Utilization of Symmetry Properties for the Pattern Analysis of Mutually Coupled Patch Radiators

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Abstract

The influence of mutual coupling effects on the radiation pattern of microstrip antennas is investigated by introducing symmetry properties in the surface current distributions. The standard method of moments is used, which requires only the upper half of three x− and y− direction offset patch radiators to be solved, while the symmetric lower half is incorporated after the determination of the current distribution. Furthermore, the Toeplitz symmetry of the impedance matrix is taken into account, and the singular value decomposition is used to invert the resulting non-square matrix in a numerically stable way. Both the symmetry property and the Toeplitz symmetry procedures reduce storage and CPU-time requirements in the analysis of broadside microstrip arrays. The pattern characteristics calculated with and without mutual coupling interactions are found to differ up to 10 dB in sideloge levels which clearly demonstrates that the pattern can be as severely affected by mutual coupling as the commonly investigated input impedance. The predicted results are in good agreement with measured data on single patches.

1. Introduction

Patch radiators in standard and monolithic microwave integrated circuitry are well known for their excellent performance in light-weight and low-cost array architectures, e.g. [1,2]. A most comprehensive collection of papers on the analysis and/or design of single and coupled microstrip elements can be found in [3].

Most of the theoretical models presented, however, firstly assume that the patch is fed by a coaxial cable from below the ground plane, which complicates the manufacturing process and considerably reduces the achievable level of integration for feeding networks; secondly and more important, the effects of mutual coupling between elements are restricted to the determination of input and mutual coupling impedances, input reflection and transmission coefficients or other quantities defined in terms of circuit parameters rather than by electromagnetic field and pattern parameters. Methods have been presented, e.g. [4,5], to include the - from a network - integrational point of view - more appropriate feed mechanism of a microstrip line instead of a coaxial cable. The problems associated with the influence of mutual coupling effects on the radiation pattern of a set of circular microstrip antennas have been rigorously solved, e.g. [6]; however, as the mathematics involved are completely different, these solutions cannot be applied to rectangular patches.

A rigorous solution to the problem of a rectangular microstrip antenna fed by a microstripline was presented in [7]; the currents on the feed line and the patch are expanded in a suitable set of modes, and a moment method solution is formulated in the spectral domain. Similarly, finite phased arrays of rectangular microstrip patches have been treated [8] by the spectral domain moment method; a theory was developed for the active input impedance, as well as the active element pattern of the array.

Most of the theoretical models currently available to analyze rectangular patch antenna structures require substantial computer resources to handle and reliably solve the large matrix systems involved. Therefore, this paper focuses on a simplified method-of-moments algorithm to reduce storage and CPU-time requirements in the analysis of patch antenna configuartions. Symmetry properties in both the structure (c.f.Fig. 1) and the resulting impedance matrix are efficiently utilized to analyze the characteristics of single and multiple patches including mutual coupling effects.

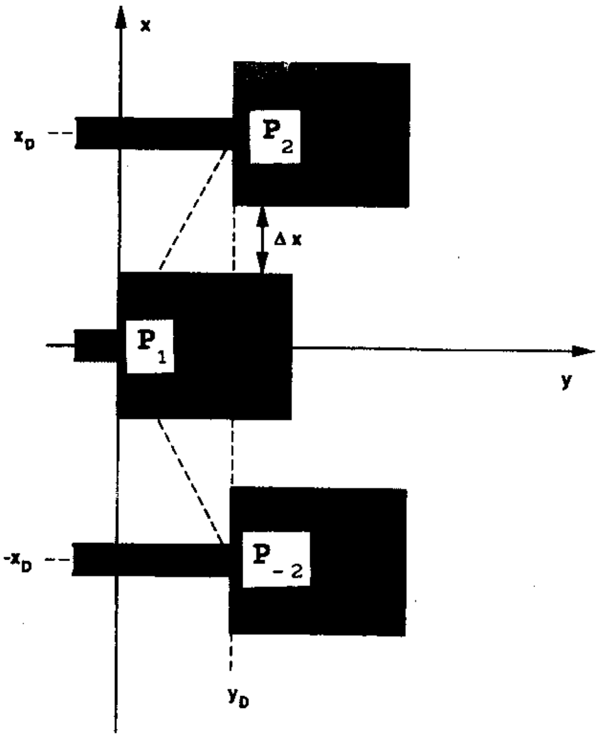

Fig. 1 Basic configuration of three coupled patch radiators.

