DESIGN OF THE DECOMPOSED STATE MODEL OF AN ONE-DIMENSIONAL PIECEWISE-LINEAR SYSTEM

Zdeněk Kolka
Institute of Radioelectronics
Technical University of Brno
Antonínská 1, 662 09 Brno
Czech Republic

Abstract

Present paper deals with the problem of automatic design of the implicit state model (in its decomposed parametric form) of any one-dimensional piecewise-linear (PWL) system. This model consists of the internal and the external parts which provides the possibility to find the model parameters, i.e. to solve separately the two main problems of an one-dimensional PWL model design:

(i) existence of the individual breakpoints of the resultant PWL characteristic (determined by the internal block);

(ii) setting of the breakpoint co-ordinates (determined by the external block).

Keywords:

piecewise-linear, modelling, state model

1. Introduction

In [1],[2] van Bokhoven presented a PWL model using state variables to determine for which part of the domain space the mapping equations are valid, however it is difficult to find its parameters and the general algorithm is not known yet. The parameters are usually found using explicit models [2],[3] but in such a case the advantages of the implicit state model are not fully utilized. To avoid these difficulties the *decomposed parametric* modification of the *state model* has been developed in the following form [4],[5]

$$y = A'u + B'j + f', \tag{1}$$

$$x = C'u + D'j + g', \tag{2}$$

$$0 = Mu + Nj + q, \tag{3}$$

$$u \geq 0, j \geq 0, u^T j = 0, \tag{4}$$

representing the PWL mapping $f: R^n \to R^m, x \to f(x)$,

where

$$A' \in R^{m \times k}, \quad B' \in R^{m \times k}, \quad f' \in R^m,$$

$$C' \in R^{n \times k}, \quad D' \in R^{n \times k}, \quad g' \in R^n,$$

$$M \in R^{(k-n) \times k}, \quad N \in R^{(k-n) \times k}, \quad q \in R^{(k-n)}.$$

Similarly as in the basic form [1], [2] vectors u and j are the state vectors. Relation (4) formulates the linear complementary condition for state variables u and j. Set of n eqns (2) and $(k-n)$ eqns (3) describe together the boundaries of the polytope and the linear mappings are expressed in m eqns (1). On the contrary eqns (1),(2) and eqns (3) represent the descriptions of the *external* and *internal* blocks, respectively.

The main difference between the basic and decomposed forms is that the *external* variables (y,x) are considered dependent and the *internal* ones (u,j) independent. This is the reason why eqns (1), (2) express the $f(x)$ in the *parametric* form where for each partial state the corresponding nonzero state variables act as parameters. As the total number of equations $(m+n)$ must remain the same formula (6) represents $(k-n)$ equations containing only state variables u and j. The decomposed form can evidently be derived only for $k \geq n, (k-n \geq 0)$ so that for $n=1$ this condition is valid for any k. Such a description provides some new possibilities namely for the state model design.

The main advantage of the decomposed description is a very simple relation between the parameters of the matrices in eqns (1),(2) and the co-ordinates of the individual breakpoints of the resultant y-x characteristic (*external part* of the model). It also entails a simple design algorithm.The parameters of the matrices in eqns (3) have no direct relation to the shape of PWL characteristic because these equations do not contain external variables , however, they determine the existence of the individual breakpoints of the PWL characteristic, i.e. also its type (*internal part* of the model). The corresponding design algorithm is more complicated than in the previous case.

2. Internal block design

Any complementary pair of state variables $(u_r, j_r, r = 1, 2, ..., k)$ can be in one of their two basic states, i.e. the total number of all the possible states is generally 2^k. Each breakpoint of the resultant PWL characteristic corresponds to such a transient state between two adjoining states when only one complementary pair changes its state

($u_r = 0, j_r = 0$) while the remaining (k-1) pairs are in one of their two basic states so that just (k-1) state variables have nonzero values. As the internal block is described by (k-1) eqns (3) expressing the relations among the state variables only. These equations determine the existence of the individual breakpoints of the resultant PWL characteristic, i.e also the existence of the corresponding particular states. Then the initial step in the systematic design of the decomposed state model consists in the derivation of M,N and q, that correspond to the required type of PWL characteristic. Such a procedure enables us not only to utilize all the possible particular states but also to derive all the attainable types of the modelled PWL characteristic. (The results for k=2 and k=3 are available in paper [5].)

The symbolic representation (Fig. 1 , k= 1 to 3) is here very useful because each next diagram can easily be derived from the previous one by the systematic hierarchical procedure. Each existing segment of the PWL characteristic can evidently be defined either by two breakpoints (inner segment) or by one breakpoint and the slope (outer segment). If none of the possible k passages between any particular state and the adjoining ones exist then the corresponding segment does not exist as well [7].

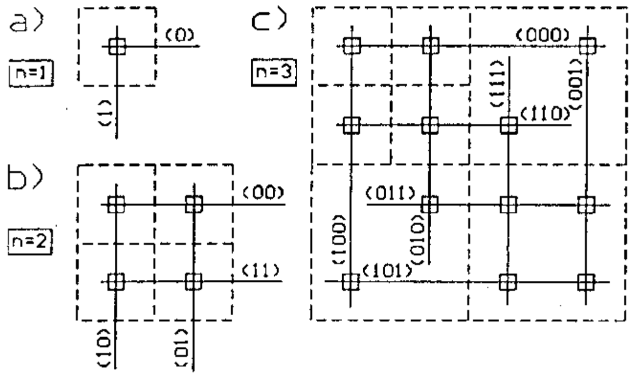


Fig. 1
Symbolic representation of all the particular states.

In the next parts the systematic design of matrices M,N and q for all single and multiple symmetric PWL characteristics, both in the close and open form, is suggested.

