Utilization of Symmetry Properties for the Pattern Analysis of Mutually Coupled Patch Radiators

Martin GIMERSKY and Jens BORNEmann
Laboratory for Lightwave Electronics
Microwaves and Communications (LLiMiC)
Department of Electrical and Computer Engineering
University of Victoria, P.O. Box 3055, Victoria, B.C.,
CANADA V8W 3P6

Abstract
The influence of mutual coupling effects on the radiation pattern of microstrip antennas is investigated by introducing symmetry properties in the surface current distributions. The standard method of moments is used, which requires only the upper half of three x- and y-direction offset patch radiators to be solved, while the symmetric lower half is incorporated after the determination of the current distribution. Furthermore, the Toeplitz symmetry of the impedance matrix is taken into account, and the singular value decomposition is used to invert the resulting non-square matrix in a numerically stable way. Both the symmetry property and the Toeplitz symmetry procedures reduce storage and CPU-time requirements in the analysis of broadside microstrip arrays. The pattern characteristics calculated with and without mutual coupling interactions are found to differ up to 10 dB in sidelobe levels which clearly demonstrates that the pattern can be as severely affected by mutual coupling as the commonly investigated input impedance. The predicted results are in good agreement with measured data on single patches.

1. Introduction
Patch radiators in standard and monolithic microwave integrated circuitry are well known for their excellent performance in light-weight and low-cost array architectures, e.g. [1,2]. A most comprehensive collection of papers on the analysis and/or design of single and coupled microstrip elements can be found in [3].

Most of the theoretical models presented, however, firstly assume that the patch is fed by a coaxial cable from below the ground plane, which complicates the manufacturing process and considerably reduces the achievable level of integration for feeding networks; secondly and more important, the effects of mutual coupling between elements are restricted to the determination of input and mutual coupling impedances, input reflection and transmission coefficients or other quantities defined in terms of circuit parameters rather than by electromagnetic field and pattern parameters. Methods have been presented, e.g. [4,5], to include the - from a network - integrational point of view - more appropriate feed mechanism of a microstrip line instead of a coaxial cable. The problems associated with the influence of mutual coupling effects on the radiation pattern of a set of circular microstrip antennas have been rigorously solved, e.g. [6]; however, as the mathematics involved are completely different, these solutions cannot be applied to rectangular patches.

A rigorous solution to the problem of a rectangular microstrip antenna fed by a microstrip line was presented in [7]; the currents on the feed line and the patch are expanded in a suitable set of modes, and a moment method solution is formulated in the spectral domain. Similarly, finite phased arrays of rectangular microstrip patches have been treated [8] by the spectral domain moment method; a theory was developed for the active input impedance, as well as the active element pattern of the array.

Most of the theoretical models currently available to analyze rectangular patch antenna structures require substantial computer resources to handle and reliably solve the large matrix systems involved. Therefore, this paper focuses on a simplified method-of-moments algorithm to reduce storage and CPU-time requirements in the analysis of patch antenna configurations. Symmetry properties in both the structure (cf. Fig. 1) and the resulting impedance matrix are efficiently utilized to analyze the characteristics of single and multiple patches including mutual coupling effects.

![Fig. 1 Basic configuration of three coupled patch radiators.](image-url)
The standard method-of-moments algorithm [9] is used to model the current distribution of the upper half of the array configuration, while the lower half is incorporated by utilizing a symmetric distribution. Due to the symmetry of the array configuration and Toeplitz symmetry of the impedance matrix, it is sufficient to evaluate complex elements of the upper of the impedance matrix and invert this nonsquare matrix utilizing the single value decomposition technique. The patterns of a three-element array calculated with included mutual coupling effects are compared with commonly used array factor analysis that neglects the interference of the radiators. Significant differences with respect to sidelobe levels and null positions demonstrate the necessity for field-theoretical models in the pattern analysis of microstrip antenna arrays.

2. Theory

The standard moment method algorithm, which is applied here, basically follows that outlined in [9]. Therefore, only the fundamental steps are presented here, and the reader is referred to [9] for further details. It should be noted, however, that the symmetry properties introduced are not restricted to a specific numerical model. In fact, they can be applied to any algorithm that solves for the current distribution on the patches.