The standard method-of-moments algorithm [9] is used to model the current distribution of the upper half of the array configuration, while the lower half is incorporated by utilizing a symmetric distribution. Due to the symmetry of the array configuration and Toeplitz symmetry of the impedance matrix, it is sufficient to evaluate complex elements of the upper of the impedance matrix and invert this non-square matrix utilizing the single value decomposition technique. The patterns of a three-element array calculated with included mutual coupling effects are compared with commonly used array factor analysis that neglects the interference of the radiators. Significant differences with respect

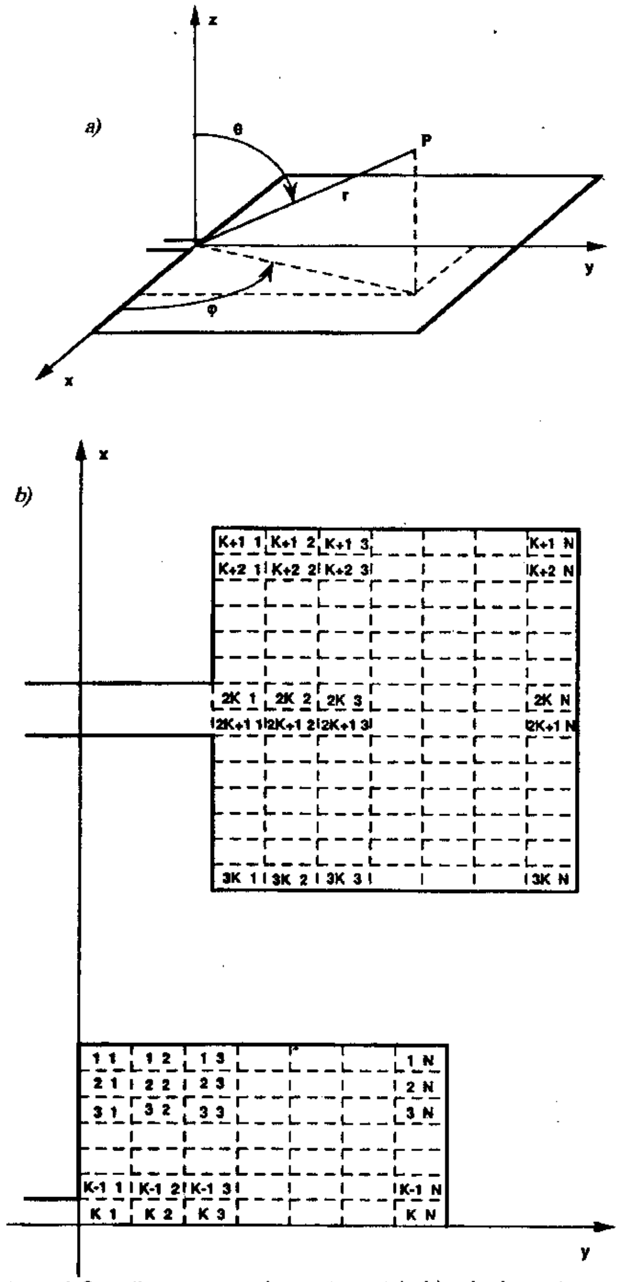

Fig. 2 a) Coordinate system for center patch; b) sub-element notation for radiators in the upper half-plane of Fig. 1.

to sidelobe levels and null positions demonstrate the necessity for field-theoretical models in the pattern analysis of microstrip antenna arrays.

2. Theory

The standard moment method algorithm, which is applied here, basically follows that outlined in [9]. Therefore, only the fundamental steps are presented here, and the reader is referred to [9] for further details. It should be noted, however, that the symmetry properties introduced are not restricted to a specific numerical model. In fact, they can be applied to any algorithm that solves for the current distribution on the patches.

