Analysis of Irregular Networks using Diakoptic Modification of Admittance Matrix Inversion

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Abstract

A special method of admittance matrix inversion is presented in this article. This method is based on diakoptic tearing of non-standard networks and then solving the subnetworks step by step with added torn branches. It is suggested for computer-aided analysis of large networks.

Keywords:
Network analysis, Diakoptics, Node voltage method

1. Introduction

In linear network analysis the node voltage method (utilizing their corresponding orientation) is currently used [1]. Then the network is described by admittance matrix **Y** and the matrix equation

$$\mathbf{I} = \mathbf{Y} \cdot \mathbf{U} \quad . \tag{1}$$

The following inversion of admittance matrix **Y** gives the possibility to determine all node voltages expressed by the formula

$$\mathbf{U} = \mathbf{Y}^{-1} \cdot \mathbf{I} \quad . \tag{2}$$

From the inverse matrix

$$\mathbf{Z} = \mathbf{Y}^{-1} \quad , \tag{3}$$

all the immittance and the transfer functions can be determined as well as all other network variables and functions. Such a method of the complete analysis is better then the classical one utilizing algebraic cofactors [1]. However the inversion $\mathbf{Y}^{-1}$ represents laborious procedure whose difficulty grows up according the order of matrix (number of network nodes n) approximately n^3 times. From this reason the diakoptic procedure introduced in [2] is very useful for the admittance matrix inversion, of course under conditions of sparse matrix **Y** only which is usually fulfilled by the real networks. (In opposite case the efficiency of this method would be even lower than the classical one).

2. Principle of diakoptic inversion method

Diakoptic method is based on the tearing of some branches represented by the chosen two terminal elements (i.e. one-ports) or by the transfer parameters of some special two-ports (Fig. 1). The corresponding (original) subnetworks can be described by diagonal admittance matrix $\mathbf{Y}_0$ whose inversion is very simple with minimum numerical errors.

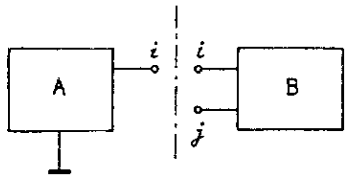

Fig. 1
Diakoptic tearing of network.

In the next step the inversion matrix $\mathbf{Y}_0^{-1}$ is modified by the following way: one of the torn branches is considered in the original description and the new inverse matrix (3) is obtained in the form

$$\mathbf{Y}_1^{-1} = \mathbf{Y}_0^{-1} + k_0 \mathbf{A}_0 \quad , \tag{4}$$

where k_0 is the correction coefficient and $\mathbf{A}_0$ is the matrix of some additional increases. The formula (4) can be generalized for any m-th step of modification, i.e.

$$\mathbf{Y}_m^{-1} = \mathbf{Y}_{m-1}^{-1} + k\mathbf{A} \quad . \tag{5}$$

The correction coefficient is generally given by the formula

$$k = \frac{-\delta y}{1 - \delta y \xi} \quad , \tag{6}$$

where δy is the increase of admittance y of the additional branch. The scalar ξ is calculated as

$$\xi = z_{ac} - z_{ad} - z_{bc} + z_{bd} \quad , \tag{7}$$

where z_{ij} are the entries of matrix $\mathbf{Z}_{m-1}$ from the previous step. Formula (7) is valid for disconnected two-port--voltage-controlled current source (VCCS) having interrupted transfer conductance g_T where the subscripts of parameter z_{ij} correspond to its connection into the chosen system of nodes (Fig. 2a). If some of the nodes (a, b, c, d) is grounded the corresponding parameter z_{ij} is zero. In case of two terminal element (R,L,C) the relations

$$a = c\ ,\ b = d\ , \tag{8}$$

must be considered. In case of direct interconnection of the separated subnetworks where the additional conductance increases to infinity ($\delta_y \to \infty$) eqn (6) has the simplified form

