CALIBRATING THE SIX-PORT REFLECTOMETER USING A MATCHED LOAD AND FOUR UNITY-REFLECTION STANDARDS

PART 1: PERFECTLY MATCHED LOAD

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ABSTRACT

An alternative six-port calibration procedure is described using an ideal matched load and four unity reflection standards. Not only offset short or open circuits can be used as the reflection standards but also arbitrary reactances, such as inductors or capacitors. A different mathematical approach is employed to obtain the calibration equations.

Keywords:
six-port reflectometer, calibration

1. Introduction

Calibration of a six-port reflectometer requires typically 4 to 7 standards [1] - [8]. The standards often used are offset short or open circuits. To attain good measurement accuracy for low reflection coefficients, a matched load should be used as one of the standards. Perhaps the most elaborate calibration method is Engen's six-port to four-port reduction [9]. However, it is rather complicated, employing iterative solution to a set of at least five simultaneous 3rd order equations. This process requires a fairly good initial guess. Therefore, simpler calibration methods are also attractive, if not for else then for providing a very good initial solution for the Engen procedure.

While open- or short-circuited line section standards are a good choice, it is not always practical to use them, especially with the advent of lumped six-ports capable of covering also of low frequencies [10], [11]. In this case, lumped reactances, such as inductors and capacitors can conveniently be used. For this purpose, the calibration method [5] is modified in the present paper to accommodate any type of unity reflection standard set. A slightly different mathematical approach is used to derive the results.

It is worth noting that a solution to this problem is actually contained in [7], where five arbitrary known standards are used. Assumption of unit reflection coefficient in the present paper however simplifies the equations and results in the decreasing of the computation time.

Another purpose of this paper is to serve as a basis for the calibration using an imperfect sliding termination in place of the ideal matched load. This problem will be treated in the accompanying paper [12].

2. Basic concepts and equations

Let the detector ports of the six-port reflectometer be numbered $i = 1, 2, 3, 4$ (no. 4 being the reference port). The power reading of the i-th detector is

$$P_i = P_o \,.\, S_i \,.\, |\Gamma - q_i|^2 \qquad i = 1, 2, 3$$

$$P_4 = P_o \,.\, S_4 \,.\, \left|c \,.\, \Gamma + 1\right|^2$$

where P_o is the power incident on the measured load,

$$\Gamma = x + jy = r \,.\, e^{j\varphi}$$

is the reflection coefficient of the load, q_i (complex) are $q-$points of the reflectometer, c (complex) is the reference port parameter, $S_i > 0$ (real) are proportionality factors. In measuring Γ, only normalized powers

$$p_i = \frac{P_i}{P_4} \qquad i = 1, 2, 3$$

are required. To simplify expressions, the following notation is introduced:

$$\frac{-1}{q_i} = x_i + jy_i \qquad i = 1, 2, 3$$

$$z_i = \left|q_i\right|^{-2} \qquad i = 1, 2, 3$$

$$c = x_4 + jy_4$$

$$z_4 = \left|c\right|^2$$

$$d = S_4 \cdot \frac{P_o}{P_4} = \left|c \cdot \Gamma + 1\right|^{-2}$$

and, in accordance with [5], the "scaling factors"

$$Y_i = \left|q_i\right|^2 \frac{S_i}{S_4} \qquad i = 1, 2, 3$$

Then the normalized powers are given by the matrix equation

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ 1 \end{bmatrix} = d. \begin{bmatrix} Y_1 & & & \\ & Y_2 & & \\ & & Y_3 & \\ & & & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2x_1 & -2y_1 & z_1 \\ 1 & 2x_2 & -2y_2 & z_2 \\ 1 & 2x_3 & -2y_3 & z_3 \\ 1 & 2x_4 & -2y_4 & z_4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ x \\ y \\ r^2 \end{bmatrix} \qquad (1)$$

or, in a shortened notation

$$\mathbf{P} = d \cdot \mathbf{Y} \cdot \mathbf{B} \cdot \mathbf{R} = d \cdot \mathbf{C} \cdot \mathbf{R} \qquad (2)$$

where

$$\mathbf{C} = \mathbf{Y} \cdot \mathbf{B}$$

From 4th row of (1), the following equation is obtained for d:

$$\frac{1}{d} = 1 + 2x_4 x - 2y_4 y + z_4 r^2 \qquad (3)$$

which is actually an expansion of the formula defining d. The measured Γ can be calculated from the inverse of (1) or (2), i.e.

