

# Optimizations of Patch Antenna Arrays Using Genetic Algorithms Supported by the Multilevel Fast Multipole Algorithm

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**Abstract.** We present optimizations of patch antenna arrays using genetic algorithms and highly accurate full-wave solutions of the corresponding radiation problems with the multilevel fast multipole algorithm (MLFMA). Arrays of finite extent are analyzed by using MLFMA, which accounts for all mutual couplings between array elements efficiently and accurately. Using the superposition principle, the number of solutions required for the optimization of an array is reduced to the number of array elements, without resorting to any periodicity and similarity assumptions. Based on numerical experiments, genetic optimizations are improved by considering alternative mutation, crossover, and elitism mechanisms. We show that the developed optimization environment based on genetic algorithms and MLFMA provides efficient and effective optimizations of antenna excitations, which cannot be obtained with array-factor approaches, even for relatively simple arrays with identical elements.

## Keywords

Antenna arrays, patch antennas, antenna optimizations, genetic algorithms, multilevel fast multipole algorithm.

## 1. Introduction

Antenna arrays often need to be optimized for desired radiation characteristics, such as lower side-lobe levels, wider or narrower beamwidths, and higher radiations at desired directions [1]–[6]. Optimizations of the directive gain for beam-steering are well known in the literature as exemplars of array optimizations [7]. Given an arrangement of antennas usually with fixed positions, a particular aim is to determine a set of source values (excitations) that provide the desired radiation pattern, e.g., maximum directive gain at a specific direction. Along this direction, optimizations using the array-factor approach and similar analytical methods are known to provide rapid designs of excitations [8], while their accuracy and reliability may deteriorate significantly in many cases, particularly when mutual couplings between antennas have significant effects on

overall radiation patterns [9],[10]. These antenna interactions can be modeled very accurately via full-wave solvers based on error-controllable applications of Maxwell's equations, while these solvers must be very efficient for reasonable optimization times [11]. In addition, the integration of these solvers into optimization mechanisms may not be trivial [12].

In this work, we present an optimization environment based on genetic algorithms [13], which are supported by full-wave simulations with the multilevel fast multipole algorithm (MLFMA) [14],[15]. Patch antenna arrays of finite extent are formulated with the electric-field integral equation (EFIE) [16] in phasor domain and solved iteratively via MLFMA that provides fast and accurate matrix-vector multiplications required for iterative solutions. Without any simplification and assumptions such as periodicity and similarity of array elements, the superposition principle [17] is used to reduce the total number of MLFMA solutions to the number of elements. Complex radiated fields obtained with MLFMA-accelerated solutions are used by genetic algorithms for optimizations of excitations to obtain desired radiation characteristics. Using MLFMA, all mutual couplings between array elements are accurately included in the optimizations. Convergences of genetic optimizations are improved significantly by modifying major operations, such as mutation, crossover, and elitism. The effectiveness of the constructed optimization environment is demonstrated on an array of  $5 \times 5$  patch antennas. We show that, despite its relatively simple geometry, such an array can be optimized very effectively with genetic algorithms and MLFMA solutions, in contrast to the array-factor approach. We further present the radiation patterns with optimal directive gains in various directions obtained by the optimizations of antenna excitations.

Details of array optimizations via a full-wave solver are considered in the next section, which also presents some important parameters of MLFMA simulations of antenna arrays. Section 3 is devoted to genetic algorithms, including improvements for efficient optimizations, followed by numerical results in Section 4 and concluding remarks in Section 5.

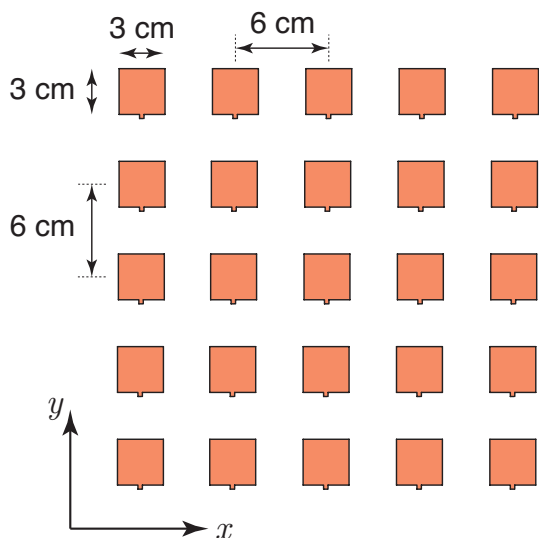


Fig. 1. A  $5 \times 5$  array involving a periodic arrangement of  $3 \text{ cm} \times 3 \text{ cm}$  patch antennas.

## 2. Optimizations of Arrays Using Full-Wave Simulations

Figure 1 presents a  $5 \times 5$  array of patch antennas, which is considered as a test problem in this paper. The patch antennas of size  $3 \text{ cm} \times 3 \text{ cm}$  are arranged periodically with  $6 \text{ cm}$  periods, and each antenna is excited from its feed port at the bottom, where the source is modeled as current injection. The antennas are oriented in the same direction, even though this is not a restriction using a full-wave solver. Using a simulation environment, our aim is to find the most appropriate source values such that the far-zone radiation of the overall array satisfies desired characteristics as much as possible. In this particular case, we maximize the directive gain in various desired directions when the frequency is fixed to  $2.45 \text{ GHz}$ . While the optimization operations, improvements, and results in this paper are demonstrated on the  $5 \times 5$  array in Fig. 1, they can be generalized to arbitrary arrays with different (and especially larger) numbers of elements, without any periodicity, infinity, or similarity assumptions.