$$k = \frac{1}{\xi}\ . \tag{9}$$

The matrix of increments between two steps of modification

$$\mathbf{A} = \xi_{z0} \cdot \xi_{0z}\ , \tag{10}$$

is given by two vectors

$$\xi_{z0} = [(z_{1c} - z_{1d}); (z_{2c} - z_{2d}); \dots (z_{nc} - z_{nd})]^T\ , \tag{11}$$

$$\xi_{0z} = [(z_{a1} - z_{b1}); (z_{a1} - z_{b1}); \dots (z_{an} - z_{bn})]\ , \tag{12}$$

where z_{ij} are again the entries of matrix $\mathbf{Z}_{m-1}$ obtained in previous step. The accuracy of calculation can easily be determined from the approach

$$\mathbf{Y} \cdot \mathbf{Z} - \mathbf{1} \to \mathbf{0}\ . \tag{13}$$

It is also necessary to take in consideration numerical errors obtained by the product of matrices.

If the response or the network functions are required for certain ports only then the analysis procedure can be modified by reduction of inverse matrices of all the separate parts so that their rows and columns corresponding to remaining nodes are deleted. Some troubles can occur in the inversion of the original set of separate subnetwork admittance matrices when some of subnetworks does not include the reference node (subnetwork B in Fig. 1). It can be improved by their fictive interconnections using two parallel branches containing mutually compensated conductances G and $(-G)$. However unsuitable conductance values G can entail increasing of numerical errors.

The more efficient inversion procedure of admittance matrix of the separated subnetwork B (Fig. 1) is based on its connection with the grounded subnetwork A through the disconnected node i utilizing the **following algorithm**:

a) To find the inverse matrix of subnetwork A:

$$\mathbf{Z}_A = \mathbf{Y}_A^{-1}\ .$$

b) To find the inverse matrix of subnetwork B (of the order n_B)

$$\mathbf{Z}_B = \mathbf{Y}_B^{-1}\ ,$$

when the node i is connected to reference node.

c) From these matrices to built up diagonal block matrix $\mathbf{Z}$

d) To write inside this matrix (over matrix $\mathbf{Z}_A$) n_B times the i-th row of matrix $\mathbf{Z}_A$.

e) To write similarly to the left side n_B times the i-th column of matrix $\mathbf{Z}_A$.

f) To add parameter z_{ij} of matrix $\mathbf{Z}_A$ to each of parameters of matrix $\mathbf{Z}_B$.

During the analysis procedure the chosen node (i), which interconnects subnetwork A and the separated subnetwork B, can suitably be changed. Then in the floating admittance matrix of subnetwork B only the rows and columns corresponding to the chosen node can be omitted.

To find relation among the parameters of inverse matrices $\mathbf{Z}_B$he corresponding nodes (i,j) can be connected with the reference node by conductances G_i and G_j . When node (i) or (j) is grounded then $G_i \to \infty$ and $G_j = 0$ or $G_i = 0$ and $G_j \to \infty$, respectively. The relation between $\mathbf{Z}_B^{(i)}$ and $\mathbf{Z}_B^{(j)}$ for any grounded node can generally be expressed in the form

$$z_{kl}^{(j)} = z_{j+k,j+l}^{(i)} = z_{jj}^{(i)} - z_{jl}^{(i)} - z_{kl}^{(i)} + z_{kl}^{(i)}\ , \tag{14}$$

where $z^{(i)}$ and $z^{(j)}$ are the parameters of inverse matrices of the separated subnetwork B having grounded node (i) and (j), respectively. Resultant matrix $\mathbf{Z}^{(i)}$ is built from matrix $\mathbf{Z}^{(j)}$ by adding i-th row and column containing zero parameters which are then replaced by the parameters according eqn (14).

