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FUNCTIONAL STABILITY THEORY AND ITS APPLICATION IN COMPLEX SYSTEMS

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Abstract

In the contribution the basic principles of the Functional Stability Theory are described, some problems discussed in more details, the Criterion of the Theory is shown, possibilities in Synergetics are mentioned and an example of the Theory application is introduced to show the effectiveness of the approach.

Keywords:
functional stability theory, criterion, applicability

1. Importance of Stability Theories in Synergetics

Stability or instability determination of individual stationary solutions of evolutionary equation of a synergetic system is just of a basic importance, refering to a possible real existence of individual systemic orders. These ones are characterized by different stationary order-parameter values and their stability depends on the constellation of investigated systemic parameters. There is a chance for the most stability theories and methods (Lyapunov's theory, the linearization method, the potential modelling method e.t.c.) to be used.

Investigation of transition conditions between the two stable system-orders at least has the same importance, certainly (more the stable stationary solutions of the evolutionary equation) or the determination of areas in the space of systemic parameters having conditioned just the respective synergetical system-order. There are many problems of this kind. Let us mention a bistable flip-flop, spontaneous and stimulated emission of a laser - assessing of the threshold voltage, space distribution of laser resonator field changes (changes of a mode structure) e.t.c.. This problem is unknown to us to be universally solved. Lyapunov's direct (second) method, bifurcation theory, Landau's phase transition theory and further theories may be applied, time by time. There are good reasons to belive that Donocik's functional stability theory might be used for the same purpose, too, or saying more exactly, the problem could be formulated as a functional stability task.

2. Principles of Functional Stability Theory

The theory assumes a model of time system behaviour in an analytical shape under consideration. The Volterra integral equation of the second order, e.g., can be used (with the nonlineary part, of course) to what all the differential equations with separable linear as well as nonlinear parts may be transformed.

The functional approach consists in a primary transformation of the total time system behaviour into the space of real numbers (of "the functional") and the subsequent investigation of this functional convergent possibilities representing the response to jump-extinction of an elementary perturbation of the system parameters. Then the statement on the functional stability is always applied to the sophisticated functional transformation and depends on the system parameters. The functional transformation is to be chosen in a way having a possibility for the result of the transformation (the functional) to be a rate of the relevant quality of an investigation system behaviour and to express the qualitative changes. The manner of the qualitative changes determination may be briefly described like this [1, 2].

The investigated system evolution is given by the Volterra integral equation

$$\phi(t) = f(t,p) + \int_0^t F(t,p,\phi(\tau)) \cdot d\tau, \qquad (1)$$

or briefly

$$\phi(t) = O_P\{\phi(t)\} \qquad (2)$$

Now this evolution will be transformed to the space of the functional (to the space of real numbers) with respect to the parameter p, for example by a functional transformation

$$\nu = S\{\phi(t)\}. \qquad (3)$$

The S-transformation is to be chosen according to the character of the investigated problem [1]. We have used for solved problems the following "convenient" functionals: the Laplace transform, the mean value, the stationary solution, one term of Fourier series e.t.c. If the functional is not selected "suitably", the changes of the system behaviour need not be taken. The situation may be interpreted by the help of an instrument used for measuring of an incorrect (or not asked) value. It is generally valid that "practise makes perfect".

The functional generally depends on the parameter p, see (2), (3), thus we have

$$\nu = Q(p) \tag{4}$$

For the functional stability p must be calculated from (4) (or n is expressed as a function of f from the very beginning), so that

$$p = q(\nu) \tag{5}$$

Now we introduce the perturbated motion $\phi_{\Delta p}(t)$ and in the relation (1) we write instead of p value $p+\Delta p$ and instead of $\phi(t)$ perturbated value $\phi_{\Delta p}$ and the equation (4) has the shape

$$\nu + \Delta\nu = Q(p+\Delta p). \tag{6}$$

The testing motion now is formed by the fact that Δp acts only within the time interval $\langle 0, t_0\rangle$, t_0 may be arbitrarily small. Then the expression $\phi^*_{\Delta p}(t)$ (testing motion) may be created from $\phi_{\Delta p}(t)$ (instead of Δp we write $\Delta p \cdot u(t_0 - t)$, where $u(t_0 - t)$ is equal 1 for $t < t_0$ and 0 for $t \geq 0$).