In any optimization problem, various sets of values for the optimization variables need to be tested. In our optimizations, each trial corresponds to the solution of a computational problem, where excitations of array elements are used to generate the overall radiation pattern. If mutual couplings between array elements (patch antennas) are to be considered, each set of excitations creates a unique distribution of the electric current on antenna surfaces. On the other hand, using the superposition principle, one can reduce the number of current-density computations to the number of array elements, without omitting mutual couplings. In each solution, only one antenna is excited with a unit source, while all

others are passive (with zero source). Hence, for the array in Fig. 1, a total of 25 solutions are actually needed. Once all these solutions are completed, far-field radiation patterns are stored in memory to be used many times during optimizations.

In order to include all mutual couplings between array elements, we use a full-wave solver based on MLFMA. Although the number of numerical solutions can be reduced significantly via the superposition principle, MLFMA is needed to accelerate solutions without sacrificing the accuracy of results. The radiation problems are formulated with EFIE, which is suitable for open surfaces with zero thickness. After the triangulation of surfaces, Rao-Wilton-Glisson functions [16] are employed to expand the electric current density. Using  $400\text{--}500$  unknowns per antenna, approximately  $10,000$  unknowns are used to model the entire array in Fig. 1. The resulting dense matrix equations are solved iteratively, where MLFMA is used to accelerate matrix-vector multiplications and to reduce the computational complexity to a linearithmic level.

MLFMA is well known in the literature [15], and details of this powerful algorithm are omitted in this paper for the sake of brevity. Nevertheless, there are several issues regarding the use of MLFMA for the optimization of arrays. In general, an iterative solution using MLFMA has two major stages, namely, setup and solution parts. During a setup part, near-zone interactions that are between nearby discretization elements are computed and stored in memory. Far-zone interactions, however, are calculated on the fly in each matrix-vector multiplication. Since excitation only affects the right-hand side of matrix equations, near-zone interactions are fixed and do not change for a given optimization problem. Therefore, only one setup part needs to be executed for the optimization of an array. In fact, if the geometry and discretization do not change, the same set of near-field interactions can be used for different optimizations of the same array. The results presented in this paper are obtained by using a single set of near-field interactions related to the  $5 \times 5$  array depicted in Fig. 1.

Using identical elements in an array may further improve the efficiency of MLFMA solutions. Since such elements can be discretized identically, near-field interactions as well as radiation/receiving patterns used during aggregation-translation-disaggregation cycles of MLFMA contain many identical numerical elements. Processing time and storage requirements can be reduced via careful indexing operations, while these may deteriorate the applicability of the solver to general problems involving non-identical and aperiodic arrays. In fact, for a relatively small array, such as depicted in Fig. 1, all required solutions can be performed in minutes (in the MATLAB [18] environment) without resorting to any such indexing operations.

Finally, we list the accuracy parameters of array simulations using MLFMA. All interactions are calculated with maximum  $1\%$  error. Near-field interactions are computed