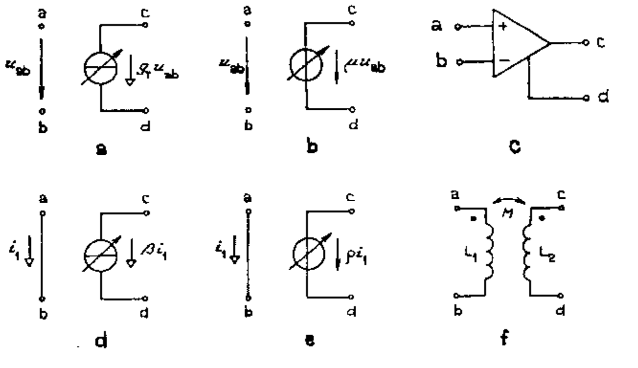

Fig. 2
Irregular two-ports.
a) voltage-controlled current source,
b) voltage-controlled voltage source,
c) ideal operational amplifier,
d) current-controlled current source,
e) current-controlled voltage source,
f) inductive coupling of two coils (transformer).

Voltage-controlled current source (VCCS) in Fig. 2 is the regular two-port described by transconductances g_T or by slope S . The following relation is valid

$$i_c = g_T\ u_{ab}\ . \tag{15}$$

Adding this source when modifying the network (i.e. the branch having transconductance g_T), the inverse matrix is modified according formula (4). The correction coefficient (6) can be rewritten for this case into the following form

$$k^{-1} = g_T^{-1} + z_{ac} - z_{ad} - z_{bc} + z_{ad} \quad . \tag{16}$$

Matrix of increments (10) can be determined from vectors (11) and (12).

3. Analysis of networks with non-standard elements

So far we have studied networks containing standard elements which can be considered as regular from the node voltage method point of view. Such networks contain only branches having finite conductances and VCCSs.

Generally, networks can contain even non-standard elements which can not be described by admittance matrix as, for example, operational amplifier, ideal voltage and current amplifiers and other blocks [1]. Networks containing these elements are usually analysed in two steps: first the regular part is described by node voltage method and then the effect of non-standard elements is expressed by a combined linear transformation according to [1] or by adding other equations as in [3]. Both procedures lead to an increase in the computer time, memory capacity and also numerical errors for higher number of mathematical operations. In further part it is shown how these elements can be included into hereby presented diakoptic modification of inverse admittance matrix.

Ideal voltage amplifier represents a **voltage-controlled voltage source** (VCVS) determined by voltage gain μ (Fig. 2b) having the internal resistance of the controlled branch (output port)

$$r = g^{-1} \to 0, \quad resp. \quad g \to \infty \quad . \tag{17}$$

For the method used the VCVS is converted in VCCS having internal conductance g , i.e.

$$i = \mu g u, \quad u_2 = \frac{1}{g}\mu g u_1 = \mu u_1 \quad . \tag{18}$$

When adding the VCVS (branch with parameter μ) to the network and considering eqns (17) the inverse matrix can be modified according to (4) with the correction coefficient in the form

$$k^{-1} = \mu(z_{ac} - z_{ad} - z_{bc} + z_{bd}) +$$

$$+(z_{cc} - z_{cd} - z_{dc} + z_{dd}) \quad , \tag{19}$$

as follows from eqns (6), (17), (18). Matrix of increments (10) is given by vector (11) and the other vector (12) is for this case modified into the form

$$\xi_{0z} = \mu[(z_{a1} - z_{b1}); (z_{a2} - z_{b2}); \ldots (z_{an} - z_{bn})] +$$

$$+[(z_{c1} - z_{d1}); (z_{c2} - z_{d2}); \ldots (z_{cn} - z_{dn})] \quad . \tag{20}$$

For an **ideal operational amplifier** (Fig. 2c) the relations derived for VCVS are valid when

$$\mu \to \infty \quad . \tag{21}$$

The effect of the operational amplifier is then expressed by the inverse matrix modification according to relation (4) where the correction coefficient is given by eqn (9) and matrix of increments (10) by vectors (11) and (12).

