

An Improved NSGA-II and its Application for Reconfigurable Pixel Antenna Design

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Abstract. Based on the elitist non-dominated sorting genetic algorithm (NSGA-II) for multi-objective optimization problems, an improved scheme with self-adaptive crossover and mutation operators is proposed to obtain good optimization performance in this paper. The performance of the improved NSGA-II is demonstrated with a set of test functions and metrics taken from the standard literature on multi-objective optimization. Combined with the HFSS solver, one pixel antenna with reconfigurable radiation patterns, which can steer its beam into six different directions ($\theta_{DOA} = \pm 15^\circ, \pm 30^\circ, \pm 50^\circ$) with a 5 % overlapping impedance bandwidth ($S_{11} < -10$ dB) and a realized gain over 6 dB, is designed by the proposed self-adaptive NSGA-II.

Keywords

Elitist non-dominated sorting genetic algorithm (NSGA-II), multi-objective, pixel antenna, radiation pattern reconfigurability, self-adaptive.

1. Introduction

In real-world optimization applications, it is often necessary to optimize multi-objective in one problem synchronously. With more than one objective, the multi-objective optimization is quite different from single-objective optimization for these objectives which may conflict each other and should be evaluated simultaneously. Usually, no single solution in multi-objective optimization problems can satisfy all the required objectives at the same time. This phenomenon leads to a set of non-dominated solutions, known as Pareto optimal solutions [1].

During the past two decades, multi-objective evolutionary algorithms (MOEAs) have attracted an increasing attention among optimization community mainly because of the fact that they can be suitably applied to deal with a set of possible solutions simultaneously. Some MOEAs have been developed, such as Fonseca and Fleming's MOGA [2], Srinivas and Deb's NSGA [1], Horn's NPGA [3], Zitzler and Thiele's SPEA [4], Knowles and Corne's PAES [5], Deb's NSGA-II [6], Coello's MOPSO [7] and so on. Adaptability is one of the most important and pro-

ducing research areas in evolutionary computation. Several adaptive multi-objective algorithms have been proposed [8]-[10]. From those above mentioned works, the adaptive techniques can improve the performances of multi-objective algorithms.

In this paper, we propose an improved NSGA-II with self-adaptive crossover and mutation operators to obtain good optimization performance. The effectiveness of self-adaptive NSGA-II is validated using several benchmark functions reported in the specialized literature and compared with the conventional NSGA-II and multi-objective particle swarm optimization (MOPSO). Test results show its good spread of solutions and good convergence near the true Pareto-optimal front. As an application to electromagnetics, the proposed self-adaptive NSGA-II is employed to design a pixel antenna with reconfigurable radiation patterns. Simulation results of the antenna verify the accuracy and efficiency of our proposed algorithm.

2. Self-Adaptive NSGA-II

2.1 Definition and Notation of Multi-objective Optimization

A general multi-objective minimization (maximization) optimization problem can be defined as follows:

Find the optimal solution vector $\mathbf{x}^* = (x_1^*, x_2^*, \dots, x_N^*)$ which minimizes (maximizes) the vector function $f(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_M(\mathbf{x})]$, where $\mathbf{x} = (x_1, x_2, \dots, x_N)$ is the vector of decision variables.

The domination is an important concept in multi-objective optimization, in which a solution x_i is said to dominate another solution x_j when both of the following conditions are met:

- The solution x_i is not worse than x_j in all objectives.
- The solution x_i is better than x_j in at least one objective.

2.2 Proposed Self-Adaptive NSGA-II

The conventional NSGA-II uses a fast non-dominated sorting approach with computational complexity $O(MN_p^2)$

(where M is the number of objectives and N_p is the population size), an elitist-preserving approach that creates a mating pool by combining the parent and offspring populations and selecting the best (with respect to fitness and spread) N_p solutions, and a crowded-comparison approach that does not require any user-defined parameter for maintaining diversity among population members [6].