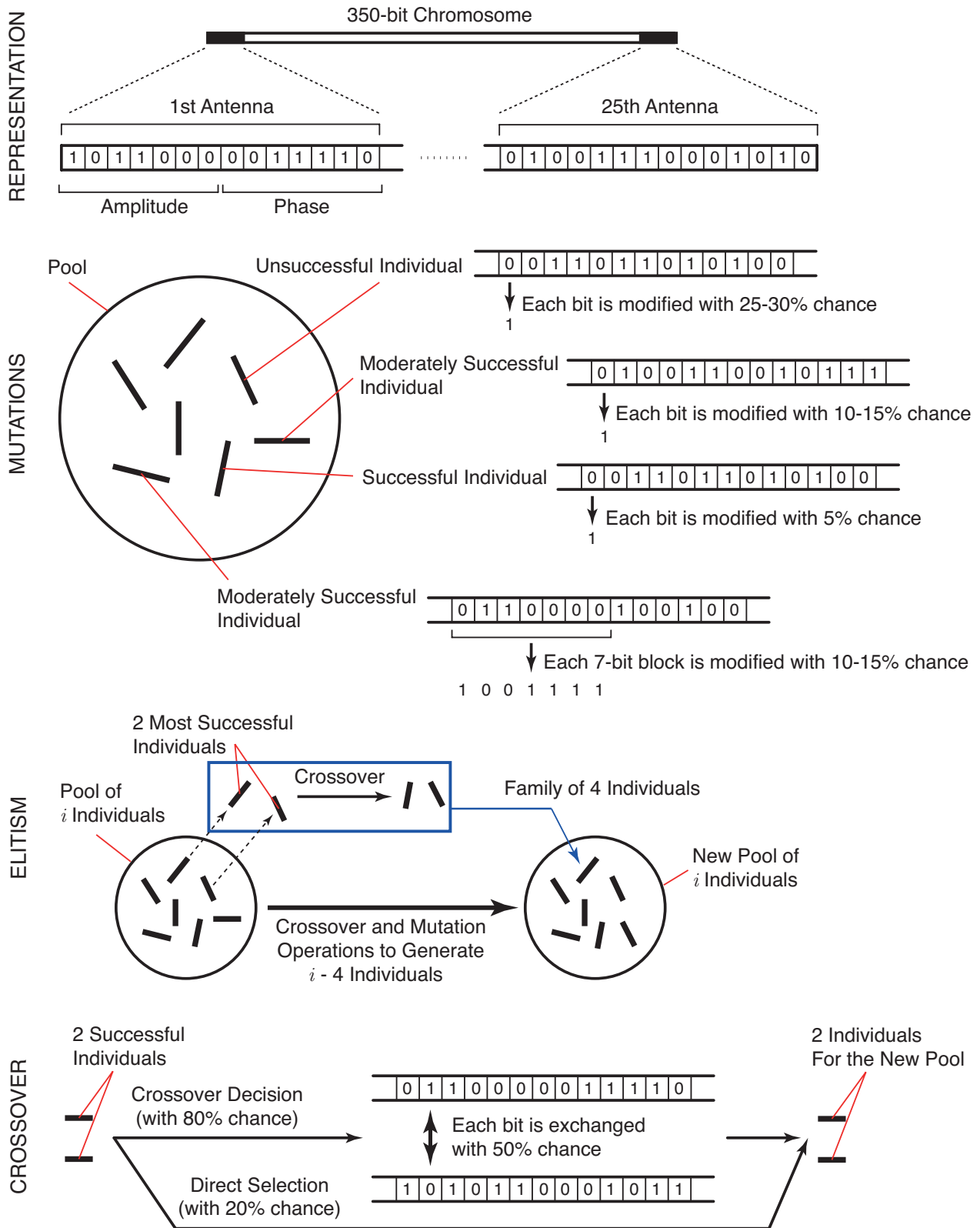


Fig. 2. Description of major operations and probability rates used in the improved genetic algorithms.

by extracting the singularity of the Green's function and dividing triangular integrations into analytical and computa-

tional parts. In far-zone interactions, excess bandwidth formulas [14] are used to determine truncation numbers. Be-

tween levels, Lagrange interpolations with  $2 \times 2$  stencils are used to match sampling rates of radiated and incoming fields. Iterative solutions are performed with 0.001 target residual errors, which can be reached in 70–100 iterations using GMRES [19] without preconditioning for the  $5 \times 5$  array in Fig. 1.

### 3. Genetic Algorithms for Array Optimizations

Being among most popular heuristic optimization techniques, genetic algorithms are suitable particularly for electromagnetic problems by providing great flexibility in terms of cost functions [13]. Similar to other heuristic algorithms, however, performance of genetic algorithms may vary significantly depending on their parameters, and they must be carefully designed for a given problem. In general, genetic algorithms work on pools of individuals, each of which represents a trial for the optimization. The trial information is coded as a chromosome, which is used in evolution operations, such as crossover, mutation, elitism, and in general, creating new generations. Each trial also has a success rate (fitness), which corresponds to the directive gain in a desired direction in our optimizations.

Using genetic algorithms, a critical point is to generate a suitable map between optimization parameters and chromosomes. For the initial optimizations presented in this paper, real and imaginary parts of complex antenna excitations are allowed to be in the  $[-1, 1]$  range. Both ranges are divided into 127 equal intervals, leading to a 7-bit coding per variable. Hence, for the  $5 \times 5$  array in Fig. 1, each chromosome contains  $25 \times 2 \times 7 = 350$  bits (zeros and ones) to account for all antennas, amplitudes, and phases. As discussed in Section 4, choosing amplitudes equally may significantly reduce the decision space, leading to more efficient optimizations. In these optimizations, phases of excitations are sampled in the range from 0 to  $360^\circ$ , leading to 175-bit chromosomes. We note that different chromosomes may correspond to the same scenario; for example, in the constant-amplitude optimizations, all antennas may have the same phase and this can occur in 128 different ways.

Obviously, the setup described above leads to huge decision spaces. Using 350 bits, there are  $2^{350} \approx 2.3 \times 10^{105}$  different combinations for the excitations of antennas. Making excitation amplitudes constant reduces this number to  $4.8 \times 10^{52}$ , which is still very challenging for high-quality optimizations. Based on many experiments with the genetic algorithms, we systematically improve optimizations by considering convergence characteristics and modifying optimization operations accordingly. Some important modifications, especially in contrast to a *conventional genetic algorithm*, are listed below and depicted in Fig. 2.

1. Mutations: Instead of a single mutation rate (that is fixed to 5% in our conventional algorithm), we use

different mutation rates in the same pool to accelerate convergences. Specifically, heavy, moderate, and light mutation rates are applied to individuals, depending on their success rates. Individuals with low success rates are exposed to heavy mutations with 25–30% rates, i.e., each chromosome bit is changed with 25–30% probability, while more successful individuals are mutated less (e.g., 5% rates for the most successful individuals) to maintain the stability. For moderately successful individuals, we also use collective mutations, e.g., changing a portion of the chromosome rather than bit-by-bit mutations, so that badly excited antennas can be directly eliminated.

2. Crossover Operations: Instead of a popular one-point crossover scheme, we use bit-by-bit crossover operations between selected parents to generate children. Specifically, after a crossover is decided (with 80% rate) for a given pair of individuals, all corresponding bits are exchanged with 50% probability. This way, the variety of individuals in the pool increases significantly, leading to more efficient optimizations.
3. Elitism: In addition to reserving the most successful individuals for next generations, we also force them to mate. Specifically, for a given pool at a specific generation, we let two individuals and their children survive and exist in the next generation. This approach guarantees the quality of the pool during the entire optimization process.