Fig. 2a shows the location of a single element in the coordinate system. If the surface currents in the \( x \)-direction are assumed to have a negligible influence compared to those in the \( y \)-direction, then the far field has only \( E_\theta \) and \( E_\phi \) components in the \( \phi = 0 \) and \( \phi = \pi/2 \) planes, respectively. The tangential electric field on the surface of the patches

\[ \mathbf{E} = j\omega \mathbf{A} + \frac{\partial \Phi}{\partial y} \hat{y} \quad \text{on } S \]  

is derived from well-known potential functions

\[ \mathbf{A} = \frac{1}{\pi} \iint_S \frac{\mathbf{e}^{jkr}}{4\pi r} dS \]  

\[ \Phi = \frac{1}{\varepsilon} \iint_S \frac{1}{4\pi r} dS \]

with the charge distribution

\[ \sigma = -\frac{1}{j\omega} \frac{dl}{ds} \]

In (1)-(4), \( S \) is the area variable at the antenna surface and \( R \) is the distance measured from the center of an antenna sub-element to a field point in space. \( k_g \) is the propagation constant of a microstrip line of width \( w \), substrate height \( h \) and permittivity \( \varepsilon_0 \varepsilon_{r} \). The frequency dependence of \( k_g = 2\sqrt{\varepsilon_r} \) is taken into account according to [10].

The division into sub-elements for the upper-half-plane patches of Fig. 1 is shown in Fig. 2b. The sub-element notation corresponds to the location of elements of the impedance matrix to be derived. Similar to [9], the potential function

\[ \psi(m, n) = \frac{1}{4\pi \Delta n} \int_{2n}^{n+1} e^{-jk_g R_m} dy = \]

\[ = \frac{1}{4\pi \Delta n} \int_{2n}^{n+1} \left( \frac{1}{R_m} - jk_g - \frac{k_g^2 R_m}{2} + \ldots \right) dy \]  

with its approximation by a Maclaurin series is used. For \( m = n \) the consideration of the first two real terms in (5) provides sufficient accuracy, and the corresponding integral
\[ \psi(n, n) = \frac{1}{4\pi \Delta n} \int_{-\Delta n}^{\Delta n} \left[ \frac{1}{R_m} - \frac{k^2}{2} R_m \right] dy - \frac{jk}{4\pi} \]  

(6)

is evaluated numerically. Since all sub-elements are of identical dimensions, (6) has to be computed only once. For \( m \neq n \), \( R_m \) is considered constant, and hence

\[ \psi(m, n) = \frac{e^{-jkR_{mn}}}{4\pi R_{mn}} \]  

(7)

where \( R_{mn} \) is the distance between sub-elements \( m \) and \( n \). The resulting matrix equation [9] for the upper half of the structure (Fig. 2b)

\[ [Z] [I] = [V] \]  

(8)

needs to be solved for the current vector \([I]\). Due to the Toeplitz symmetry of \([Z]\) only one half of the impedance needs to be considered. The resulting non-square matrix is inverted using singular value decomposition. The radiation pattern of the complete structure is finally calculated by duplicating the related columns of the matrix \([V] = [Z]^{-1}\) to account for the Toeplitz symmetry and the subelements in the lower half-plane of Fig. 2b.

When the complete structure is divided into 250 elements, the matrix-inversion time reduction factor due to the current distribution symmetry is 10.03; furthermore, additional time reduction factor of 2.73 can be achieved when the Toeplitz-symmetry properties are utilized, which altogether leads to the total matrix-inversion time reduction factor of 27.37. It should be noted, however, that the initial phases of patches \( P_2 \) and \( P_{-2} \) (c.f. Fig. 1) are assumed to be identical due to the symmetry property utilized. Therefore, the method as presented here is applicable to broadside microstrip arrays only.

3. Results

At the example of a single patch, Fig. 3 and Fig. 4 show a comparison of the results of this theory with measured pattern and input reflection data. Under the limitations of the standard method of moments applied, which does not consider dispersion currents and rooftop function [11],

![Fig. 3 Comparison of the pattern obtained by this method with measured data provided by Spar Aerospace Ltd., Quebec, Canada; solid line: this theory in the dash-dotted line: measured in the dashed line: this theory in the xz-plane. Dimensions: \( w = l = 1.712 \, \text{cm}, \varepsilon_r = 2.33, f = 5.3 \, \text{GHz} \)]

The incorporation of these symmetry conditions reduces the number of matrix elements by a factor of eight which makes the software operational on modern workstations.