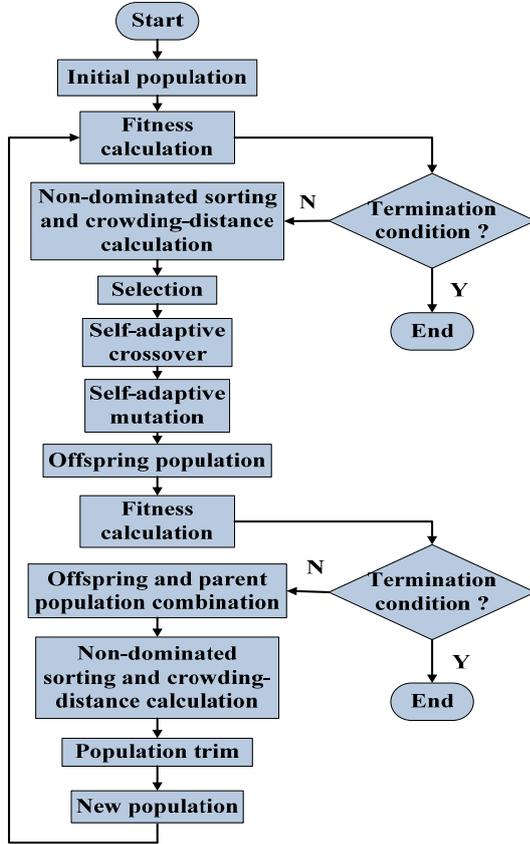


Fig. 1. Optimization process of self-adaptive NSGA-II.

In genetic algorithm, crossover and mutation operators are important factors to determine algorithm's performance. The bigger crossover and mutation probabilities are, the more new individuals will be generated along with the diversity of population. If the probabilities are too big, good genes will be destroyed easily. At the same time, if the probabilities are too small, it is not conducive to generate new individuals and the search speed will slow down. In order to obtain better optimization performance, we proposed the self-adaptive crossover and mutation operators in the conventional NSGA-II for better spread of solutions and convergence near the true Pareto-optimal front. They are defined, respectively, as follows:

$$p_c = \frac{(M+1)K_1}{1 + e^{\sigma_1} + e^{\sigma_2} + \dots + e^{\sigma_m} + \dots + e^{\sigma_M}}, \quad (1)$$

$$p_m = K_2 \times \sin\left(\frac{\pi}{2} \times \frac{1}{e^{(\sigma_1 + \sigma_2 + \dots + \sigma_m + \dots + \sigma_M)}}\right) \quad (2)$$

where $\sigma_m = \left(\sum_{i=1}^{N_p} (f_{i,m} - f_{ave,m})^2\right) / N_p$ ($m = 1, 2, \dots, M$), $f_{i,m}$

is the value of i th individual's objective function for m th objective, $f_{ave,m}$ is the average value of all individuals' objective functions for m th objective, and K_1 and K_2 are positive numbers in the range of $[0, 1]$. Equations (1) and (2) are based on all objectives to calculate population's diversity. When population's diversity reduces or the algorithm traps in a local optimum, p_c and p_m will become bigger. When population's diversity is relatively good, in order to preserve good genes, p_c and p_m will become smaller. The flowchart of self-adaptive NSGA-II is depicted in Fig. 1.

