Abstract

A novel type of signal-flow graph is presented creating minimum number of nodes for independent voltages only. The graph will be derived by inspection of the network using special transforming T-graphs. The method is useful for the analysis of medium-size active and switched networks.

Keywords:
networks, circuit theory, graph theory

Introduction

Signal-flow graph (SFG) technique is very useful tool for the hand analysis of small and medium-size networks and/or subnetworks of larger systems. Last days the SFG's have been effectively used for the switched capacitor (SC) networks too. Many types of SFG's and different methods of their derivation have already been proposed, but still relatively complicated and cumbersome. Many of them are also restricted on some class of networks, specially of SC.

The method described in this paper is quite general and can be applied to all types of networks, that may contain any active elements - op.amps, voltage amplifiers, controlled sources, unity gain buffers, immittance converters etc.

This method requires neither equivalent circuit, nor auxiliary network transformation, nor tables of subnetworks and nor other intermediate steps. The derivation of the SFG is straightforward by inspection of the SC network, using special transforming T-graph.

The foundation of this method was given in [1]. However novel improvement, generalization, simplification and extension of this method is introduced in this paper. Furthermore the SFG will contain minimum number of nodes, for independent voltages in particular phases only.

This graph method is primarily effective for the hand analysis, but it can be used for a reverse process of synthesis as well. This is done by first deriving the SFG corresponding to a given transfer or immittance net- work functions. Then from the SFG a desired network is constructed.

Construction of signal-flow graphs using T-graphs

An ingenious technique to obtain uniformly the SFG of arbitrary network will be now given in this section. This method is based on node analysis. The derivation of the SFG is straightforward by inspection of the analyzed network, using special transforming T-graph. It describes active elements, functional blocks and switching ingeniously and uniformly.

Observance 1: The SFG of any network can be obtained by transforming the subgraphs of particular subnetworks (one-ports or two-ports $Y_i$).

Definition 1: The transforming T-graph is an auxiliary graph describing relationship between the variables of subnetworks (the separated admittances $Y_i$) denoted $V_i$ and others $V_n$ for the whole circuit.

Example 1: Consider a most simplest case, namely the admittance $Y_1$ (described by the selfloop $Y_1$, voltage $V_1'$ and current $I_1'$ respectively) is connected to the regular node $(n)$ of larger network. The given variables are in following relation

\begin{align}
V_1' &= V_n \quad (1a) \\
I_1' &= I_n \quad (1b)
\end{align}

that can be rewritten in this form

\begin{align}
V_1' &= a \cdot V_n = 1 \cdot V_n \quad (2a) \\
I_n &= b \cdot I_1' = 1 \cdot I_1' \quad (2b)
\end{align}

The eq's (2) can be portrayed by simple T-graph of the Fig.1b, where $a = b = 1$.

Definition 2: The T-graph (Fig.1b) consists of two branches, namely a voltage and a current one. The voltage T-branch is port rating eqn (2a) and has transmittance $(a)$. The current T-branch portraying eqn (2b) has transmittance $(b)$ respectively.

Remark: Note that the transmittances $a = b = 1$ in this case of Example 1 only.

Proposition 1: Generally, the transmittances of the voltage and the current T-branches have the same value $(a = b)$ for passive networks only. It is not valid for the networks containing active elements (op.amp's, VCVS's), irregular to nodal analysis methods.

Furthermore using this T-graph the branches $(y_{ij})$ and/or the selfloops $(y_{ii})$ of subnetworks $(Y_i)$ are transformed in the resulting SFG $(y_{km}$ or $y_{mn})$ as shown at the bottom of Fig.1d.
Definition 3: The branch (selfloop) of the SFG \((y_{km}, y_{mm})\) replaces the path between two nodes of SFG \((m, k \text{ or } m, m)\). This path is creating by one voltage T-branch \((a)\), one branch (selfloop) of subgraph \((y_{ji} \text{ or } y_{jj})\) and one current T-branch \((b)\) only. Not more! The transmittance of SFG branch is given by formula

\[
y = \pm a \cdot y' \cdot b,
\]

what is particularly given in Fig.1d.