![Fig. 4 Comparison of the input reflection coefficient obtained by this method with measured results in [12] for a single patch; solid line: this theory; dashed line: measured. Dimensions: \( w = 3.18 \, \text{cm}; l = 3.85 \, \text{cm}; h = 1.568 \, \text{mm}, \varepsilon_r = 2.34 \), a) magnitude, b) phase.]
dielectric losses or finite ground planes, good agreement is obtained for the $yz$-plane radiation pattern (Fig. 3). The magnitude (Fig. 4a) and the phase (Fig. 4b) of the input reflection coefficient agree to a somewhat lesser degree. This is not only a result of the points mentioned above, but especially the frequency shift can be attributed to the different feed mechanisms. While the measured patch in [12] is fed by coaxial cable, microstrip feeding is assumed in our model. Frequency dependence of the effective dielectric constant was computed utilizing the technique presented in [10]. Full wave analyses of the structure might lead to more precise values of the effective dielectric constant.

At the example of a wide rectangular patch, Fig. 5 compares a measured (dots) $yz$-plane pattern presented in [5] with the results obtained by this theory (solid line). Excellent agreement between this model and the measured data can be observed within angles up to 60 degrees. For higher angular values, measured and calculated patterns diverge due to the simplicity of this model, which necessarily leads to a $yz$-plane pattern null at 90 degrees.

Fig. 6 show the $xz$-plane pattern characteristics of three aligned patches with the vertical patch separation as parameter. As the distance between the radiators increases from 3 cm (Fig. 6a) to 12 cm (Fig. 6c), the displayed results for included (solid lines) and neglected (dashed lines) mutual coupling seem to merge towards each other as expected. Of more practically oriented importance, however, is the fact that mutual coupling significantly increases the sidelobe levels. If the separation distance is reduced from approximately one half of the patch width (Fig. 6c) over one third (Fig. 6b) to one eighth (Fig. 6a), the corresponding first-sidelobe difference between included and neglected coupling effects increases from 7.1 dB to 10 dB; similar behavior is apparent for higher order sidelobes. Furthermore, the beamwidth and the first null position are reduced by mutual coupling. These results strongly support the necessity for rigorous field-theory-based procedures in microstrip an-
Fig. 7 yz-plane patterns of three patches with fixed vertical separation at Δx = 3 cm and y0 as parameter. Solid lines: mutual coupling included; dashed lines: mutual coupling neglected. Dimensions as in Fig. 5. a) Δy = 0, b) Δy = 8 cm.

Fig. 8 yz-plane phase patterns of three patches with fixed vertical separation at Δx = 3 cm and included mutual coupling: dash-dotted line: Δy = 0; dotted line: Δy = 8 cm.

4. Conclusions

A simplified moment method algorithm for radiation pattern calculation of mutually coupled patch antennas is presented. The utilization of symmetry properties in the surface current distribution and Toeplitz symmetry of the impedance matrix of mutually coupled microstrip antennas reduces storage and CPU-time requirements for the analysis of array radiation characteristics. Due to the symmetry properties considered, the software is operational on modern workstations. Good agreement is obtained when this method is compared with measured data on radiation characteristics and input impedance. The influence of mutual coupling effects on the patterns of patch antennas is found to significantly increase sidelobe levels and to reduce the beamwidth and the first null position. The results demonstrate first that the pattern characteristics can be as severely affected by mutual coupling as the commonly investigated input impedance and, second, these effects must be incorporated in microstrip array design procedures.

Acknowledgment

The authors would like to thank the Satellite and Communications Systems Division of spae Aerospace Ltd., Quebec, Canada for providing the measured data presented in Fig. 3.

References

About authors, ...

Martin Gimersky was born in Presov, Czechoslovakia, on May 23, 1965. He received the Ing. Degree in electrical engineering from the Czech Technical University of Prague, Czechoslovakia, in 1988. Currently he has been working towards the Ph.D. degree in electrical engineering at the University of Victoria, Canada. His research interests include microwave and millimeter wave antennas and numerical methods in electromagnetic theory.

Jens Bornemann was born in Hamburg, West Germany, on May 26, 1952. He received the Dipl.-Ing. and the Dr.-Ing. degrees, both in electrical engineering, from the University of Bremen, West Germany, in 1980 and 1984, respectively. From 1980 to 1983, he was a Research and Teaching Assistant in the Microwave Department at the University of Bremen, working on quasi-planar waveguide configurations and computer-aided E-plane filter design. After a two year period as a consulting engineer, he joined the University of Bremen again, in 1985, where he was employed at the level of Assistant Professor. Since April 1988 he has been an Associate Professor at the University of Victoria, Canada. His current research activities include microwave system design and problems of electromagnetic field theory in integrated circuits and radiating structures. Dr. Bornemann was one of recipients of the A.F. Bulgin Premium of the Institution of Electronic and Radio Engineers in 1983. He serves on the editorial board of the Institute of Engineering Transactions on Microwave Theory and Techniques and has authored and coauthored more than 50 technical papers.