2.3 Test Functions and Performance Measures

Problem	N	Variable bound	Objective function	Optimal solution	Nature
SCH	10	$[-5, 5]$	$f_1 = \frac{1}{40} \sum_{i=1}^{10} x_i$ $f_2 = \frac{1}{40} \sum_{i=1}^{10} (x_i - 2)^2$	$x_1 \in [0, 2]$ $x_i = x_1$ $i = 2, \dots, N$	convex
FON	3	$[-4, 4]$	$f_1 = 1 - \exp(-\sum_{i=1}^3 (x_i - \frac{1}{\sqrt{3}})^2)$ $f_2 = 1 - \exp(-\sum_{i=1}^3 (x_i + \frac{1}{\sqrt{3}})^2)$	$x_1 = x_2 = x_3$ $\in [-1/\sqrt{3}, 1/\sqrt{3}]$	nonconvex
DEB	2	$x_1 \in [0.1, 1]$ $x_2 \in [0, 1]$	$f_1 = x_1$ $f_2 = g(x_2) / x_1$ $g(x_2) = 2 - \exp[-\frac{(x_2 - 0.2)^2}{0.004}]$ $-0.8 \times \exp[-\frac{(x_2 - 0.6)^2}{0.4}]$	$x_1 \in [0.1, 1]$ $x_2 = 0.2$	convex
ZDT1	30	$[0, 1]$	$f_1 = x_1$ $f_2 = g(x) [1 - \sqrt{x_1 / g(x)}]$ $g(x) = 1 + 9(\sum_{i=2}^N x_i) / (N-1)$	$x_1 \in [0, 1]$ $x_i = 0$ $i = 2, \dots, N$	convex
ZDT2	30	$[0, 1]$	$f_1 = x_1$ $f_2 = g(x) [1 - (x_1 / g(x))^2]$ $g(x) = 1 + 9(\sum_{i=2}^N x_i) / (N-1)$	$x_1 \in [0, 1]$ $x_i = 0$ $i = 2, \dots, N$	nonconvex
ZDT3	30	$[0, 1]$	$f_1 = x_1$ $f_2 = g(x) [1 - \sqrt{x_1 / g(x)} - \frac{x_1}{g(x)} \sin(10\pi x_1)]$ $g(x) = 1 + 9(\sum_{i=2}^N x_i) / (N-1)$	$x_1 \in [0, 1]$ $x_i = 0$ $i = 2, \dots, N$	convex, disconnected
ZDT6	10	$[0, 1]$	$f_1 = 1 - \exp(-4x_1) \sin^6(6\pi x_1)$ $f_2 = g(x) [1 - (f_1 / g(x))^2]$ $g(x) = 1 + 9(\sum_{i=2}^N x_i) / (N-1)^{0.25}$	$x_1 \in [0, 1]$ $x_i = 0$ $i = 2, \dots, N$	nonconvex, nonuniformly spaced
UF1	10	$x_1 \in [0, 1]$ $x_i \in [-1, 1]$ $i = 2, \dots, N$	$f_1 = x_1 + \frac{2}{ J_1 } \sum_{i \in J_1} [x_i - \sin(6\pi x_i + \frac{i\pi}{N})]^2$ $f_2 = 1 - \sqrt{x_1} + \frac{2}{ J_2 } \sum_{i \in J_2} [x_i - \sin(6\pi x_i + \frac{i\pi}{N})]^2$ $J_1 = \{i i \text{ is odd and } 2 \leq i \leq N\}$ $J_2 = \{i i \text{ is even and } 2 \leq i \leq N\}$	$x_1 \in [0, 1]$ $x_i = \sin(6\pi x_1 + \frac{i\pi}{N})$ $i = 2, \dots, N$	convex

Tab. 1. Test problems used in this study (minimization of both objectives).

In order to illustrate the effectiveness of algorithm, the test functions [11]–[14] in Tab. 1 were used to test the performances of the proposed self-adaptive NSGA-II, con-

ventional NSGA-II and MOPSO. The table also shows the number of variables, their bounds, the Pareto-optimal solutions, and the nature of the Pareto-optimal front for each problem.

There are two goals in a multi-objective optimization: 1) convergence to the true Pareto-optimal front and 2) maintenance of diversity in solutions of the true Pareto-optimal front. So two performance metrics, convergence metric c [15] and spacing metric s [16], are introduced to evaluate the performance of the proposed algorithm. They are defined as follows:

$$c = \frac{\sum_{i=1}^A d_i}{A}, \tag{3}$$

$$s = \sqrt{\frac{1}{A-1} \sum_{i=1}^{A-1} (p_i - \bar{p})^2} \tag{4}$$

where A is the number of Pareto-optimal solutions obtained with the algorithm, d_i is the Euclidian distance in the objective space between the i th solution and the closest solution from 1000 uniformly spaced members of the true Pareto front, p_i is the Euclidean distance between two consecutive solutions in the obtained non-dominated set of solutions, and \bar{p} is the average of all distances p_i . It is clear that the lower values of the convergence metric c and spacing metric s represent the better convergence ability and diversity character.