Fig. 2a shows the location of a single element in the coordinate system. If the surface currents in $x-$direction are assumed to have a negligible infuence compared to those in $y-$direction, then the far field has only E_φ and $E\theta$ components in the $\varphi = 0$ and $\varphi = \pi/2$ planes, respectively. The tangential electric field on the surface of the patches

$$\vec{E} = j\omega\vec{A} + \frac{\partial\Phi}{\partial y}\vec{y}_0 \quad on\ S \tag{1}$$

is derived from well-known potential functions

$$\vec{A} = \mu \iint_S \vec{J}\,\frac{e^{-jkR}}{4\pi R}\,dS \tag{2}$$

$$\Phi = \frac{1}{\varepsilon}\iint_S \sigma\,\frac{e^{-jk_gR}}{4\pi R}\,dS \tag{3}$$

with the charge distribution

$$\sigma = -\frac{1}{j\omega}\frac{dI}{dS}$$

In (1)-(4), S is the area variable at the antenna surface and R is the distance measured from the center of an antenna sub-element to a field point in space. k_g is the propagation constant of a microstrip line of width w, substrate height h and permittivity $\varepsilon_0\,\varepsilon_{reff}$. The frequency dependence of $k_g = 2\pi/\lambda_g$ is taken into account according to [10].

The division into sub-elements for the upper-half-plane patches of Fig. 1 is shown in Fig. 2b. The sub-element notation corresponds to the location of elements of the impedance matrix to be derived. Similar to [9], the potential function

$$\psi(m,n) = \frac{1}{4\pi\Delta l_n}\int_{n^-}^{n^+}\frac{e^{-jk_gR_m}}{R_m}\,dy =$$

$$= \frac{1}{4\pi\Delta l_n}\int_{n^-}^{n^+}\left(\frac{1}{R_m} - jk_g - \frac{k_g^2}{2}R_m + \ldots\right)dy \tag{5}$$

with its approximation by a Maclaurin series is used. For $m = n$ the consideration of the first two real terms in (5) provides sufficient accuracy, and the corresponding integral

$$\psi(n,n) \approx \frac{1}{4\pi\Delta l_n} \int_{n^-}^{n^+} \left[\frac{1}{R_m} - \frac{k_g^2}{2} R_m \right] dy - \frac{jk_g}{4\pi} \tag{6}$$

is evaluated numerically. Since all sub-elements are of identical dimensions, (6) has to be computed only once. For $m \neq n$, R_m is considered constant, and hence

$$\psi(m,n) \approx \frac{e^{-jk_g R_{mn}}}{4\pi R_{mn}} \tag{7}$$

where R_{mn} is the distance between sub-elements m and n. The resulting matrix equation [9] for the upper half of the structure (Fig. 2b)

$$[\mathbf{Z}].[\mathbf{I}] = [\mathbf{V}] \tag{8}$$

needs to be solved for the current vector $[\mathbf{I}]$. Due to the Toeplitz symmetry of $[\mathbf{Z}]$ only one half of the impedance needs to be considered. The resulting non-square matrix is inverted using singular value decomposition. The radiation pattern of the complete structure is finally calculated by duplicating the related columns of the matrix $[\mathbf{Y}] = [\mathbf{Z}]^{-1}$ to account for the Toeplitz symmetry and the subelements in the lower half-plane of Fig. 2b.