$$d \cdot \mathbf{R} = \mathbf{D} \cdot \mathbf{P} \qquad (4)$$

where the matrix

$$\mathbf{D} = \mathbf{C}^{-1} = \mathbf{B}^{-1} \cdot \mathbf{Y}^{-1} \qquad (5)$$

is called calibration matrix. Expressing d from 1st row of (4), Γ is given by the known formula

$$\Gamma = \frac{\left(\mathbf{D}_{2\cdot} + j\,\mathbf{D}_{3\cdot}\right) \cdot \mathbf{P}}{\mathbf{D}_{1\cdot}\ \mathbf{P}}$$

where the notation D_i is used for i−th row of matrix **D**. Graphically, this solution corresponds to the familiar intersection of the 3 common chords of six-port impedance-locating circles (q−circles).

3. Calibration

In the process of calibration, five known loads (standards) are connected in place of device under test and the corresponding detector powers are recorded. Four of the standards, numbered $j = 1, 2, 3, 4$, have reflection coefficient modulus equal to unity:

$$\Gamma_j = e^{j\varphi_j} = c_j + j\,s_j \qquad j = 1, 2, 3, 4$$

The fifth standard, the matched load, has $\Gamma_5 = 0$. The power reading at the i−th detector when the j−th standard is connected, will be denoted as P_{ij}; the normalized powers are

$$p_{ij} = \frac{P_{ij}}{P_{4j}}$$

The calibration process can be divided in 3 steps:
1. determining of the scaling factors Y_i;
2. determining of the reference port parameter c;
3. determining of the q-points in terms of x_i, y_i $(i = 1, 2, 3)$;

These steps will now be pursued successively.

3.1 Scaling factors

Using the powers for the matched load ($j = 5$), the scaling factors are determined from (1) after substituting $x = y = r = 0$:

$$Y_i = \left[p_i\right]_{\Gamma = 0} = p_{i5} = \frac{P_{i5}}{P_{45}} \qquad i = 1, 2, 3$$

3.2 Reference port parameter

To determine the reference port parameter $c = x_4 + j\,y_4$, the powers for the reflecting standards are used together with the now known scaling factors Y_i. From (1) and (3), the equations are obtained

$$Q_{ij}\left(1 + z_4 + 2c_j x_4 - 2s_j y_4\right) =$$

$$= 1 + z_i + 2c_j x_i - 2s_j y_i$$

$$i = 1, 2, 3; \qquad j = 1, 2, 3, 4 \qquad (6)$$

where Q_{ij} are known quantities defined as

$$Q_{ij} = \frac{p_{ij}}{Y_i}$$

The twelve equations (6) are grouped to three sets (corresponding to ports $i = 1, 2, 3$) of four equations. Each group will be treated separately. Our immediate goal is to eliminate x_i, y_i, z_i from each of the groups so that only equations in terms of the unknowns of interest x_4, y_4, z_4 remain. These equations will then be solved.

Considering the i-th group, the four equations are processed as follows: The first three of them (corresponding to the first three standards) can be rewritten in the matrix form

$$\begin{bmatrix} Q_{i1} & & \\ & Q_{i2} & \\ & & Q_{i3} \end{bmatrix} \cdot \begin{bmatrix} 1 & c_1 & -s_1 \\ 1 & c_2 & -s_2 \\ 1 & c_3 & -s_3 \end{bmatrix} \cdot \begin{bmatrix} 1+z_4 \\ 2x_4 \\ 2y_4 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & c_1 & -s_1 \\ 1 & c_2 & -s_2 \\ 1 & c_3 & -s_3 \end{bmatrix} \cdot \begin{bmatrix} 1+z_i \\ 2x_i \\ 2y_i \end{bmatrix}$$

or, in the shortened notation

$$\mathbf{Q}_i \,.\, \mathbf{F} \,.\, \mathbf{Z} = \mathbf{F} \,.\, \mathbf{X}_i \tag{7}$$