2.4 Simulation Results and Discussion

Population size was $N_p = 50$ and maximum iteration number was 50 for three algorithms. In the conventional

NSGA-II, the crossover probability of $p_c = 1/N$ and mutation probability of $p_m = 1/N$ (where N is the number of decision variables) were used. In self-adaptive NSGA-II, $K_1 = 1$ and $K_2 = 0.3$. In MOPSO, the mutation rate is 0.5 and 15 divisions for the adaptive grid. Each test problem was calculated for 100-times. The best values, the mean values and the standard deviations of convergence metric c and spacing metric s are displayed in Tab. 2 and Tab. 3, respectively.

From Tab. 2, we can see that the proposed self-adaptive NSGA-II has better convergence ability than the conventional NSGA-II and MOPSO in all problems except FON problem, where MOPSO has better convergence. From Tab. 3, we can see that self-adaptive NSGA-II's solutions are able to spread better in all problems except in DEB and UF1 problems, where the conventional NSGA-II or MOPSO takes on better convergence.

3. Reconfigurable Pixel Antenna Design and Optimization

3.1 Antenna Structure

As the adaptable structures for reconfiguration, the pixel patch antennas have been proposed in [17]-[19]. A typical reconfigurable pixel antenna generally consists of uniform-size electrically small metallic patches which are interconnected by RF-switches. By controlling the states of switches, the pixel patches can synthesize a rich variety of antenna shapes flexibly. However, because of requiring a large number of patches and switches, the reconfigurable pixel antenna has an extremely large complexity, which

Problem	NSGA-II			Self-adaptive NSGA-II			MOPSO		
	Best	Mean	Std. Dev	Best	Mean	Std. Dev	Best	Mean	Std. Dev
SCH	0.0179	0.0358	1.20×10^{-4}	0.0138	0.0255	4.71×10^{-5}	0.0165	0.0433	4.05×10^{-4}
FON	0.0057	0.0084	2.12×10^{-6}	0.0045	0.0090	3.44×10^{-6}	0.0004	0.0023	1.89×10^{-6}
DEB	0.0002	0.5359	1.73×10^{-1}	0.0002	0.3956	5.09×10^{-2}	0.0033	0.5152	4.33×10^{-2}
ZDT1	0.0675	0.1589	1.40×10^{-3}	0.0193	0.0729	6.94×10^{-4}	2.0785	2.7478	5.67×10^{-2}
ZDT2	0.0824	0.1982	2.13×10^{-3}	0.0093	0.0871	2.09×10^{-3}	2.0680	2.7822	7.10×10^{-2}
ZDT3	0.0215	0.1042	2.30×10^{-3}	0.0073	0.0561	7.54×10^{-4}	1.4321	2.0050	4.64×10^{-2}
ZDT6	0.0000	0.0008	2.84×10^{-5}	0.0000	0.0004	2.70×10^{-6}	4.5575	5.9191	1.47×10^{-1}
UF1	0.0129	0.0313	9.50×10^{-5}	0.0134	0.0285	5.88×10^{-5}	0.0505	0.0933	4.30×10^{-4}

Tab. 2. Best, mean and standard deviation of the convergence metric. (Bold fonts indicate the best results.)