All modifications described above (and shown in Fig. 2), as well as the given mutation rates and crossover probabilities, can be used for efficient optimizations of various arrays involving different numbers of elements. The pool size and the number of generations, however, depend on the processing time allowed for optimizations. In the optimizations of the  $5 \times 5$  array in Fig. 1, we use pools of 80 individuals, where the most successful individual represents the state of the optimization. The number of generations is limited to 2000, leading to a maximum of 160,000 trials. An optimization with 2000 generations takes around 45 minutes (on a single processor in the MATLAB environment), thanks to the superposition principle that allows for combinations of radiation patterns to efficiently obtain the overall radiation characteristics of the array. Specifically, for a given set of complex excitation coefficients  $e_n$  for  $n = 1, 2, \dots, 25$ , the radiation pattern of the array is obtained as

$$\bar{f}_A(\theta, \phi) = \sum_{n=1}^{25} e_n \bar{f}_n(\theta, \phi) \quad (1)$$

where  $\bar{f}_n(\theta, \phi)$  represents the radiation pattern of the  $n$ th antenna while other antennas are parasitic. Then, by calculating the power density as

$$p_A(\theta, \phi) = |\hat{\theta} \cdot \bar{f}_A(\theta, \phi)|^2 + |\hat{\phi} \cdot \bar{f}_A(\theta, \phi)|^2 \quad (2)$$

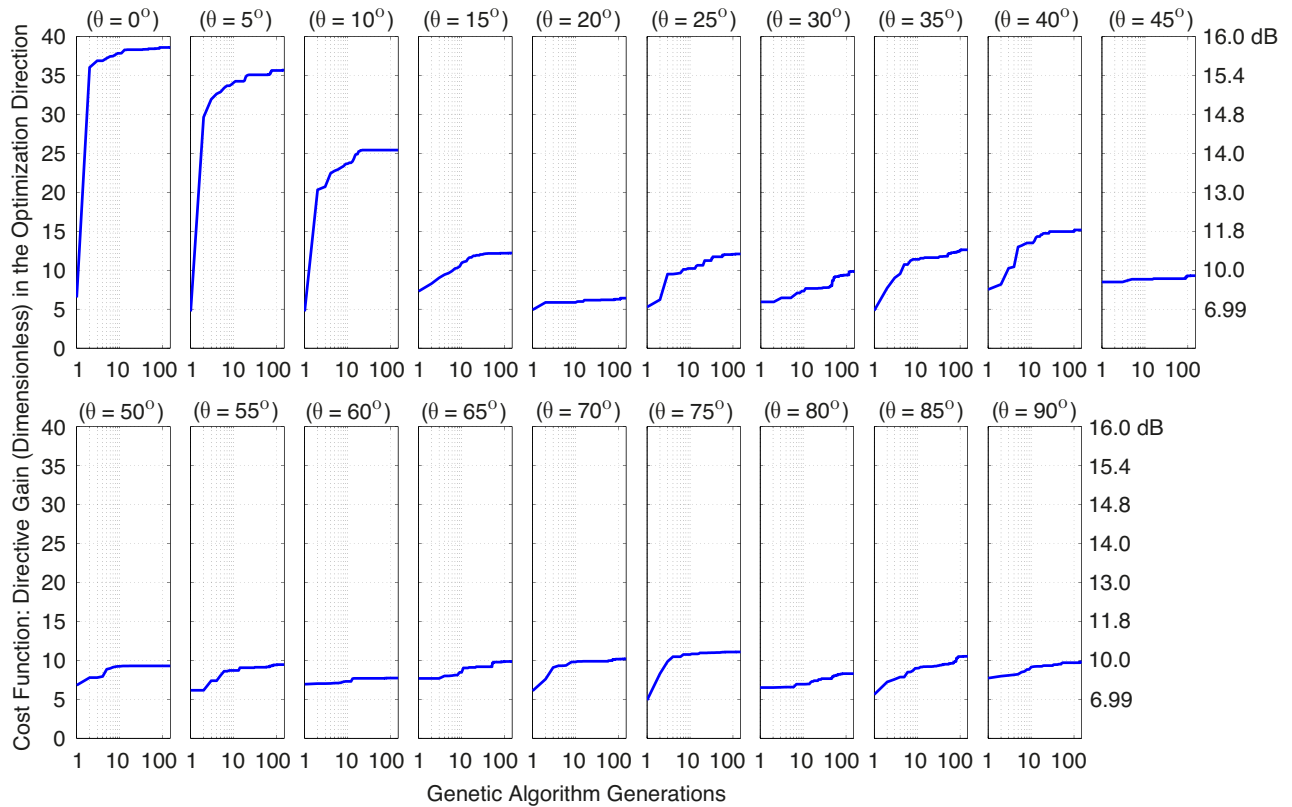


Fig. 3. Cost functions with respect to number of generations when the conventional genetic algorithm is used for the  $5 \times 5$  array in Fig. 1 to maximize its directive gain at various directions. The conventional genetic algorithm is used once to generate each plot.

we obtain the directive gain at any desired direction  $(\theta_0, \phi_0)$  as

$$d_A(\theta_0, \phi_0) = \frac{4\pi p_A(\theta_0, \phi_0)}{I_{total}} \quad (3)$$

where

$$I_{total} = \int_0^\pi \int_0^{2\pi} d\theta d\phi \sin\theta p_A(\theta, \phi). \quad (4)$$

