SIGNAL-FLOW GRAPH NETWORK ANALYSIS

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Abstract

A novel type of signal-flow graph is presented creating minimum number of nodes for independent voltages only. The graph will be derived by inspection of the network using special transforming T-graphs. The method is useful for the analysis of medium-size active and switched networks.

Keywords:
networks, circuit theory, graph theory

Introduction

Signal-flow graph (SFG) technique is very useful tool for the hand analysis of small and medium-size networks and/or subnetworks of larger systems. Last days the SFG's have been effectively used for the switched capacitor (SC) networks too. Many types of SFG's and different methods of their derivation have already been proposed, but still relatively complicated and cumbersome. Many of them are also restricted on some class of networks, specially of SC.

The method described in this paper is quite general and can be applied to all types of networks, that may contain any active elements - op.amps, voltage amplifiers, controled sources, unity gain buffers, immittance converters etc.

This method requires neither equivalent circuit, nor auxiliary network transformation, nor tables of subnetworks and nor other intermediate steps. The derivation of the SFG is straightforward by inspection of the SC network, using special transforming T-graph.

The foundation of this method was given in [1]. However novel improvement, generalization, simplification and extension of this method is introduced in this paper. Furthermore the SFG will contain minimum number of nodes, for independent voltages in particular phases only.

This graph method is primarily effective for the hand analysis, but it can be used for a reverse process of synthesis as well. This is done by first deriving the SFG corresponding to a given transfer or immittance network functions. Then from the SFG a desired network is constructed.

Construction of signal-flow graphs using T- graphs

An ingenious technique to obtain uniformly the SFG of arbitrary network will be now given in this section. This method is based on node analysis. The derivation of the SFG is straightforward by inspection of the analyzed network, using special transforming T-graph. It describes active elements, functional blocks and switching ingeniously and uniformly.

Observance 1: The SFG of any network can be obtained by transforming the subgraphs of particular subnetworks (one-ports or two-ports Y_i).

Definition 1: The transforming T-graph is an auxiliary graph describing relationship between the variables of subnetworks (the separated admittances Y_i) denoted V_i' and others V_n for the whole circuit.

Example 1: Consider a most simplest case, namely the admittance Y_1 (described by the selfloop Y_1, voltage V_1' and current I_1' respectively) is connected to the regular node (n) of larger network. The given variables are in following relation

$$V_1' = V_n \tag{1a}$$

$$I_1' = I_n \tag{1b}$$

that can be rewritten in this form

$$V_1' = a \,.\, V_n = 1 \,.\, V_n \tag{2a}$$

$$I_n = b \,.\, I_1' = 1 \,.\, I_1' \tag{2b}$$

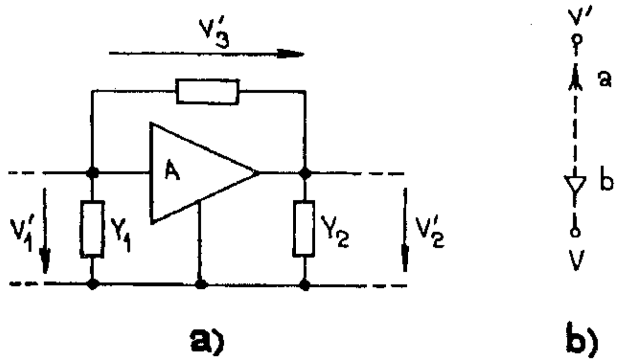
The eq's (2) can be portrayed by simple T-graph of the Fig.1b, where $a = b = 1$.

Definition 2: The T-graph (Fig.1b) consists of two branches, namely a voltage and a current one. The voltage T-branch is port raying eqn (2a) and has transmittance (a). The current T-branch portraying eqn (2b) has transmittance (b) respectively.

Remark: Note that the transmittances $a = b = 1$ in this case of Example 1 only.

Proposition 1: Generally, the transmittances of the voltage and the current T-branches have the same value ($a = b$) for passive networks only. It is not valid for the networks containing active elements (op.amp's, VCVS's), irregular to nodal analysis methods.

Furthermore using this T-graph the branches (y_{ji}') and/or the selfloops (y_{ii}') of subnetworks (Y_i') are transformed in the re sulting SFG (y_{km} or y_{mm}) as shown at the bottom of Fig.1d.