**T-graphs of active SC networks**

Up-to-date networks usually contain active elements too, namely op.amps, voltage amplifiers (VCVS), unity gain buffers etc., irregular to nodal analysis methods.

Proposition 2: There are other transformations of voltage and current respectively, owing to special properties of active elements. Consequently the T-graph has other transmittances \((a = b)\), if the subnetwork \(Y_1\) is connected at the port of this element (Fig.1a).

**Operational amplifier**

Connecting the admittance \(Y_1\) at the input of the op.amp (Fig.1a), the voltage \(V' = 0\) (that is the characteristic of the nullator). Therefore the output voltage \(V_2\) is taken as one independent voltage only. Consequently

\[
V_1' = 0 = a \cdot V_2 = 0 \cdot V_2
\]

and than the coefficient \(a = 0\) in the table of Fig.1c.

The current \(I_1'\) is transformed without any change \((I_1' = I_1)\), because \(I_{inp} = 0\) of the op.amp. Consequently

\[
I_1' = b \cdot I_2 = 1 \cdot I_2
\]

and therefore \(b = 1\) in the table of Fig.1c.

Similarly the other values of the transmittances \((a, b)\) in the table of Fig.1c can be proved for \(V_2\) and \(V_3\) too. Connecting the admittance \(Y_2\) at the output of the op.amp (Fig.1a) corresponding T-graph has \(a = 1, b = 0\). For the admittance \(Y_3\) connected between \(y_0\) of the op.amp, there is a combination of the foregoing T-graphs and eqn

\[
V_3' = V_1'' - V_2''
\]

**Voltage amplifier**
Furthermore if the active element is a voltage amplifier, VCVS and a unity gain buffer, the T-graphs (Fig.1b) should be similarly used, using corresponding transmittances $a$, $b$ from the bottom of the table (Fig.1c).

However the given T-graph technique will be illustrated by the following example.

**Illustrative examples - analogue network ARC**

Example 2: Consider an active RC filter (biquad) in Fig.2a, containing two op.amp's (OA1, OA2) and unity gain buffer ($A = -1$). There is a set of independent node voltages $V_1, V_2$ and $V_6$. Others are dependent on them what is directly denoted in Fig.2a. Therefore the SFG can be containing nodes $V_1, V_2, V_6$ only.

Furthermore an associated T-graph is constructed in Fig.2b (dashed lines) using the given Proposition 2. The voltage on the capacitor $C_1$ can be expressed as follows.

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**Fig.2**

Illustrative example - analogue ARC network

a) circuit diagram,

b) construction of the signal-flow graph.
\[ V'_1 = V_1 - V''_2 = V_1 - 0 . V_3 \]  \hspace{1cm} (7)

The eqn (4) is portrayed by the T-graph (part: \( V_1, V'_1, V''_2, V_3 \)) in Fig.2b. Using this T-graph the selloop \((pC_1)\) is transformed in resulting SFG as shown at the bottom of the Fig.2b.

The capacitors \( C_2 \) and \( C_3 \) are connected between i/o of the OA1 and the OA2 respectively. Therefore the T-graphs have the form of \( Y_3 \) in Fig.1.

The voltage on the admittance \( Y_4 \) should be expressed as

\[ V'_4 = V''_2 - V''_6 = 0 . V_3 - 1 . V_6 \]  \hspace{1cm} (8)

what is portrayed by corresponding T-graph (part: \( V_3, V''_2, V_4, V''_6, V_6 \)) in Fig.2b.

Similarly for conductance \( G_3 \) (Fig.2a)

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Fig.3

A biphase switched capacitor
a) a toggle-switched capacitor (between phases A and B),
b) equivalent circuit for the phase B (passing A to B),
c) corresponding subgraph,
d) the T-subgraph for the passing B to A,
e) construction of the SFG
f) a simplified form of the T-graph.
\[ V_S' = V_4'' - V_5'' = (-1) \cdot V_3 - 0 \cdot V_6 \]  

\[ V_A' = V_B - z^{-1/2} \cdot V_A, \quad Q_B' = Q_B \]  

Switched capacitor networks

The presented method is applicable to all types of switched capacitor networks too. They can be containing any active elements and functional blocks, biphase and multiphase switching. The T-graph describing the switching will have the same form.

