SWITCHED CAPACITOR EQUIVALENT OF FREQUENCY DEPENDENT NEGATIVE RESISTOR

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Abstract

A new circuit of frequency dependent negative resistor (FDNR), derived using an almost linear s-z transformation in a four phase switched capacitor (SC) network is presented.

Keywords:
circuit theory, switched capacitor networks, frequency dependent negative resistors

1. Introduction

The FDNR given in [1] has been designed using the bilinear (BL) $s-z$transformation in an ingenious four phase SC network. However, the BL transformation is nonlinear and causes the frequency response of SC filter to differ from an analogue prototype owing to the known frequency warping effect. Therefore the construction of the FDNR using the almost linear $s-z$ transformation will be sometimes required. Than the frequency axis of discrete-time filter will agree well with that of analogue prototype counterpart over the wide frequency range.

2. Main results

One this transformation so called MDI (mixed discrete integrator) has been given in [2]. The MDI transformation is defined by following relation in the z-domain (for 4-phase SC switching specialy)

$$sT = \frac{2\cdot(1-z^{-4})}{k\cdot(1+z^{-4})+2\cdot(1-k)\cdot z^{-2}} \qquad (1)$$

where $z^{-4}=e^{-sT}$, $z^{-2}=e^{-sT/2}$ and T is the clock cycle.

Changing the parameter k (in the range $0 \le k \le 1$), the presented MDI transformation changes the character between the LDI (loss-less discrete integrator) and the BL (bilinear).

From eqn.(1) we can obtain the frequency transformation property given by

$$\Omega T = \frac{2\cdot\sin\left(\frac{\omega T}{2}\right)}{k\cdot\cos\left(\frac{\omega T}{2}\right)+1-k} \qquad (2)$$

where Ω is the frequency variable in analogue circuit and ω in the discrete equivalent. An optimum (where $\omega = \Omega$) of the parameter k, in the range $\omega R = 0-\pi$, has the value of $k_0 = 0.35855$.

Since half clock delay is not permitted in the SC system including closed loops, the transformation according to the eqn.(1) can not be realized directly and an approximation of the discrete variable is carried out as follows

$$sT = \frac{2\cdot(1-z^{-4})}{2-k+kz^{-4}} \qquad (3)$$

where the coefficient z^{-2} in eqn.1 is transferred and added to the constant term.

This transformation is changed between the BD (backward difference) and BL respectively, by changing the parameter k in the range $0 \le k \le 1$. However for certain value k_0 we can obtain the other almost linear (AL) transformation.

3. Realization of FDNR

The charge of the analog prototype of the FDNR is described by following relation in the s-domain

$$Q_0(s) = sDV_0(s) = \frac{D}{T}sTV_0(s) \qquad (4)$$

where D is the constant factor for the FDNR with resistance $R_e = 1/s^2D$.

Substituting eqn.(3) into eqn.(4), the charge equation in the z-domain has the form of

$$Q_0(z) = \frac{2D}{(2-k)T}\frac{1-z^{-4}}{1+\frac{k}{2-k}z^{-4}}V_0(z) \qquad (5)$$

and for switched capacitor than

$$Q_0(z) = C_0\frac{1-z^{-4}}{1+\frac{k}{2-k}z^{-4}}V_0(z) \qquad (6)$$

Comparing these eqns. (5) and (6) we obtain

$$D = \frac{2-k}{2}C_0T \qquad (7)$$

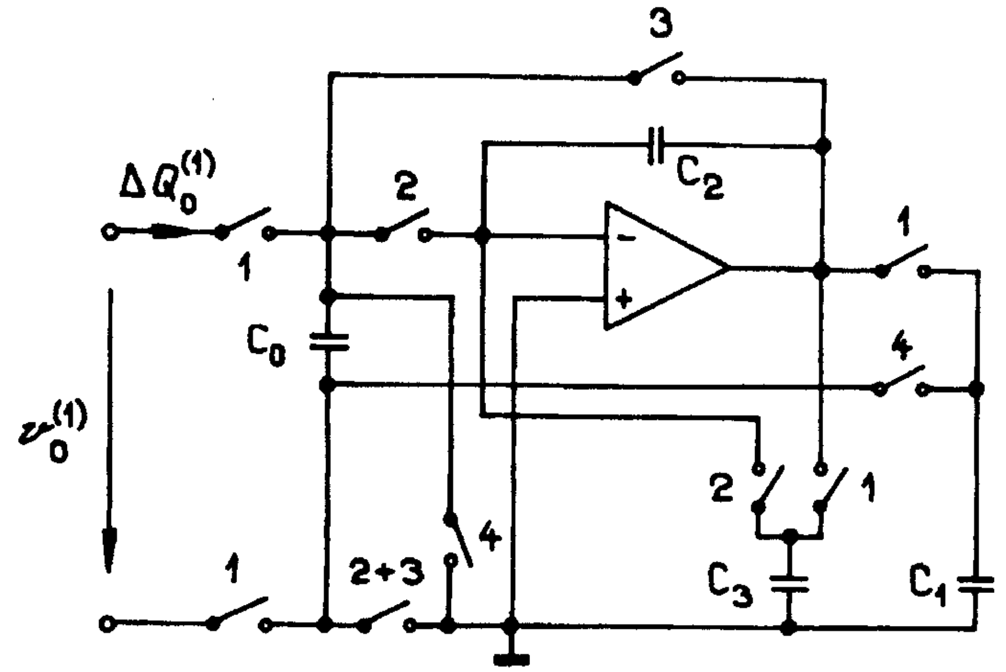

Fig.1
Circuit simulating the frequency depended negative resistor

The incremental of the charge can be written as

$$\Delta Q_0^{(1)}(z) = (1 - z^{-4})Q_0(z) = C_0 \frac{(1 - z^{-4})^2}{1 + \frac{k}{2-k} z^{-4}} V_0(z) \quad (8)$$