The fourth equation (corresponding to the fourth standard, $j = 4$) is from (6) similarly

$$Q_{i4} \,.\, \begin{bmatrix} 1 & c_4 & -s_4 \end{bmatrix} \cdot \mathbf{Z} = \begin{bmatrix} 1 & c_4 & -s_4 \end{bmatrix} \cdot \mathbf{X}_i$$

or

$$Q_{i4} \,.\, \mathbf{F}_4 \,.\, \mathbf{Z} = \mathbf{F}_4 \,.\, \mathbf{X}_i \tag{8}$$

The vector $\mathbf{X}_i$ will now be eliminated from the two matrix equations (7), (8). To achieve this, Eq. (7) is left-multiplied by $\mathbf{F}^{-1}$ (the inverse matrix to $\mathbf{F}$) and $\mathbf{X}_i$ is expressed as

$$\mathbf{X}_i = \mathbf{F}^{-1} \,.\, \mathbf{Q}_i \,.\, \mathbf{F} \,.\, \mathbf{Z}$$

$\mathbf{X}_i$ is substituted to (8) from which the equation is obtained

$$\left(\mathbf{F}_4 \,.\, \mathbf{F}^{-1} \,.\, \mathbf{Q}_i \,.\, \mathbf{F} - Q_{i4} \,.\, \mathbf{F}_4\right) \cdot \mathbf{Z} = 0 \tag{9}$$

The expression in the parentheses is a 3-dimensional row vector with all its elements known. Three such vectors are obtained for $i = 1, 2, 3$. These can be stacked to form a 3×3 matrix designated $\mathbf{A}$ so that the matrix equation is obtained

$$\mathbf{A} \,.\, \mathbf{Z} = 0 \tag{10}$$

with the unknown row vector $\mathbf{Z}$. The matrix equation (10) then re presents three equations of the form

$$a_{i1}\left(1 + x_4^2 + y_4^2\right) + 2a_{i3}y_4 = 0 \qquad i = 1, 2, 3 \tag{11}$$

in the unknowns x_4, y_4. (note that $z_4 = x_4^2 + y_4^2$). A method of solving (11) is described in [13] and will not be discussed here. At this point, the reference port parameter $c = x_4 + j\,y_4$ can be considered known.

An expansion of (9) useful for the practical purposes of computation is described in Appendix 1.

3.3 Parameters x_i, y_i

In this state of the calibration, the left-hand sides of Eqs. (6) are completely known. The equations can now be rewritten as

$$2c_j x_i - 2s_j y_i + z_i = W_{ij} - 1 \tag{12}$$

where the known quantities W_{ij} are

$$W_{ij} = Q_{ij}\left(1 + z_4 + 2c_j x_4 - 2s_j y_4\right) \tag{13}$$

For each i is thus obtained a set of four linear equations ($j = 1, 2, 3, 4$) in three unknowns x_i, y_i, z_i, or, more conveniently, in the unknowns $2x_i$, $-2y_i$, z_i which directly enter the matrix $\mathbf{B}$ in Eq. (1). All three equation sets can be condensed to the form

$$\begin{bmatrix} c_1 & s_1 & 1 \\ c_2 & s_2 & 1 \\ c_3 & s_3 & 1 \\ c_4 & s_4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2x_1 & 2x_2 & 2x_3 \\ -2y_1 & -2y_2 & -2y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} = \begin{bmatrix} W_{11}^{-1} & W_{21}^{-1} & W_{31}^{-1} \\ W_{12}^{-1} & W_{22}^{-1} & W_{32}^{-1} \\ W_{13}^{-1} & W_{23}^{-1} & W_{33}^{-1} \\ W_{14}^{-1} & W_{24}^{-1} & W_{34}^{-1} \end{bmatrix} \tag{14}$$

with the square matrix unknown. The system can be solved using a standard least squares method. Then all the available information will optimally be used.

