\mathrm{THam}} \cdot R$ in (9) and (13), then equation (21) will be as follows:

$$\mathrm{e}^{-2\cdot\left(\frac{\delta_{\mathrm{Gam}}}{\Theta}\right)^2}=\mathrm{e}^{-2\cdot\left\{\frac{\delta_{\mathrm{THam}}-K_2\cdot R^{\mathrm{c}-1}\cdot\delta^{\mathrm{c}}_{\mathrm{THam}}}{[1-K_2\cdot(\Theta\cdot R)^{\mathrm{c}-1}]\cdot\Theta}\right\}^2} .$$

By further modifications, we get:

$$\delta_{\mathrm{Gam}}=\frac{\delta_{\mathrm{THam}}-K_2\cdot D^{\mathrm{c}-1}\cdot\delta^{\mathrm{c}}_{\mathrm{THam}}}{[1-K_2\cdot(\Theta\cdot D)^{\mathrm{c}-1}]} . \quad (22)$$

It is clear from (22) that the Top Hat beam angular deflection limit has to be solved using an iterative method. The angular deflection limit δ_{Gam} can be calculated from (22) for several distances within the interval of $R \in \langle 100\ \mathrm{m}, 2000\ \mathrm{m}\rangle$ and for gradually changing values of the deflection δ_{THam}. The result is the value of δ_{THam} for which the following equation is true:

$$\delta^{(22)}_{\mathrm{Gam}}=\delta^{(19)}_{\mathrm{Gam}}$$

where $\delta^{(22)}_{\mathrm{Gam}}$ is the angular deflection limit of the Gaussian beam calculated from (22), $\delta^{(19)}_{\mathrm{Gam}}$ is the angular deflection limit of the Gaussian beam calculated from (19). The divergence and the transmitted power for the Gaussian beam and the Top Hat beam have to be the same.

For our example in Subsection 2.3, the angular deflection limits of both beams are arranged in table Tab. 1. Fig. 7 shows these deflections graphically, depending on distance. Both the table and the picture were set for $K_2 = 2.4$, c = 1.5 and the atmosphere was considered to be a lossless environment.

3.3 The Consequences of an Optical Beam Power Drop

If a drop in the optical beam power occurs along the transmission route, both the power budget and the angular deflection limits change for both beam types. Also, the distance for which the equation $\delta_{\mathrm{THam}} = \delta_{\mathrm{Gam}}$ is true changes. The reason for this could be signal loss due to either impurities of the transmitter or receiver cover windows or due to atmospheric interference [1], [10], [11].

The drop in power can be expressed by additional losses using different expression suited to their origin.

This article focuses only on atmospheric losses such as radiation absorption and scattering. The losses are usually expressed either by the attenuation α_{a} (dB · km^{-1}) or by the transmittance τ_{a} (–), using the extinction coefficient β_{a} (km^{-1}). The transmittance τ_{a} is then used to assess the effect of the power drop on the angular deflection limit. Assuming that the transfer path of the length R (km) is homogeneous, then the transmittance is as follows [1], [10]:

$$\tau_{\mathrm{a}}=\mathrm{e}^{-\beta_{\mathrm{a}}\cdot R} . \quad (23)$$

The actual transmission route transmittance τ_{a} has an equivalent effect on the system as when the transmitted power drops down to the level of $P_{\mathrm{t}}\cdot\tau_{\mathrm{a}}$.

Then formula (19) can be expressed as follows:

$$\delta_{\mathrm{Gam}}=\sqrt{-0.5\cdot\Theta^2\cdot\ln\frac{S_{\mathrm{r}}\cdot(\Theta\cdot D)^2}{0.185\cdot\pi\cdot D^2_{\mathrm{POS}}\cdot P_{\mathrm{t}}\cdot\tau_{\mathrm{a}}}} . \quad (24)$$

	R (m)										
Θ = 0.017 rad **P_{t} = 11.67 dBm**	100	200	400	600	800	1000	1200	1400	1600	1800	2000
δ_{Gam} (rad)	0.0334	0.0302	0.0268	0.0245	0.0228	0.0213	0.0200	0.0276	0.0179	0.0169	0.0160
δ_{THam} (rad)	0.0252	0.0242	0.0226	0.0214	0.0205	0.0196	0.0189	0.0182	0.0175	0.0169	0.0163
Θ = 0.025 rad **P_{t} = 14.77 dBm**											
δ_{Gam} (rad)	0.0491	0.0444	0.0393	0.0359	0.0228	0.0311	0.0293	0.0276	0.0260	0.0245	0.0232
δ_{THam} (rad)	0.0376	0.0358	0.0333	0.0315	0.0301	0.0288	0.0277	0.0266	0.0256	0.0247	0.0238

Tab. 1. Angular deflection limits for the Gaussian beam and the Top Hat beam – without any atmospheric interference.

β_{a} = 0.8 km^{-1}	**R (m)**										
Θ = 0.017 rad **P_{t} = 11.67 dBm**	100	200	400	600	800	1000	1200	1400	1600	1800	2000
δ_{Gam} (rad)	0.0332	0.0299	0.0259	0.0231	0.0206	0.0184	0.0163	0.0140	0.0117	0.0089	0.0051
δ_{THam} (rad)	0.0251	0.0240	0.0221	0.0206	0.0192	0.0179	0.0165	0.015	0.0133	0.0112	0.0078
Θ = 0.025 rad **P_{t} = 14.77 dBm**											
δ_{Gam} (rad)	0.0488	0.0439	0.0380	0.0337	0.0302	0.0268	0.0236	0.0202	0.0166	0.0123	0.0060
δ_{THam} (rad)	0.0374	0.0355	0.0326	0.0303	0.0282	0.0262	0.0241	0.0218	0.0192	0.0158	0.0099

Tab. 2. Angular deflection limits for the Gaussian beam and the Top Hat beam – with atmospheric interference.