		Y_1	Y_2	Y_3
OA ($A \to \infty$)	a	0	1	-1
	b	1	0	1
VCVS (A)	a	1	A	1 - A
	b	1	0	1

c)

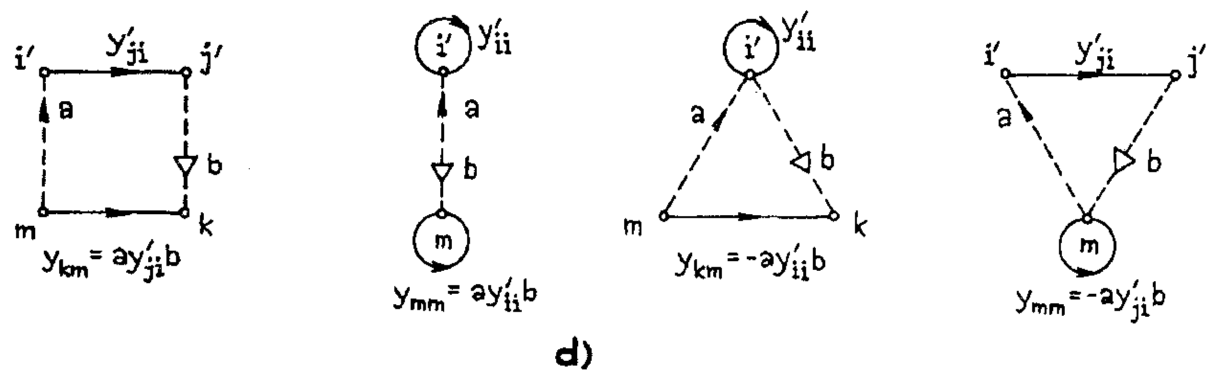


Fig.1
Transforming graphs of active elements
a) connection of subnetworks (admittances) Y to an active element,
b) simple form of the T-graph,
c) coefficients of T-branches,
d) basic transformation of subgraphs.

Definition 3: The branch (selfloop) of the SFG (y_{km}, y_{mm}) replaces the path between two nodes of SFG (m, k or m, m). This path is creating by one voltage T-branch (a), one branch (selfloop) of subgraph (y_{ji}' or y_{ii}') and one current T-branch (b) only. Not more! The transmittance of SFG branch is given by formula

$$y = \pm a . y' . b \quad , \tag{3}$$

what is particularly given in Fig.1d.

T-graphs of active SC networks

Up-to-date networks usualy contain active elements too, namely op.amps, voltage amplifiers (VCVS), unity gain buffers etc., irregular to nodal analysis methods.

Proposition 2: There are other transformations of voltage and current respectively, owing to special properties of active ele ments. Consequently the T-graph has other transmittances ($a = b$), if the subnetwork Y_i is connected at the port of this element (Fig.1a).

Operational amplifier

Connecting the admittance Y_1 at the input of the op.amp (Fig.1a), the voltage $V' = 0$ (that is the character of the nullator). Therefore the output voltage V_2 is taken as one independent volta ge only. Consequently

$$V_1' = 0 = a . V_2 = 0 . V_2 \tag{4}$$

and than the coefficient $a = 0$ in the table of Fig.1c.

The current I_1' is transformed without any change ($I_1' = I_1$), because $I_{inp} = 0$ of the op.amp. Consequently

$$I_1' = b . I_2 = 1 . I_2 \quad , \tag{5}$$

and therefore $b = 1$ in the table of Fig.1c.

Similarly the other values of the transmittances (a, b) in the table of Fig.1c can be proved for Y_2 and Y_3 too. Connecting the admittance Y_2 at the output of the op.amp (Fig.1a) corresponding T-graph has $a = 1$, $b = 0$. For the admittance Y_3 connected between i/o of the op.amp, there is a combination of the foregoing T-graphs and eqn

$$V_3' = V_1'' - V_2'' \tag{6}$$

Voltage amplifier

Furthermore if the active element is a voltage amplifier, VCVS and a unity gain buffer, the T-graphs (Fig.1b) should be similarly used, using correspondig transmittances a, b from the bottom of the table (Fig.1c).