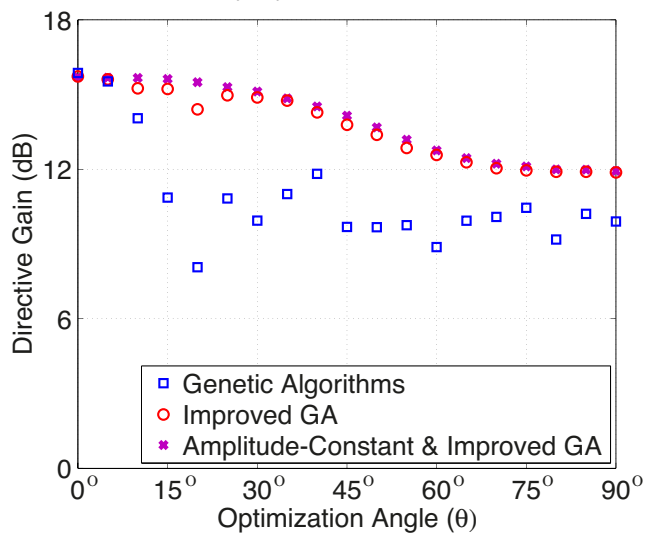


Fig. 5. Final optimization results for the  $5 \times 5$  array in Fig. 1.

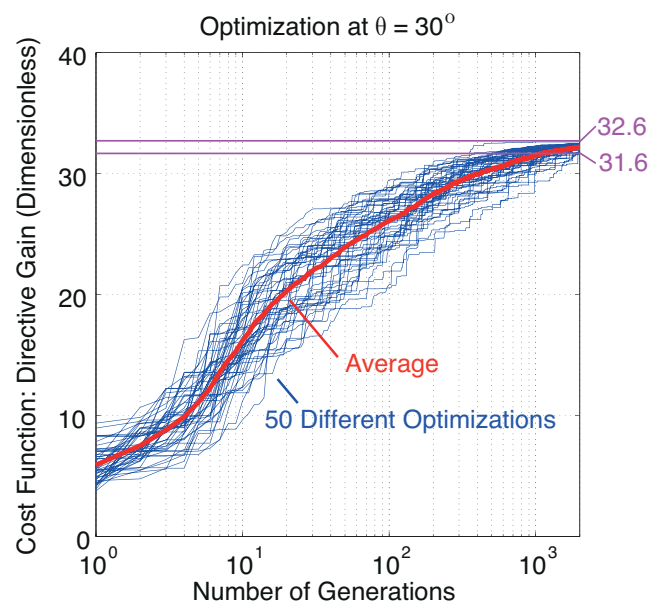


Fig. 6. Optimization histories for 50 different trials of the improved genetic algorithm to optimize the directive gain of the  $5 \times 5$  array in Fig. 1 at  $\theta = 30^\circ$ .