Biphase switched capacitor

Consider a toggle-switched capacitor \( C_1 \) in Fig.3a. There is a basic biphase switching with a clock period \( T_c \) or a sampling period \( T_S = \frac{T_c}{2} \) respectively. However the \( T_S \) seems to be more convenient means to express delay factor \( z^{-d} \).

The capacitor is charged at \( Q_{1A} , V_{1A} \) from the input port in the phase \( A \). Due to a charge conservation these \( Q_{1A} , V_{1A} \) are transferred in the phase \( B \) (the delay factor \( z^{-1/2} \)), that is represented by a source \( z^{-1/2} \cdot V_{1A} \). The equivalent circuit of Fig.3b. There the voltage \( V_B' \) and the charge \( Q_B \) of the capacitor \( C_1 \) in phase \( B \) can be evaluated by following eqns

\[ V_B' = V_B - z^{-1/2} \cdot V_A, \quad Q_B' = Q_B \]  

A T-subgraph portraying the eqns (10) is shown in Fig.3c. Similarly the other T-subgraph in Fig.3d describes the oposite switching - passing \( B \) to \( A \). Furthermore the T-graph in Fig.3e portraying the whole biphase switching is a combination of two subgraphs given above. A construction of the resulting SFG is shown there too, using the Definition 3. The T-graph (Fig.3e) can be redrawn (grouping the nodes \( V_A \) and \( V_B \)) in a simplified form of Fig.3f, henceforth that will be better to be used.

Proposition 3: The biphase switched capacitor \( C_1 \) is described by the T-graph in Fig.3f. The selfloop \( C_1 \) is transformed in the SFG using the Definition 3 to obtain the static branches (for one phase only). The transmittance of the dynamic branches (between nodes of other phases \( V_A \rightarrow V_B \) and \( V_B \rightarrow V_A \) respectively) must be multiplied by the delay factor \( z^{-1/2} \), that is noted in the T-graph of Fig.3f.

Multiphase switching

For this case the Proposition 3 can be generalized and then a procedure of the SFG construction will be given.

Procedure 1: Independent voltage nodes in particular phases may be taken to create the SFG only and to determine all other voltages in the SCN.

The static part of the SFG (for all phases separately) is constructed using the Definition 3.

The transmittance of the dynamic branches between the consequent phases: \( A \rightarrow B \), \( B \rightarrow C \), ..., \( (N - 1) \rightarrow N \) and back \( N \rightarrow A \), must be multiplied by the delay factor \( (-z^{-1/2}) \).

Any branch between nonconsequent phases (e.g. \( A \rightarrow C, B \rightarrow E \) etc.) may be omitted, because the charges are not directly transferred there.

Intermediate switching

A special switching with intermediate unemploy phase is considered in Fig.4a. There a capacitor \( C_1 \) was charged in the phase \( A \) and then is passing over \( B \) (without any change of the charge ) and will be switched at the output port in phase \( C \). This is described by eqns

\[ V_C' = V_{2B} - z^{-2b}, V_{1A}, \quad Q_{2C} = Q_{C'} \]  

for the equivalent circuit and corresponding T-graph in Fig.4b.

Proposition 4: The voltage \( V_{10} \) on the capacitor \( C_1 \) in the phase \( B \) is not taken in the SFG if this phase is unemploy (intermediate) namely the \( C_1 \) transfers the charge only (without any change of charge value in this phase). The transmittance of dynamic branches between the phases \( A \rightarrow C \) must be multiplied by the delay factor \( (-z^{-1/2}) \).