However the given T-graph technique will be illustrated by following example.

Illustrative examples - analogue network ARC

Example 2: Consider an active RC filter (biquad) in Fig.2a, containing two op.amp's (OA1, OA2) and unity gain buffer ($A = -1$). There is a set of independent node voltages: V_1, V_3 and V_6. Others are dependent on them what is directly denoted in Fig.2a. Therefore the SFG can be containing nodes V_1, V_3, V_6 only.

Furthermore an associated T-graph is constructed in Fig.2b (dashed lines) using the given Proposition 2. The voltage on the capacitor C_1 can be expressed as follows

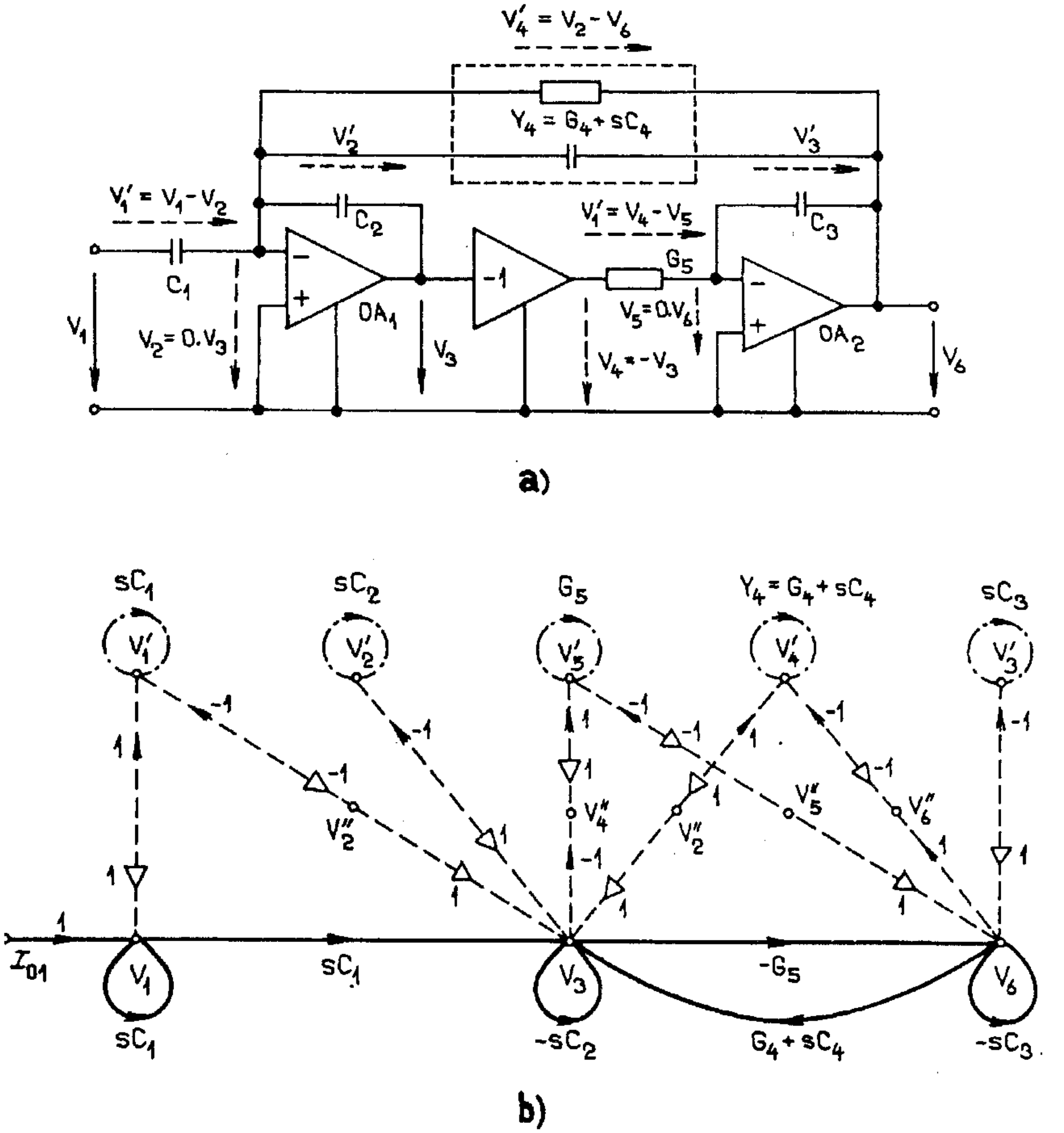


Fig.2
llustrative example - analogue ARC network
a) circuit diagram,
b) construction of the signal-flow graph.