Problem	NSGA-II			Self-adaptive NSGA-II			MOPSO		
	Best	Mean	Std. Dev	Best	Mean	Std. Dev	Best	Mean	Std. Dev
SCH	0.0089	0.0177	3.23×10^{-5}	0.0087	0.0170	4.11×10^{-5}	0.0064	0.0171	2.76×10^{-5}
FON	0.0147	0.0197	5.34×10^{-6}	0.0133	0.0192	5.53×10^{-6}	0.0146	0.0206	6.11×10^{-6}
DEB	0.1242	0.2625	6.86×10^{-2}	0.1848	0.4502	1.86×10^{-2}	0.1015	0.5718	7.50×10^{-2}
ZDT1	0.0488	0.0841	4.19×10^{-4}	0.0254	0.0472	9.63×10^{-5}	0.0632	0.2183	8.70×10^{-3}
ZDT2	0.0474	0.1690	4.70×10^{-3}	0.0297	0.0860	9.47×10^{-4}	0.0213	0.0930	3.50×10^{-3}
ZDT3	0.0732	0.1421	8.54×10^{-4}	0.0431	0.0792	3.99×10^{-4}	0.0429	0.0819	1.20×10^{-3}
ZDT6	0.0383	0.0740	1.40×10^{-3}	0.0346	0.0683	3.50×10^{-4}	0.0021	0.1039	5.80×10^{-3}
UF1	0.0401	0.0723	1.46×10^{-4}	0.0378	0.0584	9.13×10^{-5}	0.0228	0.0471	1.82×10^{-4}

Tab. 3. Best, mean and standard deviation of the spacing metric. (Bold fonts indicate the best results.)

significantly impacts the antenna cost and efficiency. And in the uniform-size pixel antenna, the contribution of each pixel patch or switch to the antenna reconfiguration may be different. The contribution decreases when the distance from each patch or switch to the RF-port increases. To overcome this drawback, the multi-size pixel technique is proposed in [18].

In this paper, we proposed a three-size pixel antenna with reconfigurable radiation patterns. Its configuration is depicted in Fig. 2. All the patches are interconnected by switches and printed on an FR4 substrate with a relative permittivity of 4.4, a thickness of h and a loss tangent of 0.02. The antenna is fed by a 50 Ω probe with a distance d away from the center point. The values of all parameters are given in Tab. 4.

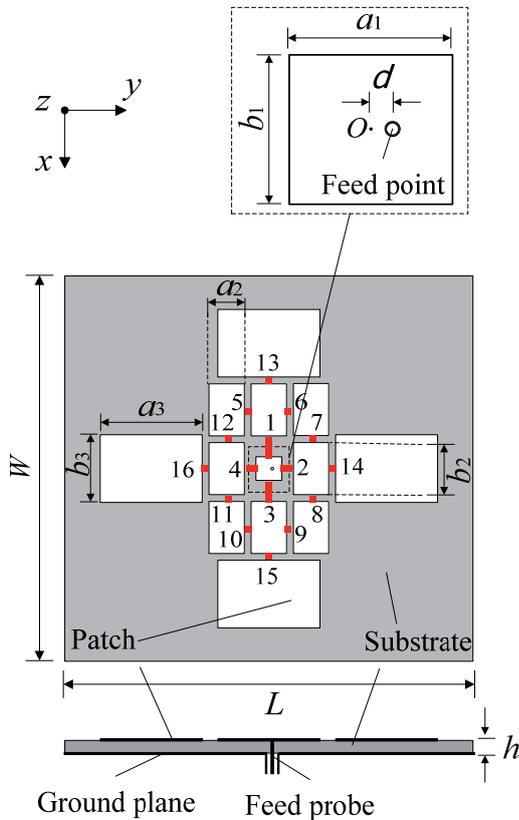


Fig. 2. Configuration of the proposed antenna.

Parameters	Value(mm)	Parameters	Value(mm)
a_1	3.8	b_3	10
b_1	3.5	h	1.9
a_2	5.2	d	0.5
b_2	7.7	W	85
a_3	15	L	90

Tab. 4. Parameter values of the proposed antenna.

3.2 Antenna Optimization

Combined with the HFSS solver, the proposed self-adaptive NSGA-II was employed to optimize the antenna. The interface between Matlab and HFSS was implemented through the Matlab-API (application program interface)

files [20]-[21]. In the optimization process, the optimized parameters were the states of all interconnecting switches which are placed between adjacent metallic pixel patches. These parameters were encoded in a binary string, in which one gene represented the state of one switch. "1" denoted the state of "ON" of a switch, and "0" denoted the state of "OFF". The proposed antenna consists of 16 switches, so there are 2^{16} possible permutations of switch states to be investigated in search space. Self-adaptive NSGA-II program was the main body of the whole calculation, which produced optimized parameters of switch states of next generation and controlled HFSS with VBScript by sending HFSS these parameters. With the VBScript, HFSS returned its results to self-adaptive NSGA-II for calculating the fitnesses after finishing the simulation of the present generation. The two processes were being run alternately until the program was terminated.