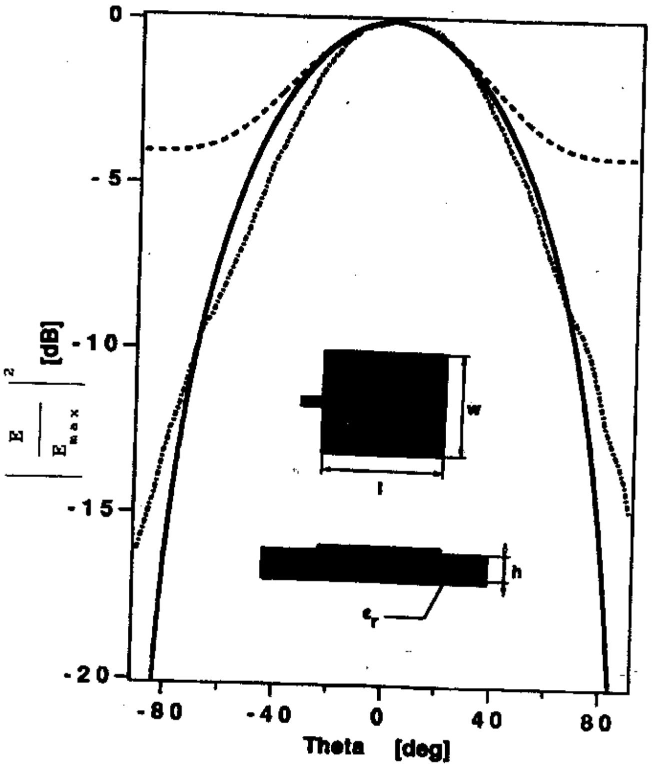

Fig. 3 Comparison of the pattern obtained by this method with measured data provided by Spar Aerospace Ltd., Quebec, Canada; solid line: this theory in the dash-dotted line: measured in the dashed line: this theory in the xz−plane. Dimensions: w = l = 1.712 cm, ε_r = 2.33, f = 5.3 GHz

The incorporation of these symmetry conditions reduces the number of matrix elements by a factor of eight which makes the software operational on modern workstations. When the complete structure is divided into 250 elements, the matrix-inversion time reduction factor due to the current distribution symmetry is 10.03 ; furthermore, additional time reduction factor of 2.73 can be achieved when the Toeplitz-symmetry properties are utilized, which altogether leads to the total matrix-inversion time reduction factor of 27.37. It should be noted, however, that the initial phases of patches P_2 and P_{-2} (c.f.Fig.1) are assumed to be identical due to the symmetry property utilized. Therefore, the method as presented here is applicable to broadside microstrip arrays only.

3. Results

At the example of a single patch, Fig.3 and Fig.4 show a comparison of the results of this theory with measured pattern and input reflection data. Under the limitations of the standard method of moments applied, which does not consider dispersion currents and rooftop function [11],

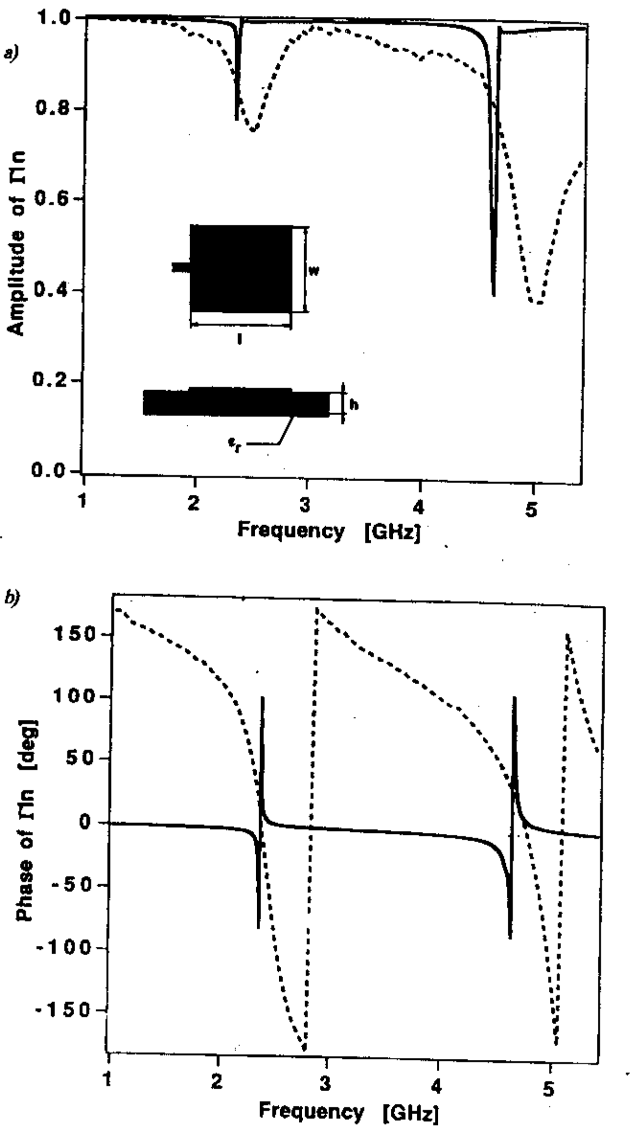

Fig. 4 Comparison of the input reflection coefficient obtained by this method with measured results in [12] for a single patch; solid line: this theory; dashed line: measured [12]. Dimensions: w = 3.18 cm, l = 3.85 cm, h = 1.568 mm, ε_r = 2.34. a) magnitude, b) phase.