$$V_1' = V_1 - V_2'' = V_1 - 0 \,.\, V_3 \quad (7)$$

The eqn (4) is portrayed by the T-graph (part: V_1, V_1', V_2'', V_3) in Fig.2b. Using this T-graph the selfloop (pC_1) is transformed in resulting SFG as shown at the bottom of the Fig.2b.

The capacitors C_2 and C_3 are connected between i/o of the OA1 and the OA2 respectively. Therefore the T-graphs have the form of Y_3 in Fig.1.

The voltage on the admittance Y_4 should be expressed as

$$V_4' = V_2'' - V_6'' = 0 \,.\, V_3 - 1 \,.\, V_6 \quad (8)$$

what is portrayed by corresponding T-graph (part: V_3, V_2'', V_4', V_6'', V_6) in Fig.2b.

Similarly for conductance G_5 (Fig.2a)

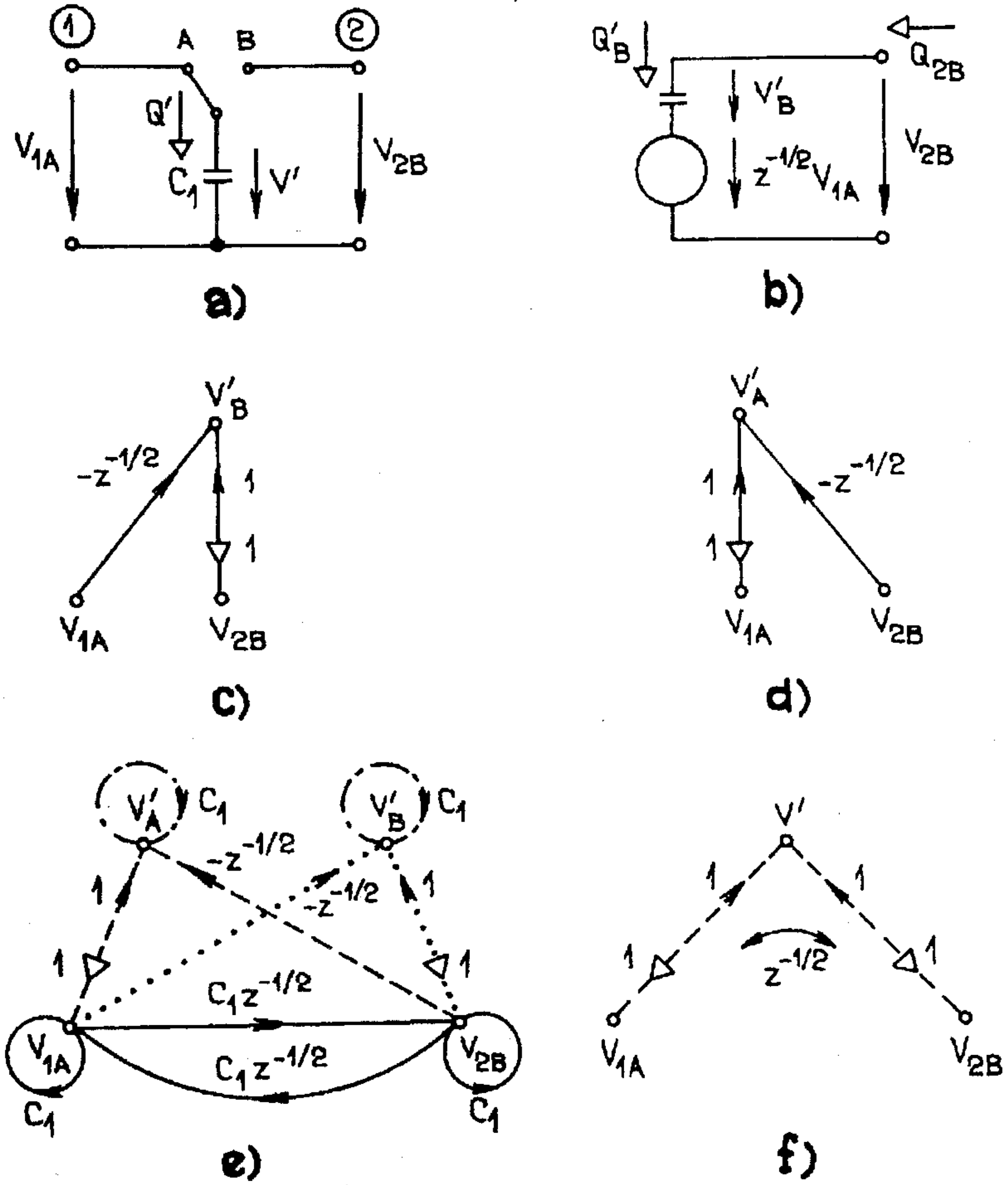


Fig.3

A biphase switched capacitor

a) a toggle-switched capacitor (between phases A and B),

b) equivalent circuit for the phase B (passing A to B),

c) corresponding-subgraph,

d) the T-subgraph for the passing B to A,

e) construction of the SFG

f) a simplified form of the T-graph.

$$V_5' = V_4'' - V_5'' = (-1) \,.\, V_3 - 0 \,.\, V_6 \qquad (9)$$

and corresponding T-graph (part: $V_3, V_4'', V_5', V_5'', V_6$) in Fig.2b.

Switched capacitor networks

The presented method is applicable to all types of switched capacitor networks too. They can be containing any active elements and functional blocks, biphase and multiphase switching. The T-graph describing the switching will have the same form.

Biphase switched capacitor

Consider a toggle-switched capacitor C_1 in Fig.3a. There is a basic biphase switching with a clock period (T_c) or a sampling period ($T_s = \frac{T_c}{2}$) respectively. However the T_c seems to be more convenient means to express delay factor z^{-d}.

The capacitor is charged at Q_{1A}, V_{1A} from the input port in the phase A. Due to a charge conservation these Q_{1A}, V_{1A} are transferred in the phase B (the delay factor $z^{-1/2}$), that is represented by a source ($z^{-1/2} \,.\, V_{1A}$) in the equivalent circuit of Fig.3b. There the voltage V_B' and the charge Q_B' of the capacitor C_1 in phase B can be evaluated by following eqns