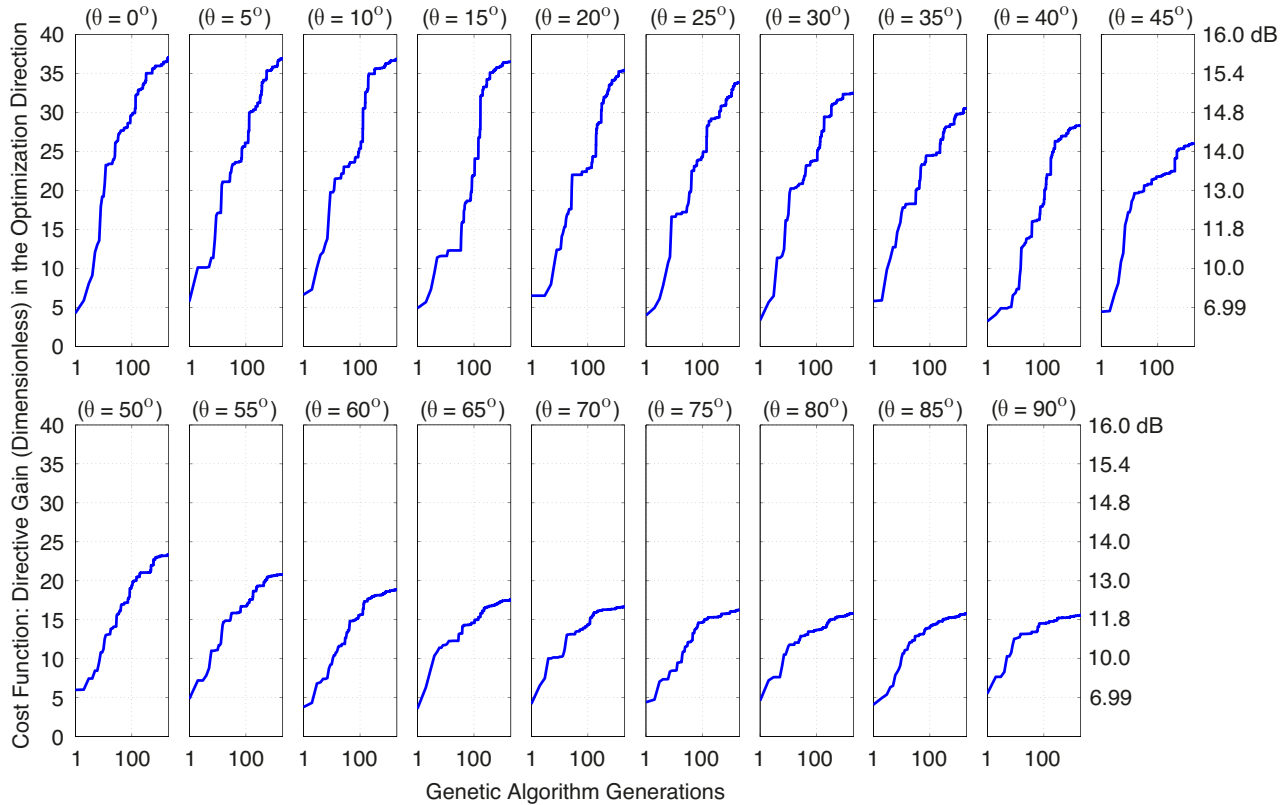


Fig. 4. Cost functions with respect to number of generations when the improved genetic algorithm is used for the  $5 \times 5$  array in Fig. 1 to maximize its directive gain at various directions. The improved genetic algorithm is used once to generate each plot.

## 4. Numerical Results

Figure 3 presents the results of the optimizations using the conventional genetic algorithm (described in Section 3) for the  $5 \times 5$  array in Fig. 1. The directive gain of the array is maximized at various directions on the  $z$ - $x$  plane from  $\theta = 0$  to  $90^\circ$ . For each optimization, the directive gain in the optimization direction is plotted with respect to the generations from 1 to 150. We note that the conventional genetic algorithm uses pools of 80 individuals, single-point crossover operations, 5% fixed mutation rate, and a standard elitism with two individuals. Both amplitudes and phases of excitations are considered as optimization parameters. We also note that all optimizations are stopped far below the 2000 limit due to the convergence of individuals (to each other). It can be observed that some optimizations, e.g., those for smaller optimization angles, are quite efficient with significant enhancements of the directive gain. In many cases, however, the genetic algorithm does not seem to improve the quality of the pools sufficiently. For example, at  $\theta = 45^\circ$ , the directive gain increases only from 8.49 to 9.30, where the optimization stagnates.

Figure 4 presents the results of similar optimizations using the improved genetic algorithm for the  $5 \times 5$  array. As described in Section 3, the improved algorithm uses bit-by-bit crossover operations, success-based mutation rates, and a

family elitism involving two individuals and their children, without a change in the pool size (80 individuals). In addition, in the optimizations presented in Fig. 4, only phases of antenna excitations are considered; but, this only stabilizes the optimizations (see below) while major improvements are due to the modification of the optimization algorithm. It can be observed that the optimizations are improved significantly, especially for problematic cases with higher optimization angles. In general, each optimization dramatically increases the directive gain at the desired direction, at least 3 times of the best case in the initial pool.

Figure 5 depicts the comparison of the final optimization results using the conventional and improved genetic algorithms. The maximized directive gain is plotted with respect to the optimization angle from  $0$  to  $90^\circ$ . Obviously, the improved genetic algorithm provides much better results than the conventional one, especially at higher optimization angles. Selecting constant excitation amplitudes and optimizing only excitation phases further stabilize optimizations by limiting the decision space. This is mostly visible at  $20^\circ$ , where an unusual dip for the improved genetic algorithm is eliminated in the amplitude-constant case.

Genetic algorithms are based on randomly generated pools and their evolutions via random operations. There-