The antenna performance of the gains in specific directions and bandwidth for $S_{11} < -10$ dB at the operating frequency were involved. So the objective functions were defined as follows:

$$obf_{\text{Gain}}(f_0, \theta_{\text{DOA}}, \phi_{\text{DOA}}) = \begin{cases} -1, & \text{Gain} > 8 \text{ dB} \\ -\text{Gain}/8, & 0 \text{ dB} < \text{Gain} < 8 \text{ dB} \\ 0, & \text{Gain} < 0 \text{ dB} \end{cases} \quad (5)$$

$$obf_{\text{BW}}(f_0, S_{11}) = -(f_H - f_L) \quad (6)$$

where f_0 is the expected frequency, f_H and f_L represent the higher frequency and lower frequency respectively for the range of an acceptable value of S_{11} , θ_{DOA} and ϕ_{DOA} describe the main lobe direction, and S_{11} represents the magnitude of reflection coefficient. In our optimization process, $f_0 = 6$ GHz, $\theta_{\text{DOA}} = \pm 15^\circ, \pm 30^\circ, \pm 50^\circ$ and $\phi_{\text{DOA}} = 0^\circ$ were considered. Population size was $N_p = 50$, maximum iteration number was 50, $K_1 = 1$, and $K_2 = 0.3$.

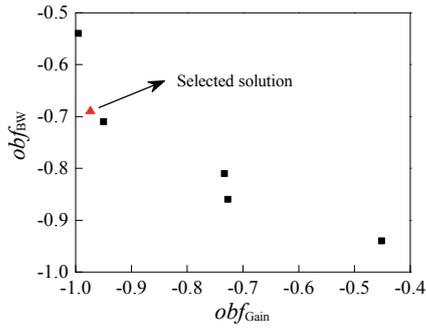
3.3 Results and Discussion

Because of the symmetrical feature of the antenna, once the mode with the tilt angle θ is found, the other mode of $-\theta$ can be obtained by mirroring the switch states along the y -axis. So three tilt directions, $\theta_{\text{DOA}} = +15^\circ, +30^\circ, +50^\circ$, were optimized and the Pareto-optimal solutions are shown in Fig. 3. Compared to those test functions used in Tab. 1, the search space of the antenna optimization problem contains fewer solutions and the nature of the Pareto-optimal front is indeterminate. In the optimization using self-adaptive NSGA-II, there are six or seven non-dominated solutions for the three tilt directions. To select the best designs from the set of Pareto optimal solutions, the overlapping impedance bandwidth and the maximum gain were taken into consideration. The selected solutions are marked in Fig. 3. Tab. 5 lists the chromosome descriptions of the switch states for six operating modes.

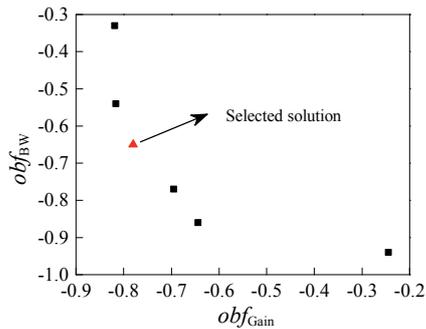
The reflection coefficients of all operating modes are shown in Fig. 4. The center frequency for all of the operating modes is around 5.98 GHz with a 5% overlapping im-