If we choose arbitrarily small $\varepsilon > 0$ and such a number $\delta > 0$ exists, so that

$$\left|S\{\phi^*_{\Delta p}(t)\} - S\{\phi(t)\}\right| < \varepsilon \tag{7}$$

whenever $|\Delta p| < \delta$, then we declare the motion $\phi(t)$ *stable with respect to the parameter p and the functional* $S\{\phi(t)\}$ and unstable if this statement is not valid. Othervise it may be said that the definition of the functionally stable motion according to (7) is equal to the statement that there is no jump at the curve $\nu = Q(p)$ if the testing motion is applied.

3. Functional Stability Criterion

The functionally stable system behaviour (as mentioned above) depends on the functional and system parameters for which the "transitional effects" have not occured in a value of the functional (it is not a question of discontinuity, only - for example type S of a characteristic, e.t.c.- see [1]).

The functionally unstable behaviour (jump effects have occured) may be defined other way round. Then we have many possibilities in an interpretation of a lot of problems concerning abrupt qualitative changes in a systemic behaviour as the functional stability task. Critical values of system parameters, after elementary overcoming of which the abrupt change of the system qualitative behaviour has been reached (the change of a synergetic system order), facilitate the application of the theory which depends on such a functional transformation choice that has fulfilled not only general conditions of the theory itself (an unambiguous correspondency of the behaviour and a joined functional to each other, e.g.) but warrant moreover, that the transitional effect of the selected functional appears just in time the examined quality of the mentioned system behaviour has been changed (or has been changed its order).

There is then a substantial fact, that the analytical expression of the functional stability criterion is at disposal having the character of the necessary and sufficient condition. Thus we have the possibility the critical parameter values of the system have been determined quite correctly, on principle (in an analytical shape at least) - [1, 2].

In [1] the derivation of the criterion has been exposed. It was found (for the one dimensional functional) that

$$c = \frac{\partial}{\partial \nu} S\{O_p\{\phi(t,\nu)\}\} \lesseqgtr 1 \ . \tag{8}$$

The signum of equality in (8) means the critical value of the continuous parameter p after a small change of which the stable or unstable motion is established. If we have calculated $c < 1$, then the motion is stable and for $c > 1$ the result gives the unstable state. The critical value $p = p_c$ at which transition or changes is a very important quantity of motion in the synergetic sense of word as well (the critical value of the order parameter).

In comparison with Lyapunov's or Thom's theory the mentioned fact that the criterion is at disposal has a very serious meaning for the systems described in an analytical shape.

In [4] the formula (8) has been mentioned in sharper form

$$\lim_{p \to p_c} \frac{\partial}{\partial \nu} S\{O_p\{\phi(t,\nu)\}\} \lesseqgtr 1 \ , \tag{9}$$

where p_c is critical parameter value.

4. Applicability of Functional Stability

From the outline mentioned follows that the general applicability of the functional stability is much more greater, than to be used only in systems, in behaviour of which synergismus may be expected but also when the relevant evolution equation of an order-parameter can be written. The method is applicable on principle to all the nonlinear dynamical systems. It is necessary to state that Haken's synergetic method based upon the evolution equation setting up complies with the applied principle mentioned above - see [5].

Problem of the evolution equation determination itself is involved, at first, in the fact that this one may serve very well in a shape of an analytical model of the investigated system depending on time, which is very strongly asked for the functional stability application.

A proper order-parameter choice in the synergetical interpretation, secondly, helps already very much to the application of the mentioned qualitative point of viev that is just being represented by a proper choice of the functional transformation, if the theory is to be applied.