dielectric losses or finite ground planes, good agreement is obtained for the yz−plane radiation pattern (Fig. 3). The magnitude (Fig. 4a) and the phase (Fig. 4b) of the input reflection coefficient agree to a somewhat lesser degree. This is not only a result of the points mentioned above, but especially the frequency shift can be attributed to the different feed mechanisms. While the measured patch in [12] is fed by coaxial cable, microstrip feeding is assumed in our

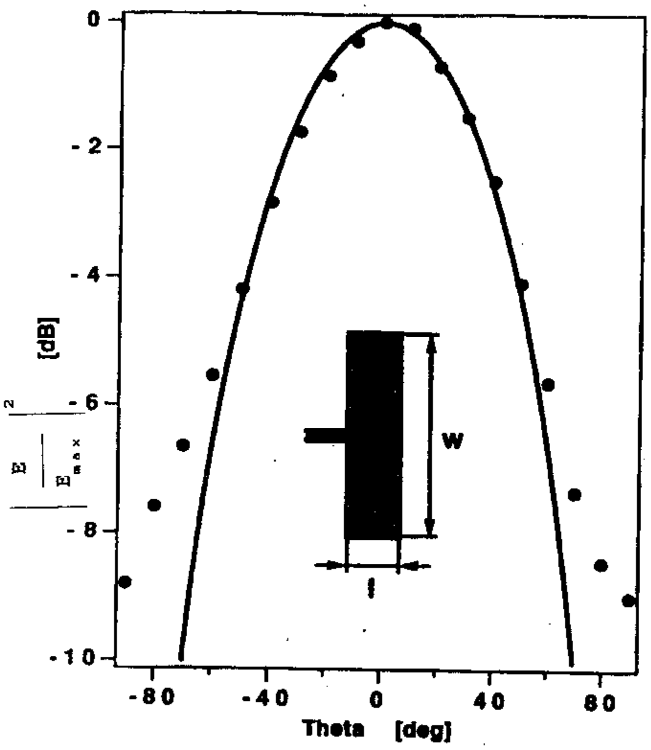

Fig.5 Comparison of the pattern obtained by this method with results in [5] for a single patch; solid line: this theory in the yz−plane, dots: measured in the yz−plane [5]. Dimensions: w = 25.32 cm, l = 3.75 cm, h = 3.18 mm, ε_r = 2.32, f = 2.35 GHz.

model. Frequency dependence of the effective dielectric constant was computed utilizing the technique presented in [10]. Full wave analyses of the structure might lead to more precise values of the effective dielectric constant.

At the example of a wide rectangular patch, Fig. 5 compares a measured (dots) yz−plane pattern presented in [5] with the results obtained by this theory (solid line). Excellent agreement between this model and the measured data can be observed within angles up to 60 degrees. For higher angular values, measured and calculated patterns diverge due to the simplicity of this model, which necessarily leads to a yz−plane pattern null at 90 degrees.

Fig. 6 show the xz−plane pattern characteristics of three aligned patches with the vertical patch separation as parameter. As the distance between the radiators increases from 3 cm (Fig. 6a) to 12 cm (Fig. 6c), the displayed results for included (solid lines) and neglected (dashed lines) mutual coupling seem to merge towards each other as expected. Of more practically oriented importance, however, is the fact that mutual coupling significantly increases the sidelobe levels. If the separation distance is reduced from approximately one half of the patch width (Fig. 6c) over one third (Fig. 6b) to one eighth (Fig. 6a), the corresponding first-