At this point, all calibration data have been obtained and the matrices $\mathbf{Y}$, $\mathbf{B}$, and $\mathbf{C}$ are known. To complete the whole process, the calibration matrix $\mathbf{D}$ is calculated by inversion of $\mathbf{C}$; cf. Eq. (5).

Note that ideally $z_i = x_i^2 + y_i^2$. However, due to inconsistencies introduced in the calibration (errors in power measurement, high contents of harmonics in the input signal, poorly defined standards, loose connectors, etc.), the relation will not exactly hold in practice. The relation may be used as a calibration consistency check.

4. Conclusions

An alternative calibration procedure has been described which uses a matched load and four arbitrary known reactances as reflection standards. The method, which is an extension to Riblet-Hansson's procedure [5], is suitable in situations where equispaced offset shorts or opens are not available or where the use of line sections is not practical (e.g. at low frequencies).

The main source of systematic errors when measuring low reflection coefficients is the residual reflection from the matched load standard. In this case, a sliding load can be used to improve the accuracy. A convenient method of calibration with a sliding load is described in the accompanying paper [12].

Appendix 1: Equation (9)

The i−th row $\mathbf{A}_{i\cdot}$ of the matrix $\mathbf{A}$ defined by (9) can be modified to a more convenient form

$$\mathbf{A}_{i\cdot} = \mathbf{F}_4 . \mathbf{F}^{-1} \left(\mathbf{Q}_i - Q_{i4} . \mathbf{E}\right) . \mathbf{F} \qquad (A1)$$

where $\mathbf{E}$ is a unit matrix. The inverse matrix $\mathbf{F}^{-1}$ can be expressed explicitly as

$$\mathbf{F}^{-1} = \frac{1}{\Delta} \cdot \begin{bmatrix} s_{23} & s_{31} & s_{12} \\ s_3 - s_2 & s_1 - s_3 & s_2 - s_1 \\ c_3 - c_2 & c_1 - c_3 & c_2 - c_1 \end{bmatrix}$$

where

$$\Delta = s_{23} + s_{31} + s_{12}$$

is the determinant of matrix $\mathbf{F}$ and

$$s_{jk} = \sin\left(\varphi_j - \varphi_k\right) = s_j c_k - c_j s_k$$

(φ_j is the phase of j−th standard reflection coefficient).

Solution to Eq. (10) will not change if each row (A1) is multiplied by the determinant Δ. The product $\Delta . \mathbf{F}_4 . \mathbf{F}^{-1}$ will then be a 3-dimensional row vector with the components

$$\begin{aligned} & s_2c_3 - c_2s_3 + c_4\left(s_3 - s_2\right) - s_4\left(c_3 - c_2\right) \\ & s_3c_1 - c_3s_1 + c_4\left(s_1 - s_3\right) - s_4\left(c_1 - c_3\right) \qquad (A2) \\ & s_1c_2 - c_1s_2 + c_4\left(s_2 - s_1\right) - s_4\left(c_2 - c_1\right) \end{aligned}$$

The second multiplier of (A1) can also be readily expressed:

$$\left(\mathbf{Q}_i - Q_{i4} . \mathbf{E}\right) . \mathbf{F} = $$

$$= \begin{bmatrix} Q_{i1}-Q_{i4} & \left(Q_{i1}-Q_{i4}\right)c_1 & -\left(Q_{i1}-Q_{i4}\right)s_1 \\ Q_{i2}-Q_{i4} & \left(Q_{i2}-Q_{i4}\right)c_2 & -\left(Q_{i2}-Q_{i4}\right)s_2 \\ Q_{i3}-Q_{i4} & \left(Q_{i3}-Q_{i4}\right)c_3 & -\left(Q_{i3}-Q_{i4}\right)s_3 \end{bmatrix} \qquad (A3)$$

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