Proposition 5: The Proposition 4 can be generalized on passing over two or more \( (m) \) intermediate unemploy phases, than the delay factor is \( (-z^{-m/2}) \).
Switching in original phase

Furthermore the other special switching is in the Fig.4 too. The capacitor $C_1$ (charged in $A$) is passing the phases $B$ and $C$ too and then it returns back in the original phase $A'$ (now $A + T$), without any change of charge. These conditions can be described by following eqns

$$V_{A'} = V_{1A} - z^{-1} . \quad V_{1A} = V_{1A} . (1 - z^{-1})$$

$$Q_{1A} = Q_{A'}$$

and an equivalent circuit in Fig.4c. The corresponding T-graph (Fig.4c) is arisen from the one of Fig.4b, grouping the nodes $V_A$ and $V_{(A + T)}$.

Proposition 6: Switching in the original phase ($A$), passing over $m = n - 1$ phases ($n$ is the number of phase), can be described by T-graph of Fig.4c. The resulting SFG contains the voltage $V_A$ only and the selfloop with transmittance $C_1 (1 - z^{-1})$, that can be interpreted as an equivalent capacitance.

Illustrative example - switched capacitor network

Using propositions given above a construction of the SFG will be expressively simplified, that will be illustrated in following example.

Example 3: Consider a biphase active switched capacitor network (Fig.5a) containing op.amps ($O_A$, $O_4$). On the base of above discussion an associated graph is constructed in Fig.5b.

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**Fig.5**

Illustrative example - switched capacitor network

a) circuit diagram,
b) construction of the signal-flow graph
Evaluation of network function from SFG

There are a gain formula and modified expression of determinants for the SFG (Coates's type) presented there. The Mason's rules allow to write down the gain $G$ (transmittance) between an input $(V_a)$ and output or other internal node $(V_k)$ in following form

$$H = \frac{V_k}{V_a} = \frac{\sum G_{ak} \cdot D_{ak}}{D_a}, \quad (13)$$

where:
- $G_{ak}$ is the gain of the $i$-th forward path from the node $V_a$ to $V_k$,
- $D_{ak}$ is the determinant of the subgraph (part of graph) nontouching the path with the gain $G_{ak}$,
- $D_a$ is the determinant of the subgraph nontouching the node $V_a$.

The determinant for any subgraph is defined by general expression

$$D = S - \sum_{i} L_1 \cdot S_1 + \sum_{j} L_2 \cdot S_2 - \sum_{k} L_3 \cdot S_3 - ... \quad (14)$$

where:
- $S$ is a product of the gains $S_i$ of the all $i$-th selfloops

$$S = \prod_{i} S_i, \quad (15)$$

$L_x$ is a product of the gains $S_x$ of $x$ nontouching loops ($x = 1, 2, 3, ...$),
- $S_x$ is the gain product (5) of all selfloops nontouching loops $L_x$.

Conclusion

A general direct method for the symbolic analysis of an analogue and/or SC networks using signal-flow graph technique has been presented in this paper. The SFG has been derived by inspection of the networks using transforming T-graph.

The presented method was quite general and could be applicable to all types of SC networks, biphase and multiphase clock driven, containing any active elements (op.amps, voltage amplifiers, VCCVs, unity gain buffers, immittance converters etc.). The method was direct, it did not require any other intermediate steps. The T-graph was describing the switching and active elements ingeniously and uniformly. The method was given for the hand analysis of small and medium-size SC networks or subnetworks as building blocks of larger SC systems.

REFERENCES


About authors...

Tomáš Dostál was born in Brno, Czechoslovakia, in 1943. He received the CSc. and DrSc. degree in electrical engineering from the Technical University of Brno in 1976 and 1989 respectively. From 1973 to 1978 and 1980 to 1984 was with Military Academy Brno, from 1978 to 1980 with Military Technical College Baghdad. Since 1984 he has been with the Technical University of Brno, where he is now Professor of Radioelectronics. His present interests are in the circuit theory, filters and switched capacitor networks.

Ján Mikula was born in Czechoslovakia on 4rd April 1930. He received the CSc. and DrSc. degree in electrical engineering from the Military Academy Brno in 1968 and 1983 respectively. In the period 1954-1965 he was with the Military Technical Institute Prague and 1965 - 1967 with the Military Technical College Cairo. Since 1984 he has been with the Military Academy Brno, where he is now Professor of Radioelectronics. His present research interests include general network theory, signals, switched capacitor networks and filters.