In case of ideal current amplifier, i.e. **current-controlled current source** (Fig. 2d) the following relation is valid

$$i_2 = \beta i_1 \quad . \tag{22}$$

When taking into consideration the zero input impedances and relation (17) the following coefficients are obtained for modification (4)

$$k^{-1} = \beta(z_{ac} - z_{ad} - z_{bc} + z_{bd}) +$$

$$+(z_{aa} - z_{ab} - z_{ba} + z_{bb}) \quad , \tag{23}$$

$$\xi_{z0} = \beta[(z_{1c} - z_{1d}); (z_{2c} - z_{2d}); \ldots (z_{nc} - z_{nd})]^T +$$

$$+[(z_{1a} - z_{1b}); (z_{2a} - z_{2b}); \ldots (z_{na} - z_{nb})]^T \quad , \tag{24}$$

where vector ξ_{0z} is given by relation (12).

For current-controlled voltage source (CCVS) in Fig. 2c the limit cases of both input and output impedances are considered, i.e.

$$u_2 = \rho i_1, \quad g_1 r_1^{-1} \to \infty, \quad g_2 = r_2^{-1} \to \infty \quad . \tag{25}$$

The effect of CCVS is interpreted by two steps in inverse matrix modification (5). In the first step we derive

$$\mathbf{Z}^{(1)} = \mathbf{Z}^{(0)} - \frac{1}{\xi^{(0)} - \rho}\xi_{z0}^{(0)} \cdot \xi_{0z}^{(0)} = \mathbf{Z}^{(0)} + k_0\mathbf{A}_0 \quad , \tag{26}$$

$$\xi^{(0)} = z_{ca} - z_{cb} - z_{da} + z_{db} \quad , \tag{27}$$

$$\xi_{z0}^{(0)} = [(z_{1a} - z_{1c}); (z_{2a} - z_{2c}); \ldots (z_{na} - z_{nc})]^T \quad , \tag{28}$$

$$\xi_{0z}^{(0)} = [(z_{c1} - z_{d1}); (z_{c2} - z_{d2}); \ldots (z_{cn} - z_{dn})] \quad , \tag{29}$$

from matrix

$$\mathbf{Z}^{(0)} = \mathbf{Y}_0^{-1} \quad , \tag{30}$$

describing the subnetwork without CCVS. In the second step we calculate from matrix (26) the coefficients $\xi^{(1)}$ (7), $\xi_{z0}^{(1)}$ (11), and $\xi_{0z}^{(1)}$ (12) for resultant inverse matrix of the complete network

$$\mathbf{Z} = \mathbf{Z}^{(1)} - \frac{1}{\xi^{(1)}}\xi_{z0}^{(1)} \cdot \xi_{0z}^{(1)} \quad . \tag{31}$$

In node voltage method the atypical elements include even transformer, i.e. **inductive coupling of two coils** (Fig. 2f). Applying the method described on this element the following expressions can be used

$$\mathbf{Z}=\mathbf{Z}^{(0)}-\xi_{z0}\cdot\xi_{00}^{-1}\cdot\xi_{0z}\ , \tag{32}$$

$$\xi_{z0}=\begin{bmatrix}(z_{a1}-z_{b1}); & (z_{a2}-z_{b2}); & \dots & (z_{an}-z_{bn})\\ (z_{c1}-z_{d1}); & (z_{c2}-z_{d2}); & \dots & (z_{cn}-z_{dn})\end{bmatrix}^{T}, \tag{33}$$

$$\xi_{0z}=\begin{bmatrix}(z_{1a}-z_{1b}); & (z_{2a}-z_{2b}); & \dots & (z_{na}-z_{nb})\\ (z_{1c}-z_{1d}); & (z_{2c}-z_{2d}); & \dots & (z_{nc}-z_{nd})\end{bmatrix}, \tag{34}$$

$$\xi_{00}=\begin{bmatrix}pL_1+z_{(a+b)(a+b)} & pM+z_{(a+b)(c+d)}\\ pM+z_{(c+d)(a+b)} & pL_2+z_{(c+d)(c+d)}\end{bmatrix}, \tag{35}$$

$$z_{(\alpha+\beta)(\gamma+\delta)}=z_{\alpha\gamma}-z_{\alpha\delta}-z_{\beta\gamma}+z_{\beta\delta}\quad , \tag{36}$$

where parameters z_{ij} are determined from matrix $\mathbf{Z}^{(0)}$.