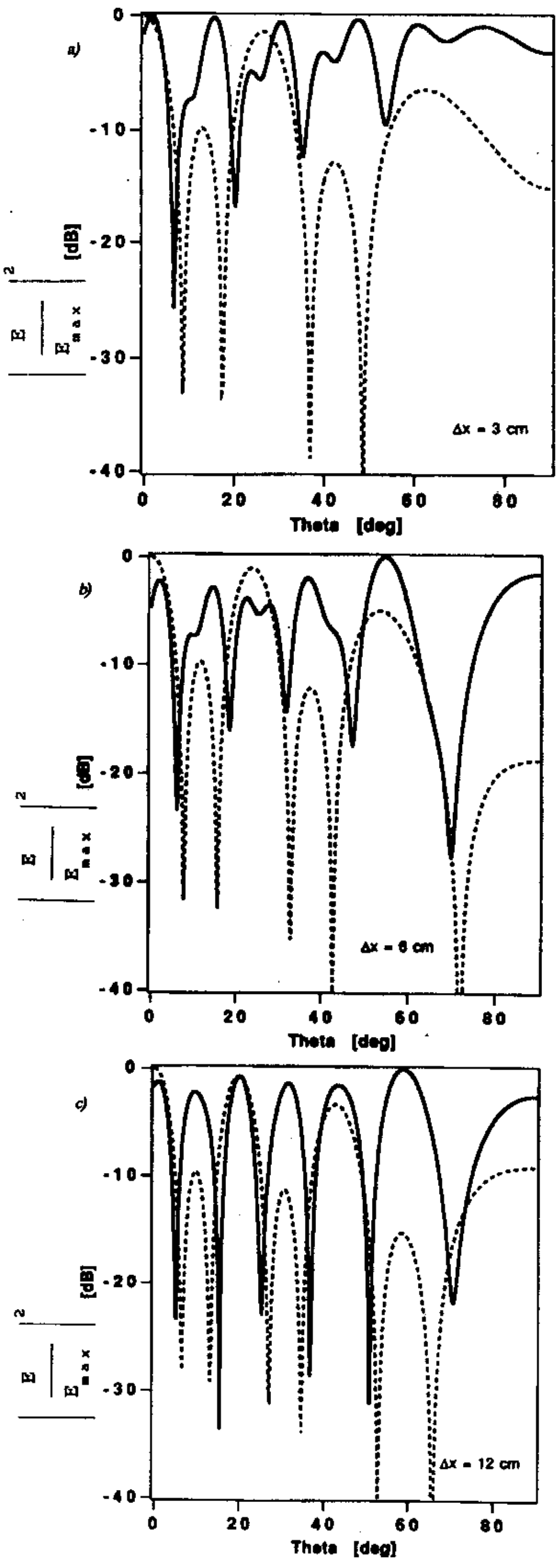

Fig. 6 xz−plane pattern of theree aligned patches (y_D = 0) with the vertical separation Δx (c.f. Fig. 1) as parameter; solid lines: mutual coupling included; dashed lines: mutual coupling neglected. Dimensions as in Fig. 5 a) Δx = 3 cm, b) Δx = 6 cm, c) Δx = 12 cm

sidelobe difference between included and neglected coupling effects increases from 7.1 dB to 10 dB; similar behavior is apparent for higher order sidelobes. Furthermore, the beamwidth and the first null position are reduced by mutual coupling. These results strongly support the necessity for rigorous field-theory-based procedures in microstrip an-