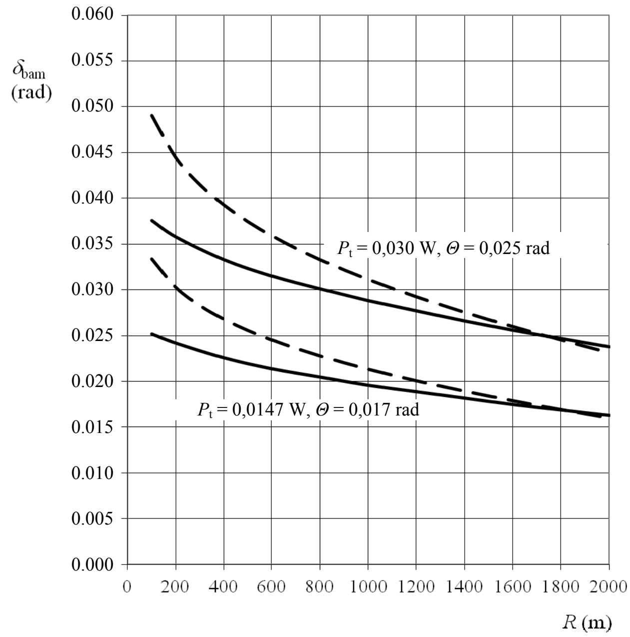

Fig. 7. Angular deflection limits of the Gaussian beam (dashed line) and the Top Hat beam (solid line) without atmospheric interference.

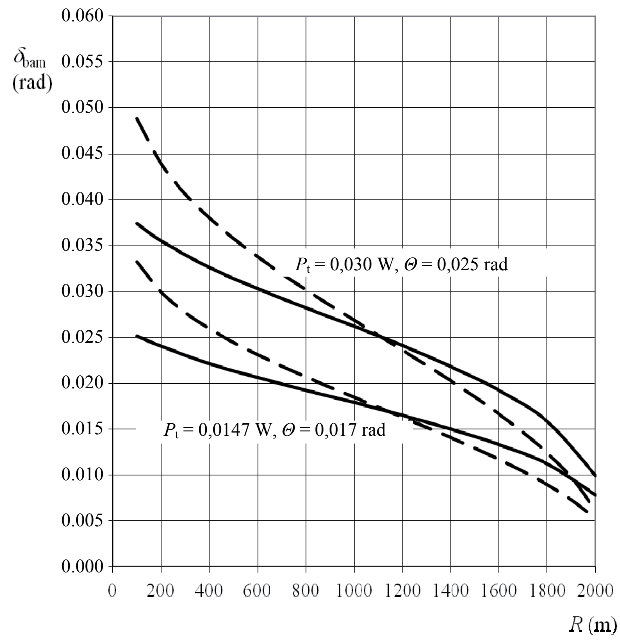

Fig. 8. Angular deflection limits of the Gaussian beam (dashed line) and the Top Hat beam (solid line) with atmospheric interference.

The angular deflection limits of the individual beams were calculated for the same link variables and parameters as in the above mentioned cases, and for coefficient $\beta_a = 0.8$ km^{-1}. The obtained deflections are listed in Tab. 2, and graphically shown in Fig. 8. For the selected coefficient β_a, the meteorological visibility is approximately 3 km, within the operating wavelengths of the FSO links.

4. Conclusion

Both, the analysis of the power budget and the received power dependency on the mutual positions of the optical beam and the receiver aperture, show that the permissible deflections of the optical beam from the ideal pointing depend on several basic factors, such as beam divergence Θ, optical intensity distribution of a specific beam type, and distance R between the link stations.

The presented analysis of received power using different beam types is applicable only if the following conditions are met:

- The optical intensity on the beam axis, at an arbitrary distance R from the transmitter and for all beam types is the same as the intensity of the Gaussian beam, in our case $I_{0RTH} = I_{0RG}$.
- The divergence of a random beam is defined the same way as the divergence of the Gaussian beam.
- The divergence is the same for all beam types, $\Theta_{TH} = \Theta_G$.
- The receiver aperture diameter is much smaller than the beam footprint diameter, $D_{RXA} << 2\Theta{\cdot}R$.

Meeting these conditions assures the applicability of the above presented conclusion when designing the link.

The condition of the identical optical intensity I_{0R} for both types of beams requires two different transmitted powers. It is clear from the optical intensity distribution profile that the transmitted power for the link using the Top Hat beam has to be greater than the power for the link using the Gaussian beam.

This analysis shows that the angular deflection limits δ_{bam} decrease with increasing distance R. When the angular deflection limits are greater than the beam divergence, it is more suitable to use the Gaussian beam than the Top Hat beam, as the following conditions apply: $I'_G = I'_{TH}$ for the radial distances $r > \Theta{\cdot}D = w$, see (9), (13), Fig. 4.

There is a certain distance R_{zv} for wich $\delta_{THam} = \delta_{Gam}$. The angular deflection limits $\delta_{THam} < \delta_{Gam}$ and $\delta_{THam} > \delta_{Gam}$ for $R < R_{zv}$ and $R > R_{zv}$, respectively. In our case, if atmospheric interference or other factors decreasing the optical power of the beam are not taken into consideration, the situation when $\delta_{THam} = \delta_{Gam}$ occurs at a distance of approximately 1800 m. If atmospheric interference or any additional attenuation is included in the power budget calculations, the situation will be different. The power budget will deteriorate, the slope of the graph for the angular deflection limits will become steeper and the distance R_{zv} will be shortened, see Fig. 8. That is why, when designing parameters and link features, the purpose of the beam type should be considered together with its expected distance interval between stations.

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