$$V_B' = V_B - z^{-1/2} \,.\, V_A, \qquad Q_B' = Q_B \qquad (10)$$

A T-subgraph portraying the eqns (10) is shown in Fig.3c. Similarly the other T-subgraph in Fig.3d describes the opossite switching - passing B to A. Furthermore the T-graph in Fig.3e portraying the whole biphase switching is a combination of two subgraphs given above. A construction of the resulting SFG is shown there too, using the Definition 3. The T-graph (Fig.3e) can be redrawn (grouping the nodes V_A' and V_B') in a simplified form of Fig.3f, henceforth that will be better to be used.

Proposition 3: The biphase switched capacitor C is described by the T-graph in Fig.3f. The selfloop C is transformed in the SFG using the Definition 3 to obtain the static branches (for one phase only). The transmittance of the dynamic branches (between nodes of other phases $V_A \to V_B$ and $V_B \to V_A$ respectively) must be multiplied by the delay factor ($z^{-1/2}$), that is noted in the T-graph of Fig.3f.

Multiphase switching

For this case the Proposition 3 can be generalized and then a procedure of the SFG construction will be given.

Proposition 4: The SFG of a multiphase toggle-switched capacitor can be obtained modifying the T-graph of Fig.3f, by the way of following procedure.

Procedure 1:

Independent voltage nodes in particular phases may be taken to create the SFG only and to determine all other voltages in the SCN.

The static part of the SFG (for all phases separately) is constructed using the Definition 3.

The transmittance of the dynamic branches between the consequent phases: $A \to B$, $B \to C, \dots, (N-1) \to N$ and back $N \to A$, must be multiplied by the delay factor $(-z^{-1})$.

Any branch between nonconsequent phases (e.g. $A \to C, B \to E$ etc.) may be omitted, because the charges are not directly transferred there.