4. Resultant network matrix

For the node voltage method it is convenient to consider currents I_0 as driving variables but networks can also be driven by voltage sources E_0 having internal resistance $r=g^{-1}\to 0$. For a simplicity suppose first this source to be grounded (connected between a node and the reference node). In the method discussed it is transformed in a current source gE_0 having internal conductance $g\to\infty$. This fact appears in the admittance element

$$y_{11}=y_{11}^{(0)}+g\ . \tag{37}$$

Similar relations can be used also for the cofactors of this matrix

$$\Delta=\Delta^{(0)}+g\Delta_{1:1}^{(0)}\ , \tag{38}$$

$$\Delta_{j,i}=\Delta_{j:i}^{(0)}+g\Delta_{1,j:1,i}^{(0)}\ . \tag{39}$$

The location of parameter g into the first column of admittance matrix (37) coheres with the location of E_0 into the first row of the driving variables vector $\mathbf{Q}$. Then the resultant node voltages can be determined from the relation

$$\mathbf{U}=\mathbf{W}\cdot\mathbf{Q}\ . \tag{40}$$

Matrix $\mathbf{W}$ can be called the resultant network matrix. It contains a unity diagonal element in the first row and the elements of the other rows in the first column represent voltage transfer ratios in accordance with

$$K=\frac{U_i}{E_0}=\lim_{g\to\infty}gW_{i1}=$$

$$=\frac{g\Delta_{1:j}^{(0)}}{\Delta^{(0)}+g\Delta_{1:1}^{(0)}}\Big|_{g\to\infty}=\frac{\Delta_{1:j}^{(0)}}{\Delta_{1:1}^{(0)}}\ . \tag{41}$$

The remaining elements of matrix $\mathbf{W}$ represent impedance elements of inverse matrix $\mathbf{Y}^{-1}$ when the input port (1) is short-circuited

$$W_{ij}=\frac{\Delta_{j:i}^{(0)}+g\Delta_{1,j:1,i}}{\Delta^{(0)}+g\Delta_{1:1}^{(0)}}\Big|_{g\to\infty}=\frac{\Delta_{1,j:1,i}^{(0)}}{\Delta_{1:1}^{(0)}}\ . \tag{42}$$

5. Conclusion

The modified method of admittance matrix inversion based on diakoptic tearing of non-standard networks has been presented. This procedure is suggested for computer-aided analysis of large networks.

6. Remark

The present results have been achieved by the co-operation with the Radioelectronics Department, Faculty of Electrical Engineering and Computer Science, Technical University of Brno, in the frame of Grant Project No.102/1993/1266, Grant Agency of the Czech Republic, "Synthesis of Electronic Networks with Special Function Blocks" (grant-holder Prof. J. Pospíšil).

7. References

[1] POSPÍŠIL,J. - DOSTÁL,T.: Electronic networks and systems I. Printed lectures, FE VUT, Brno, 1991 (in Czech).

[2] RYBIN,A.I. - DOSTÁL,T.: Analysis of compound networks by diakoptic methods. In: Bulletin of VUT Brno, Vol. A-48, 1991, pp. 129-156 (in Czech).

[3] VLACH,J. - SINGHAL, K.: Mašinnyje metody analiza i proektirovania schem. Radio i svjaz, Moskva, 1988 (transl.).

[4] MANJUK, I.J.: Programma častotnovo analiza linějnych cepej po metodu modifikacij. Radioelektronika, Kyjev, 1991, No.6, pp. 91-93 (in Russian).

About author, ...

Details are not available in time of publication.