Progressive Image Reconstruction Using Morphological Skeleton

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Abstract

This paper presents the results of a study on use of morphological skeleton to represent, progressive transmission and reconstruction of binary images. For images containing blobs and large areas, the skeleton subsets are much thinner than the original image therefore encoding of skeleton results in lower information rates than other standard coding methods.

1. Introduction

The skeleton is very interesting in image representation and we can consider two approaches to find it. First is using symetric point distance from a skeleton point to the boundary and second comes from an idea, that skeleton is the locus of centers of the maximal disks inscribable inside a filled - in image object. Second interpretation was used in a growth geometry proposed for a progressive binary image trasmission scheme [1]. A new approach in steletonization ideas is the mathematical morphology. It is a set - theoretical method for images analysis whose purpose is the quantitative description of geometrical structures. The skeleton can be obtained by morphological set transformations and we refer to it as a morphological skeleton [2],[3],[4].

In this paper we are concerned with the use of morphological skeleton for image coding and progressive transmission. In this techniques we transmit crude representation of image first and add the details later. Such image transmission is desirable in telebrowsing of image databases [1].

2. Morphological Skeleton

Let $S_n(X)$, for $n>0$, denote the $n-$th skeleton subset, then the morphological skeleton $SK(X)$ of a discrete binary image X can be expressed in the following form [2].

$$SK(X) = \bigcup_{n=0}^{N} S_n(X) \qquad n = 0, 1, 2, \ldots\ldots, N \tag{1}$$

and

$$S_n(X) = (X \varnothing nB) - (X \varnothing nB) \circ B \tag{2}$$

where

$S_n(X)$ is the $n-$th skeleton subset of X,
B is the structuring element,
nB is the structuring element of radius n,
$X \varnothing nB$ is the morphological erosion X by nB,
$^\circ B$ is the morphological opening by B.

Equation (1) implies that the skeleton $SK(X)$ of X is obtained as the finite union of $(N+1)$ skeleton subsets. A very important property of the morphological skeleton is that the discrete binary image X can be exactly reconstructed as the finite union of its $(N+1)$ skeleton subsets dilated by the structuring element of proper size :

$$X = \bigcup_{n=0}^{N} \left[S_n(X) \oplus nB \right] \tag{3}$$

where $\oplus B$ is the morphological dilation by B.

The original discrete binary image X can be exactly reconstructed in another way too [2] :

$$X = \left[\left[\left[S_N(X) \oplus B \right] \cup S_{N-1}(X) \right] \oplus B \cup \right.$$
$$\cup\, S_{N-2}(X) \ldots\ldots \oplus B \cup S_0(X) \tag{4}$$

The algorithm (4) is equivalent to algorithm (3) because dilation distributes over set union and is associative. In the algorithm (4) the reconstruction propagates from the center of the object X towards its boundary. In (3) the reconstruction can propagate in direction depending on whether we start the union of the dilated skeleton subsets from $n = N$ or from $n = 0$.

The subset of the original skeleton guaranteening the exact reconstruction of the entire image is a globally minimal skeleton. The algorithm for finding the globally minimal skeleton uses a pseudograytone function in the following form

$$\left[pgf(x) \right] (i,j) = \sum_{n=0}^{N} \sum_{(r,t) \in S_n(X)} k_n (i - r, j - t) \tag{5}$$

where

$$\underset{(r,t)\in S_n}{k_n} = \begin{cases} 1 \text{ for } (i,j) \in nB \\ \textit{otherwise } 0 \end{cases}$$

A certain point $(r, t) \in S_n(X)$ can be removed if the value of the pseudograytone function $pgf(X)$ at all the points of the region is >2 [2].

The points which cannot be removed create the globally minimal skeleton. The method searches each skeleton subset beginning from $S_1(X)$ or $S_N(X)$ and continuing in ascending or descending order of their indexes.

3. Some new considerations on morphological skeleton

Our study on the use of the morphological skeleton was made on the groupe four binary images. Fig.1 illustrates in a detailed way decomposition of binary images into its skeleton subsets. In this decomposition was used the square structuring element B. In Fig.1 proceedings from left to right columns we see progressive construction of skeleton $SK(X)$ by (1). In the first column we see the groupe binary images X and its erosions by nB. In the second column are the openings of these erosions by B. In the third column we see the skeleton subsets $S_n(X)$ which we obtained by (2). In the fourth column is the composition of the skeleton $SK(X)$ as the union of the skeleton subsets S_n (X).

The skeleton subsets of the globally minimal skeleton is in the fifth column.

The globally minimal skeleton was constructed by the pseudograytone function pgf (X).

In the sixth colums is globally minimal skeleton as the finite union of its skeleton subsets.

In the two last columns we can see the progressive reconstruction original image X. The reconstruction in the seventh column uses the algorithm (4) with structuring element B.