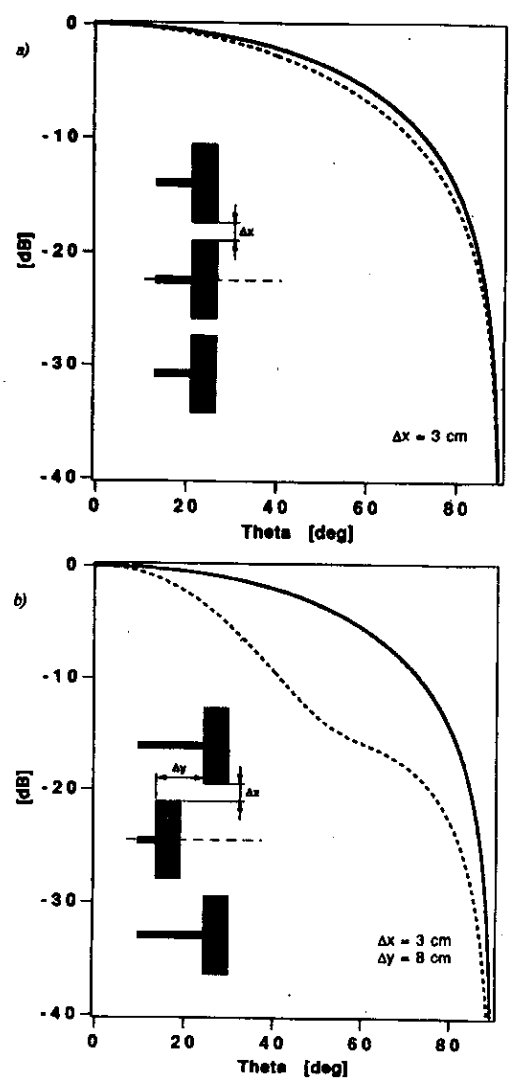

Fig. 7 yz–plane patterns of three patches with fixed vertical separation at $\Delta x = 3$ cm and y_D as parameter; solid lines: mutual coupling included; dashed lines: mutual coupling neglected. Dimensions as in Fig. 5. a) $\Delta y = 0$, b) $\Delta y = 8$ cm.

tenna array design. For the general structure in Fig. 1, Figs. 7 display the yz–plane pattern for a fixed separation distance at $\Delta x = 3\,cm$. As long as the patches are aligned (Fig. 7a), the pattern responses show no difference between the calculations including and neglecting mutual coupling. Since in this configuration coupling merely occurs in x–direction, it does not affect the pattern in the yz–plane (c.f. Fig. 2a). By offsetting the patches in the y–direction, mutual coupling only slightly affects the amplitude of the radiation pattern (Fig. 7b), while the influence on the phase pattern is much stronger (Fig. 8). Since the aperture is considerably smaller in y– than in x–direction, the mutual coupling effects on the pattern are pronounced to a much lesser degree than those in the xz–plane patterns of Figs.6.

4. Conclusions

A simplified moment method algorithm for radiation pattern calculation of mutually coupled patch antennas is presented. The utilization of symmetry properties in the surface current distribution and Toeplitz symmetry of the impedance matrix of mutually coupled microstrip antennas reduces storage and CPU-time requirements for the analysis of array radiation characteristics. Due to the symmetry properties considered, the software is operational on mod-

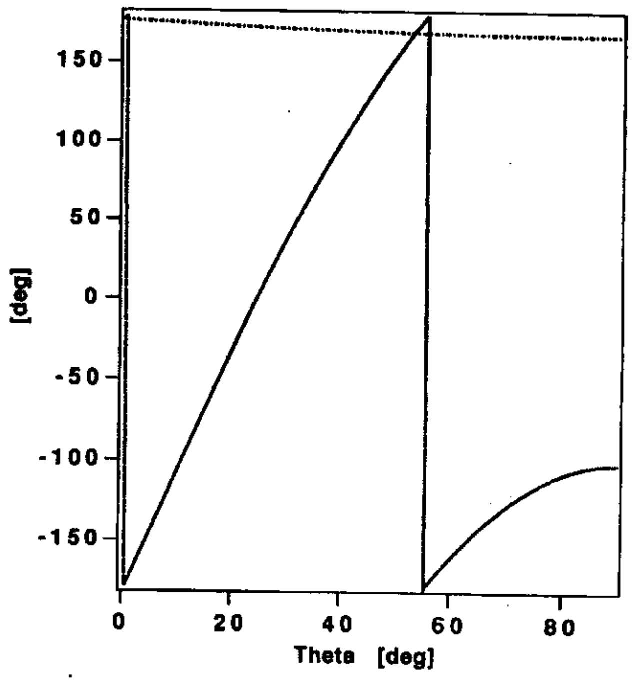

Fig. 8 yz–plane phase patterns of three patches with fixed vertical separation at $\Delta x = 3$ cm and included mutual coupling; dash-dotted line: $\Delta y = 0$; dotted line: $\Delta y = 8$ cm.

ern workstations. Good agreement is obtained when this method is compared with measured data on radiation characteristics and input impedance. The influence of mutual coupling effects on the patterns of patch antennas is found to significantly increase sidelobe levels and to reduce the beamwidth and the first null position. The results demonstrate first that the pattern characteristics can be as severely affected by mutual coupling as the commonly investigated input impedance and, second, these effects must be incorporated in microstrip array design procedures.

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