Intermediate switching

A special switching with intermediate unemployed phase is considered in Fig.4a. There a capacitor C_1 was charged in the phase A and than is passing over B (without any change of the charge) and will be switched at the output port in phase C. This is described by eqns

$$V_C' = V_{2C} - z^{-2/n} \,.\, V_{1A}, \qquad Q_{2C} = Q_C' \qquad (11)$$

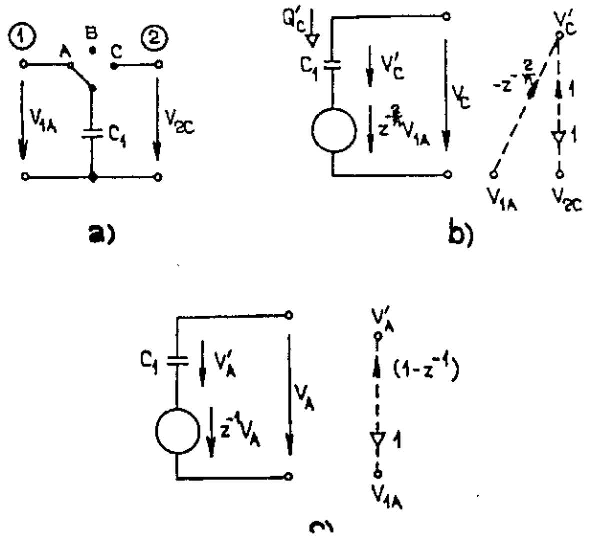


Fig.4
Special multiphase switching
a) a capacitor toggle-switched (A->C) over intermediate phase (B),
b) equivalent circuit and T-graph for passing A->C over B,
c) the same for passing A-> A' over B and C.

for the equivalent circuit and corresponding T-graph in Fig.4b.

Proposition 4: The voltage V_{iB} on the capacitor C_i in the phase B is not taken in the SFG if this phase is unemployed (intermediate) namely the C_i transfers the charge only (without any change of charge value in this phase). The transmittance of dynamic branches between the phases $A \to C$ must be multiplied by the delay factor $(-z^{(-2/n)})$.

Proposition 5: The Proposition 4 can be generalized on passing over two or more (m) intermediate unemployed phases, than the delay factor is $(-z^{-(m+1)/n})$.

Switching in original phase

Furthermore the other special switching is in the Fig.4 too. The capacitor C_1 (charged in A) is passing the phases B and C too and then it returns back in the original phase A' (now $A + T$), without any change of charge. These conditions can be described by following eqns

$$V_A' = V_{1A} - z^{-1} . V_{1A} = V_{1A} . (1 - z^{-1}) \;,$$

$$Q_{1A} = Q_A' \qquad (12)$$

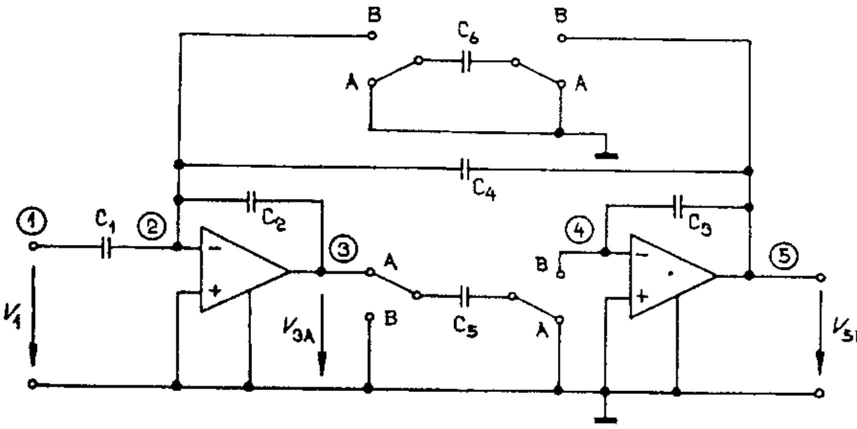
and an equivalent circuit in Fig.4c. The corresponding T-graph (Fig.4c) is arisen from the one of Fig.4b, grouping the nodes V_A and $V_{(A+T)}$.