In the last column is the progressive reconstruction original images X with the globally minimal skeleton by the algorithm (3). The same progressive reconstruction original images X we can obtain with the skeleton $SK(X)$ by the algorithm (3).

4. Progressive image reconstruction by monohological skeleton

In the Fig.1 we can see some differences between the image reconstruction from the morphological skeletons. From the point of view of progressive image reconstruction for telebrowsing the algorithm (3) is better than the algorithm (4). The image reconstruction by this algorithm is faster and the viewer can recognize the image content sooner than in the image reconstruction by algorithm (4). The image reconstruction from the globaly minimal skeleton is more effective because this skeleton has less points than $SK(X)$.

5. Conclusion

Mophological skeleton is very interesting in image processing. A finite image can be decomposed into a finite number of skelekon subsets and exactly and progressively reconstructed from them.

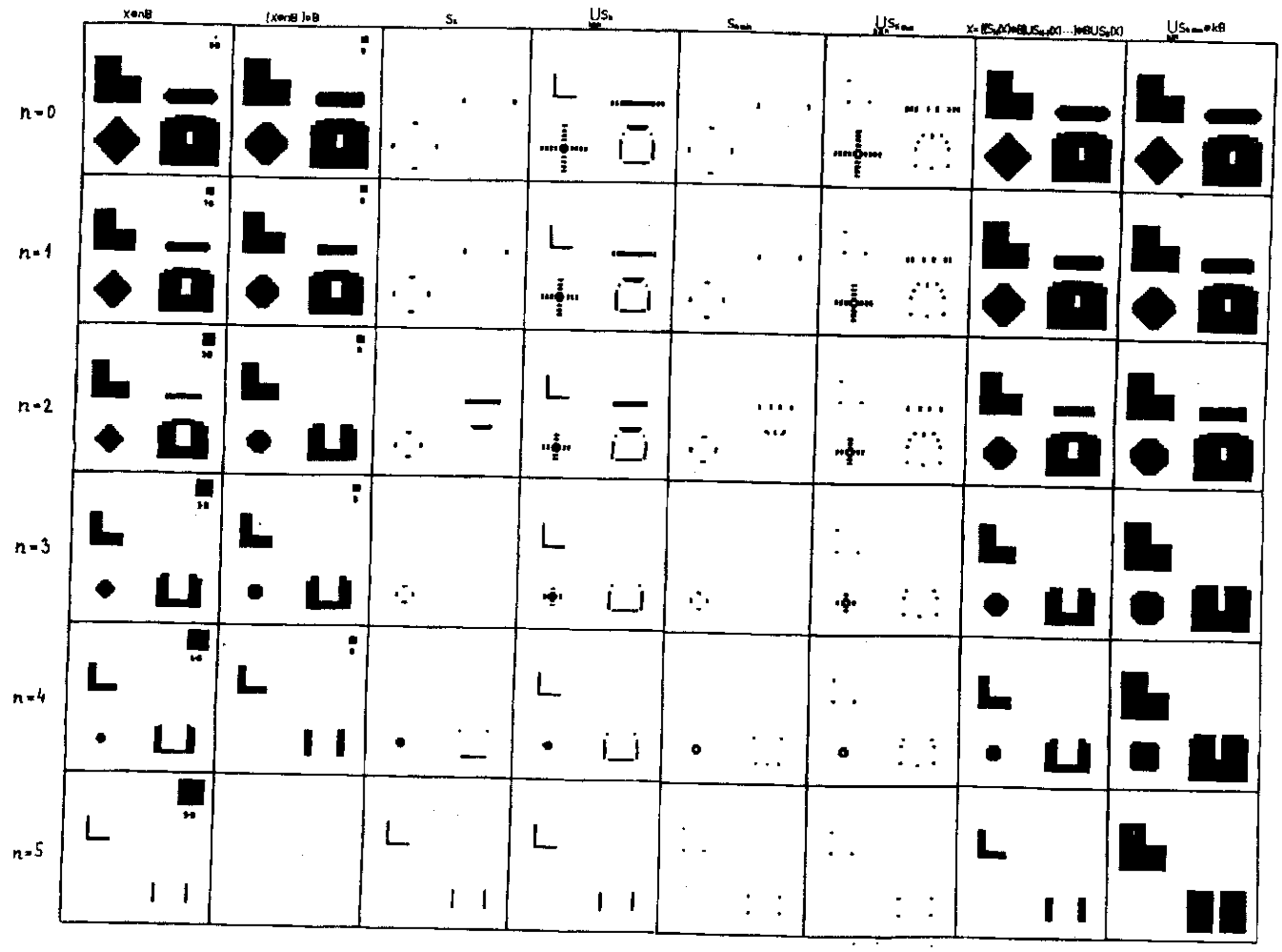

Fig.1. The skeletal decomposition of an image X and reconstruction of the image from its skeleton subsets by structuing element SQUARE.