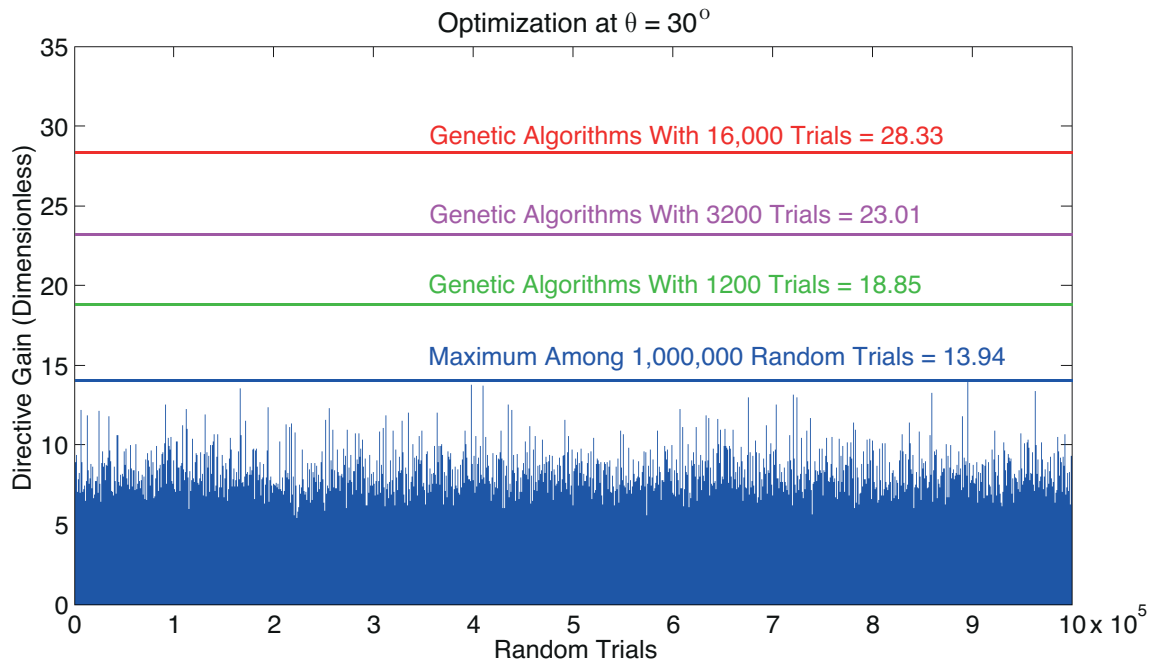


Fig. 7. Comparison of the improved genetic algorithm and random trials of constant-amplitude excitations for the optimization of the directive gain of the  $5 \times 5$  array in Fig. 1 at  $\theta = 30^\circ$ . For the genetic algorithm, average values based on 50 trials (see Fig. 6) are shown.

fore, the optimization history results in Figs. 3 and 4 tend to change in each execution, while they correctly represent the general behaviors of the conventional and improved genetic algorithms. In fact, as desired in all optimization studies, the final results in Fig. 5 change slightly for different executions. In order to demonstrate the stability of optimizations, Fig. 6 presents 50 different optimization histories when the improved genetic algorithm is used to maximize the directive gain at  $\theta = 30^\circ$ . It can be observed that, while the optimization histories may vary significantly, the final results (the most successful individuals in the converged pools) are very similar to each other. Specifically, the maximized value of the directive gain is in the range from 31.6 to 32.6, despite different qualities of pools at earlier generations.

Next, in order to demonstrate the effectiveness of genetic algorithms, Fig. 7 presents a comparison of the directive gain values obtained with the improved genetic algorithm and random selections of excitations. Among 1,000,000 random trials, the maximum directive gain at  $\theta = 30^\circ$  is only 13.94, while this value is easily exceeded by the genetic algorithm with 1200 trials (15 generations with a pool of 80 individuals). Then, with only 200 generations and a total of 16,000 trials, the directive gain is further increased to 28.33, which seems extremely difficult to reach via random trials.

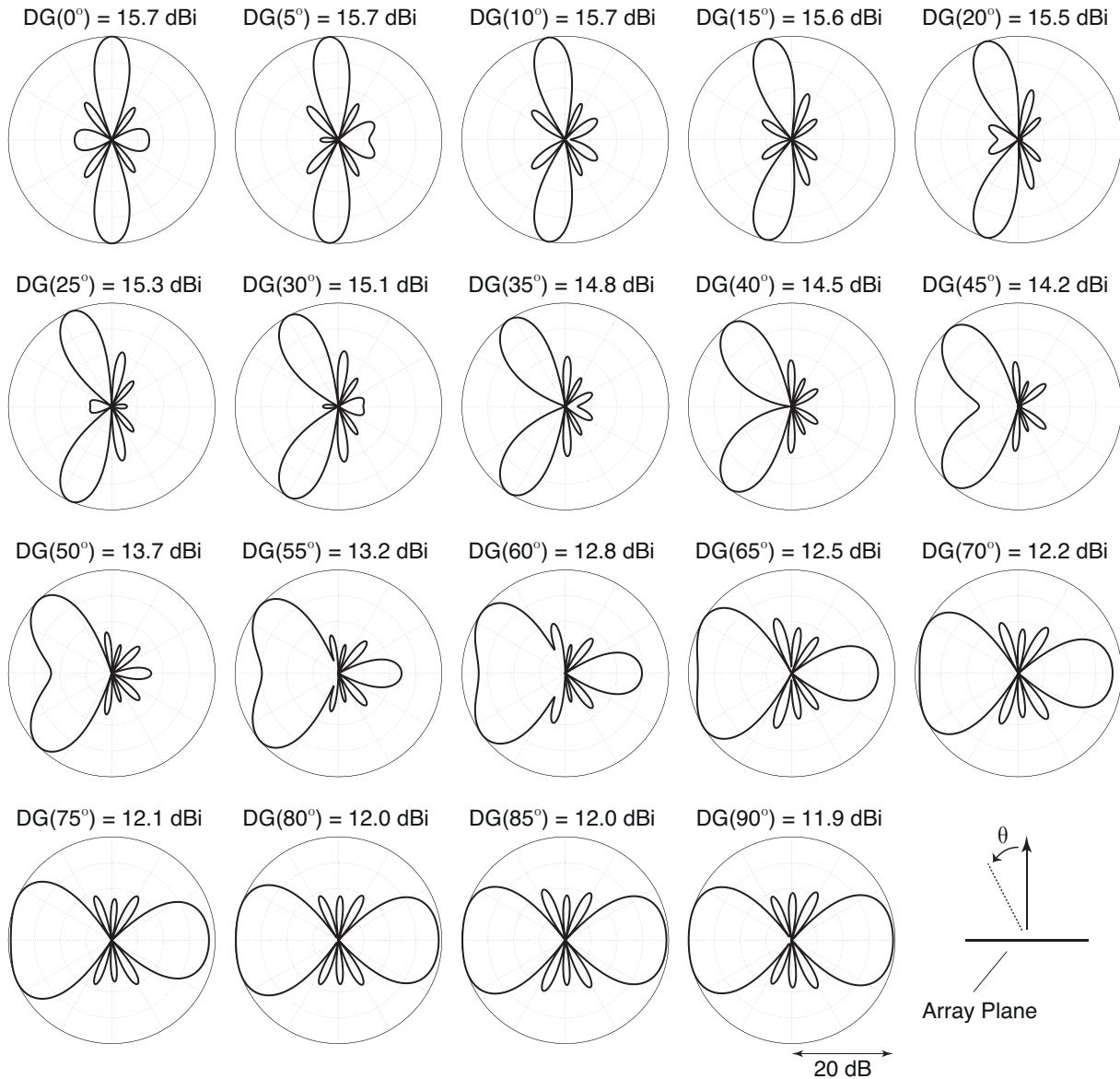
Figure 8 presents the far-zone radiation patterns of the  $5 \times 5$  array when its directive gain is optimized at different angles from  $\theta = 0^\circ$  to  $90^\circ$ . The normalized electric field