Such an order-parameter choice as wel as a successful relevant evolution equation enables, evidently, to convert the problem of more stable qualitative different system-order transition into the problem of transition conditions of individual stationary solutions of the equation. Then the problem of the formulation as the functional stability task may be facilitated.

The process of the theory application is then a following one: If there are two different stable stationary solutions of the evolution equation (constant in time), at least an abrupt change of the stationary solution of the equation (under the changed condition of the system-order with the continuous change of the systemic parameters) may consequently play a role of the jump-effect of the functional "behaviour image" of this synergetic system. As a suitable functional of the mentioned transitions the stationary solution value may be applied directly. Other functional transformations have been recommended former. The following problems have been solved, e.g., with the help of the Donocik`s theory:

- Synchronism of the phase-controlled oscillators, [1], clasical solution, see for example [6],
- Critical parameter values of Hamiltonian and Heisenberg equation, [2],
- Flip-flop separatrices in the phase space, [3],
- Synchronism of oscillating-modes in a laser resonator, [4]
- Threshold of stimulated laser emission determination, [4], e.t.c.

5. Example of Application of Functional Stability

5.1 Threshold of stimulated laser emission determination

The considered qualitative change appears when the stimulated laser emission has come for example in case that the jump-effect of the electrical field amplitude of output radiation might be observed at the investigated model of laser system. Our task lies in the fact that the critical value of exciting parameter α (so called nonsaturated gain), at what the qualitative change of laser stationary state occurs, is described by the equation

$$E+\alpha E+\beta E^3=h \tag{10}$$

where β represents the saturation constant,
h noise fluctuation and
E is the intensity of electrical field.

This relation may be transposed at the integral equation with respect to the functional stability approach. We obtain

$$E=E_0+\int_0^t (h-\alpha E-\beta E^3)\cdot d\tau. \tag{11}$$

Neglecting h the stationary solution of (10) is

$$E_{S1}=0; \quad E_{S2,3}=\pm\sqrt{\frac{-\alpha}{\beta}}. \tag{12}$$

Substituting (12) in (11) and for initial condition

$$E_0=E_S \tag{13}$$

we obtain (11) in the form

$$E_S=E_S+\int_0^t \left(-\alpha E_S-\beta E_S^3\right)\cdot d\tau. \tag{14}$$

Therefore the integral is equal zero. The meaning of the fact may be interpreted like this: The "heart" of the functional involves the information of investigated stationary state behaviour.

Mean value will be chosen as the functional so that the functional transformation is defined

$$v=S\{E_S\}=\frac{1}{t_S}\int_0^{t_S} E_S\cdot dt=E_S \tag{15}$$

The value of stationary solution is directly the functional of the investigated motion.

The functional stability criterion will be used in the form for the functionally stable solution

$$\frac{\partial}{\partial E_S}\left\{\frac{1}{t_S}\int_0^{t_S}\left[E_S+\int_0^t\left(-\alpha E-\beta E_S^3\right)\cdot d\tau\right]\cdot dt\right\}<1. \tag{16}$$

After an elementary calculation we have

$$\alpha+3\beta E_S^2>0 \tag{17}$$

and individual stationary states E_{S1} and $E_{S2,3}$ give the conditions in the shape

$$\alpha>0 \quad \text{and}\ \alpha<0 \tag{18}$$

The stationary state E_{S1} is functionally stable with respect to the chosen functional and the initial condition, namely, for the exciting parameter α greater than zero. The states $E_{S2,3}$ are functionally unstable ($\alpha<0$). The qualitative change of stationary states is just observed at the critical value of the parameter $\alpha_c = 0$. The result may be reached by classical methods as well. *The effectiveness of the functional stability is supported by the above example.*

6. Conclusion

Critical states of synergetic systems may be determined by the help of the functional stability theory, depending upon the transitions among more stable orders. A precise analytical expression is possible to find in principle for critical arrangement in the space of parameters.

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