Switch	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\theta_{DOA} = -50^\circ$	1	1	1	1	1	0	0	1	1	1	1	0	0	1	1	1
$\theta_{DOA} = -30^\circ$	1	1	1	0	1	0	1	0	0	1	1	1	1	1	0	0
$\theta_{DOA} = -15^\circ$	1	1	1	0	1	1	1	0	0	1	0	1	0	1	1	1
$\theta_{DOA} = +15^\circ$	1	1	1	0	1	0	0	1	1	1	1	0	1	1	0	1
$\theta_{DOA} = +30^\circ$	1	1	1	0	1	0	1	1	0	1	1	1	0	1	1	0
$\theta_{DOA} = +50^\circ$	1	1	1	1	1	1	1	0	0	1	0	1	1	1	0	1

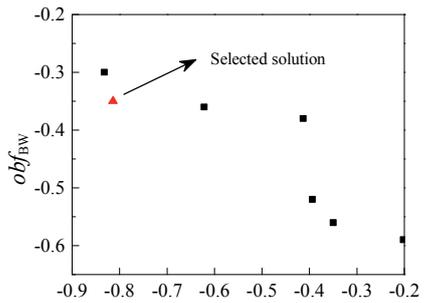
Tab. 5. Chromosome descriptions of the optimized switch states.



(a) $\theta_{DOA} = +15^\circ$



(b) $\theta_{DOA} = +30^\circ$



(c) $\theta_{DOA} = +50^\circ$

Fig. 3. Pareto optimal solutions.

pedance bandwidth ($S_{11} < -10$ dB). Because the switch states of the modes with tilt angles θ and $-\theta$ are symmetrical along the y -axis, their reflection coefficients are the same, which can be seen in Fig. 4.

Fig. 5 shows the realized gain patterns at 5.98 GHz in six different directions of x - z plane. It can be observed that the multi-size pixel patches can effectively produce significant modifications in the radiation pattern shape, with the capability of steering the main beam over a range of $\pm 50^\circ$.

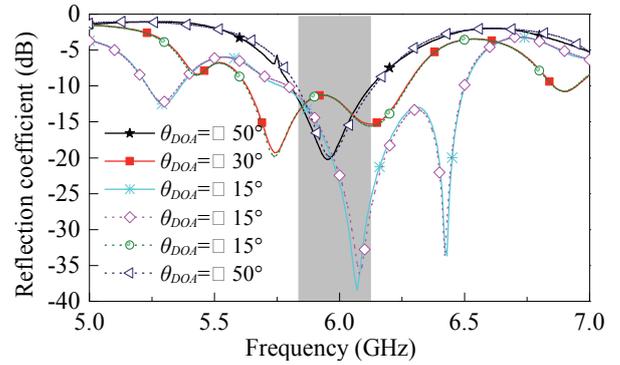


Fig. 4. Reflection coefficients of six modes of operation with a 5% overlapping bandwidth highlighted.

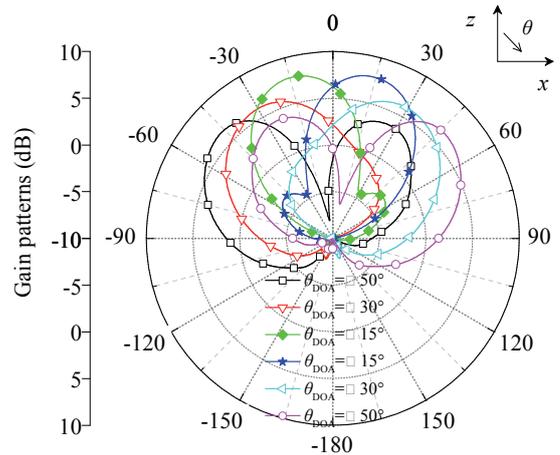


Fig. 5. Realized gain patterns in x - z plane for six different directions.

The realized gain values for all the operating modes are over 6 dB.

4. Conclusions

A self-adaptive NSGA-II, which can find better spread of solutions and better convergence near the true Pareto-optimal front, is proposed for optimization design in this paper. To show its effective improvement, several typical benchmark functions are tested. Using this self-adaptive NSGA-II, a three-size pixel antenna with reconfigurable radiation patterns is optimized and designed in electromagnetic engineering. The simulated results demonstrate the high-quality performance of the proposed algorithm.

Acknowledgements

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