intensity is plotted in a 20 dB range on the  $z$ - $x$  plane as a function of the bistatic angle. In addition, the directive gain (DG) values at the optimization angles are indicated in the dBi scale. It can be observed that, with the optimization of the directive gain, a main lobe of the radiation is directed at the desired direction. Since the array is planar and its radiation is symmetric with respect to the array plane, optimizations at smaller angles lead to two main lobes, one of which is at the desired direction ( $\theta_0$ ) and the other one is at  $180^\circ - \theta_0$ . These lobes are combined at higher optimization angles, while another lobe appears in the opposite direction due to 2-D structure of the array.

Finally, in order to demonstrate the need for full-wave solutions in the optimizations, Fig. 9 depicts an optimized radiation pattern obtained with MLFMA solutions in comparison to a corresponding pattern obtained via the array-factor approach. Excitation phase values required for maximizing the directive gain at  $\theta = 30^\circ$  (obtained with the improved genetic algorithm and MLFMA solutions) are shown on the array. Using these values along with the radiation pattern of a single antenna in the array-factor approach leads to a significantly different pattern compared to that obtained with MLFMA. In fact, even the main lobes deviate from  $\theta = 30^\circ$  and  $150^\circ$ , clearly indicating that using the array-factor approach in such an optimization would be misleading.

## 5. Conclusions

We present efficient and accurate optimizations of an-



**Fig. 8.** Optimized radiation patterns of the  $5 \times 5$  array in Fig. 1. The normalized electric field intensity in the far-zone is plotted on the  $z$ - $x$  plane as a function of the bistatic angle.

tenna arrays using genetic algorithms and full-wave solutions via MLFMA.

Given an array, the superposition principle is used to reduce the number of numerical solutions to the number of array elements, while all mutual couplings between the elements are taken into account.

MLFMA is employed to accelerate full-wave solutions, where antennas are modeled accurately without any periodicity, infinity, or similarity assumptions. Complex radiated fields computed via MLFMA are used by genetic algorithms to find the most suitable antenna excitations for any desired radiation pattern of the overall array. As a test case, we present the optimizations of a  $5 \times 5$  array of patch antennas to maximize its directive gain at var-

ious directions. We show that performances of genetic algorithms can be improved by modifying basic operations, such as mutations, crossovers, and elitism, leading to very effective optimizations of antenna excitations.

We also show that, even for simple array configurations, optimizations with full-wave solutions can provide significantly more reliable results, compared to the array-factor approach. The proposed optimization mechanism can be applied to much larger and complex array configurations, whose solutions can be performed by MLFMA.

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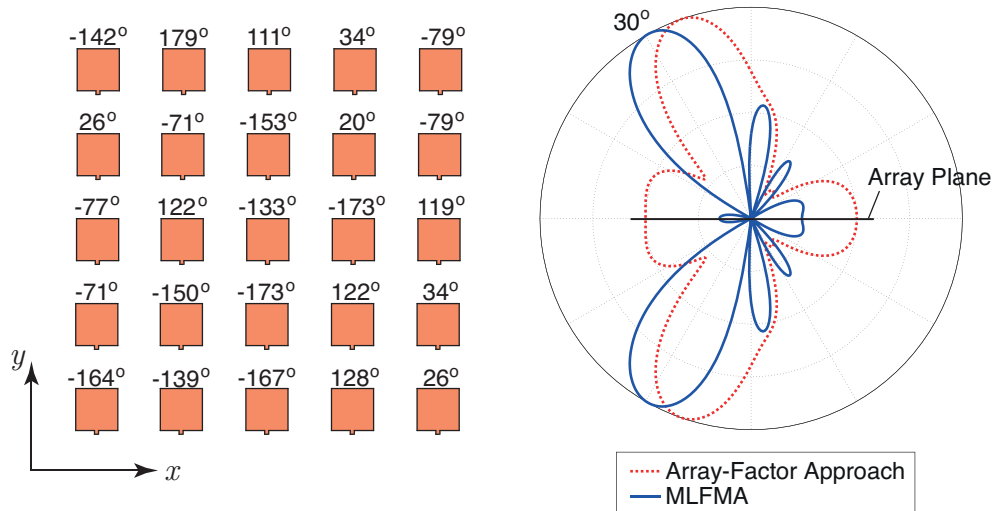


Fig. 9. Excitation phases obtained with the improved genetic algorithm to maximize the directive gain at  $30^\circ$ , and the corresponding radiation patterns obtained with MLFMA and the array-factor approach.

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