Proposition 6: Switching in the original phase (A), passing over $m = n - 1$ phases (n is the number of phase), can be described by T-graph of Fig.4c. The resulting SFG contains the voltage V_A only and the selfloop with transmittance $C . (1 - z^{-1})$, that can be interpreted as an equivalent capacitance.

Illustrative example - switched capacitor network

Using propositions given above a construction of the SFG will be expressively simplified, that will be illustrated in following example.

Example 3: Consider a biphase active switched capacitor network (Fig.5a) containing op.amps ($OA1$, $OA2$). On the base of above discussion an associated graph is constructed in Fig.5b.



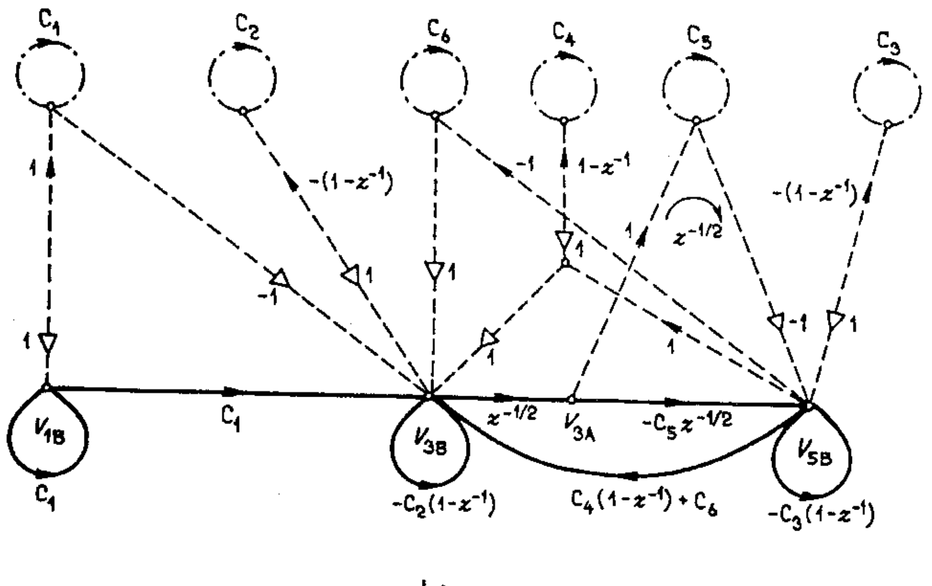


Fig.5

Illustrative example - switched capacitor network
a) circuit diagram,
b) construction of the signal-flow graph

Evaluation of network function from SFG

There are a gain formula and modified expression of determinants for the SFG (Coates's type) presented there. The Mason's rules allow to write down the gain G (transmittance) between an input (V_a) and output or other internal node (V_k) in following form

$$H = \frac{V_k}{V_a} = \frac{\sum_i G_{ak} . D_{ak}}{D_a} \quad , \tag{13}$$

where:

G_{ak} is the gain of the i-th forward path from the node V_a to V_k,

D_{ak} is the determinant of the subgraph (part of graph) nontouching the path with the gain G_{ak},

D_a is the determinant of the subgraph non-touching the node V_a.

The determinant for any subgraph is defined by general expression

$$D = S - \sum_i L_1 . S_1 + \sum_j L_2 . S_2 - \sum_k L_3 . S_3 - \dots \tag{14}$$

where:

S is a product of the gains S_i of the all i-th selfloops

$$S = \prod_i S_i \ , \tag{15}$$

L_x is a product of the gains y-th possible combination of x nontouching loops ($x = 1, 2, 3, \dots$),

S_x is the gain product (5) of the all selfloops nontouching loops L_x .

Conclusion

A general direct method for the symbolic analysis of an analogue and/or SC networks using signal-flow graph technique has been presented in this paper. The SFG has been derived by inspection of the networks using transforming T-graph.

The presented method was quite general and could be applicable to all types of SC networks, biphase and multiphase clock driven, containing any active elements (op.amps, voltage amplifiers, VCVSs, unity gain buffers, immittance converters etc.). The method was direct, it did not require any other intermediate steps. The T-graph was describing the switching and active elements ingeni ously and uniformly. The method was given for the hand analysis of small and medium-size SC networks or subnetworks as building blocks of larger SC systems.

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