Furthermore, the FDNR using the BL transformation in the 4-phase SC network has been presented in [1]. The circuit diagram of this FDNR is in Fig.1. The incremental of the charge for this network is given by

$$\Delta Q_0^{(1)}(z) = C_0 \frac{1 - \left(\frac{C_0^2}{C_2(C_0 + C_1} - \frac{C_3}{C_2} + 1 \right) z^{-4} + \frac{C_0 C_1}{C_2(C_0 + C_1)} z^{-8}}{1 + \left(\frac{C_3}{C_2} - 1 \right) z^{-4}} V_0^{(1)}(z) \quad (9)$$

Comparing eqns. (8) and (9) we obtain

$$\frac{C_3}{C_2} - 1 = \frac{k}{2-k}$$

$$\frac{C_0^2}{C_2(C_0 + C_1)} - \frac{C_3}{C_2} + 1 = 2$$

$$\frac{C_0 C_1}{C_2(C_0 + C_1)} = 1 \quad (10)$$

and from these eqns. (10) we can get

$$C_1 = \frac{2-k}{4-k} C_0$$

$$C_2 = \frac{2-k}{6-2k} C_0$$

$$C_3 = \frac{2}{6-2k} C_0 \quad (11)$$

Than obviously the capacitances for BL transformation ($k = 1$) have to be evaluated as $C_1 = \frac{1}{3} C_0$, $C_2 = \frac{1}{4} C_0$, $C_3 = \frac{1}{2} C_0$ and similarly for BD transformation ($k = 0$) as $C_1 = \frac{1}{2} C_0$, $C_2 = \frac{1}{3} C_0$, $C_3 = \frac{1}{3} C_0$.

Nevertheless, for the almost linear transformation (AL) , where $k_0 = 0.35855$, the capacitances have to be $C_1 = 0.45076 \cdot C_0$, $C_2 = 0.31071 \cdot C_0$, $C_3 = 0.37857 \cdot C_0$.

Therefore we can take a remarkable result - this FDNR can be easy realized, according to several transformation BL,

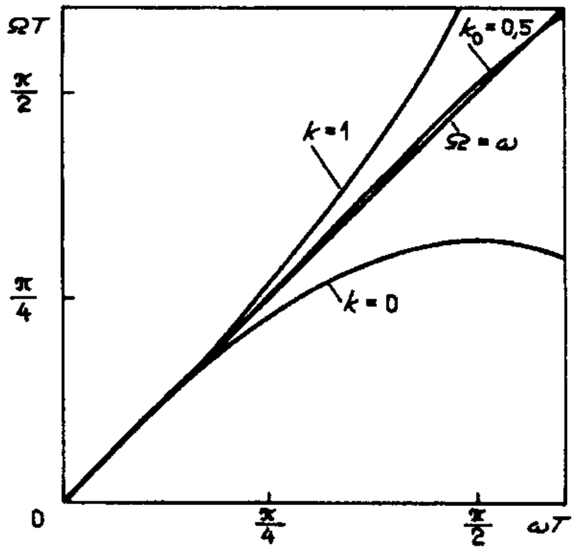

Fig.2
Frequency transformation characteristics

BD and suitable AL respectively, changing the capacitance values only.

Note that parameter $k_0 = 0.35855$ has been derived from eqn.(1), nevertheless it should be determined for the circuit in Fig.1 using the reduced eqn. (3). Considering $sT = (\Sigma + j\Omega)T$ we get accordingly

$$(\Sigma + j\Omega)T = \frac{2(1-k)\tan^2\frac{\omega T}{2} + j2\tan\frac{\omega T}{2}}{1+(1-k)^2\tan^2\frac{\omega T}{2}} \qquad (12)$$

what is purely imaginary and hence lossless, for only $k = 1$. This circuit does not present the ideal FDNR, for other values of the parameter k. A frequency transformation property resulting from eqn.(12) can be given by

$$\Omega T = \frac{2\tan\frac{\omega T}{2}}{1+(1-k)^2\tan^2\frac{\omega T}{2}} \qquad (13)$$

The parameter k has now the optimum value as $k_0 = 0.5$, in the range $\omega T = 0 - \pi/2$. The frequency transformation characteristics are given in Fig. 2, namely for BD (k=0), AL (k=0.5), BL (k=1) and ideal case ($\Omega = w$), respectively. Furthermore, the capacitances for this value of $k_0 = 0.5$ have to be evaluated as $C_1 = 0.42857 \cdot C_0$, $C_2 = 0.3 \cdot C_0$, $C_3 = 0.4 \cdot C_0$, what is slightly differ from those given above for $k_0 = 0.35855$.

Considering eqns. (3) and (7), the equivalent admittance of this FDNR can be derived in following form

$$Y_e(\omega) = s^2 D =$$

$$= -\frac{4\tan^2\frac{\omega T}{2}\left[1-(1-k)^2\tan^2\frac{\omega T}{2} - j2(1-k)\tan\frac{\omega T}{2}\right]}{T^2\left[1+(1-k)^2\tan^2\frac{\omega T}{2}\right]^2}\frac{2-k}{2}C_0 T \qquad (14)$$

what represents the admittance of the FDNR and the capacitor connected in parallel. If $k = 1$, the FDNR is ideal. Note that this capacitor determines the quality factor in the resonance circuit with FDNR and must be taken in mind for SC filter design.

4. Conclusion

The new SC-FDNR controlled by four clock phases according to the almost linear $s-z$ transformation is presented. This FDNR will find applications in the design of SC filters where the close approximation of analogue filter is required.

5. References

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