2.1 Canonical forms

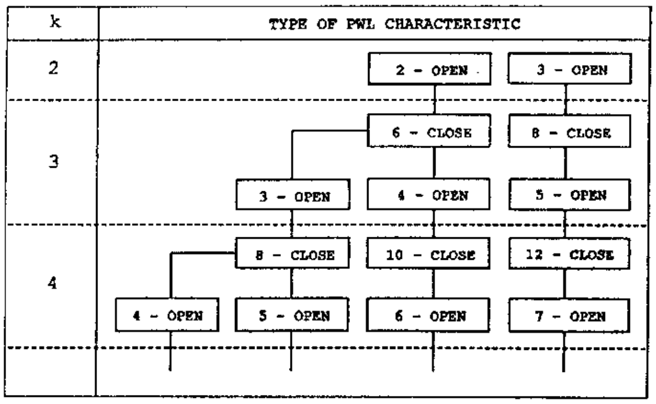
The internal block design for a given type of the resultant PWL characteristic (including close, open, and multiple types), represents a rather complicated mathematical problem. The strategy consisting in a gradual generalization of the known results for k=2 is used. First the so called canonical forms of M,N and q (each column contains maximum one nonzero parameter) are investigated.

The minimum and maximum numbers of breakpoints for the close and open types of the single or double symmetric PWL characteristics are summarized in Table 1, the corresponding forms of matrices M,N and q are introduced in Fig. 2. The systematic hierarchical development of all the attainable canonical forms is suggested in Table 2. All other details can be found in [6].

Table 1
Survey of the attainable canonical forms.

Type of PWL characteristics	Number of breakpoints		Remark
	Minimum	Maximum	
Single - close	$2k$	$4(k$-1)	even only
Single - open	k	(2k-1)	even and odd
Double - sym.	2 x any (k-1) PWL char.		

Table 2
Principle of the hierarchical development of all the attainable canonical forms.



2.2 Non-canonical forms

As follows from Table 1 the number of PWL characteristics realizable by the canonical forms is limited. The remaining possible types can be modelled by the non-canonical forms of the internal block.

The systematic hierarchical development of all these remaining types is suggested in Table 3 where the initial canonical types are represented by the full lines while the derived non-canonical ones by the broken lines.

In the corresponding matrices M and N some additional (negative or positive) parameters must be placed in suitable positions which represents the fact that the passages among some particular states cannot exist. The rules of this hierarchical procedure are based on the negative delimitation and utilize the known canonical forms and their detailed description can be found in [6]. The corresponding algorithm contains the following steps:

1. Find a path from a corresponding canonical close type to the required non-canonical one (Table 3).
2. Start from the original canonical close type.
3. Realize a "close → open - transition" on the path by canceling one or two breakpoints (a negative delimitation) in the original type.

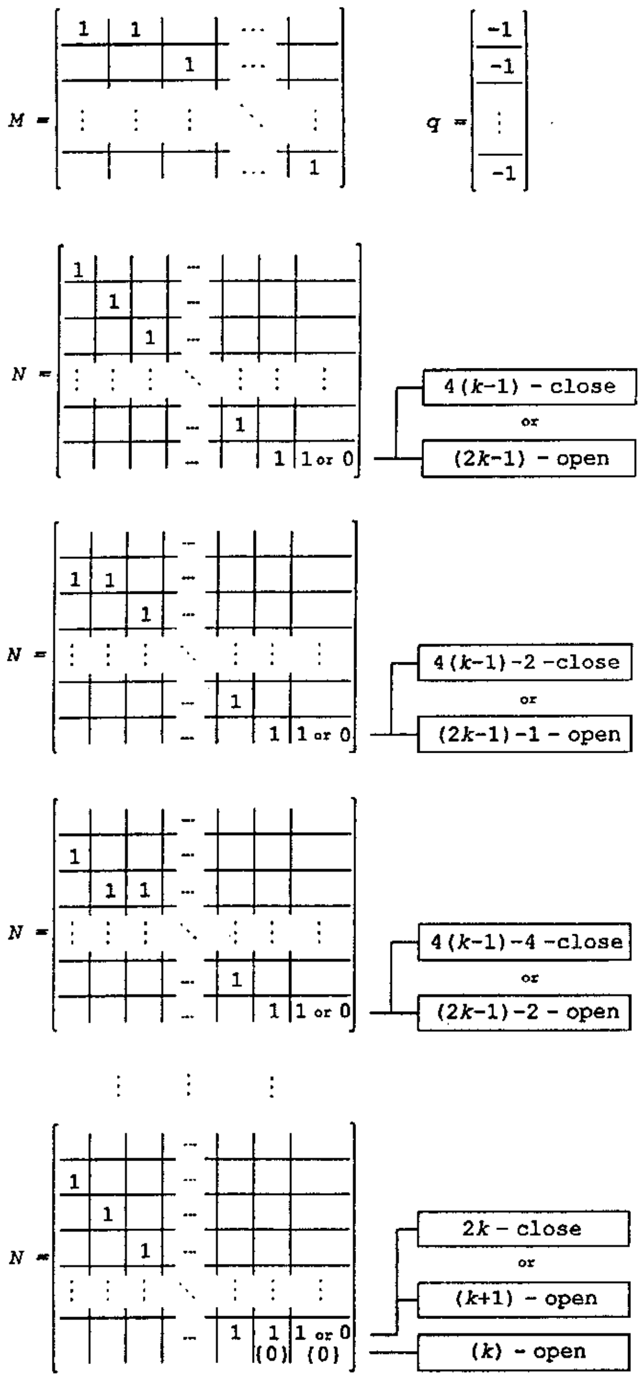


Fig. 2
Canonical forms of matrices M,N and q.

4. Realize an "open → close - transition" on the path by composing two open types (doubled previous case).

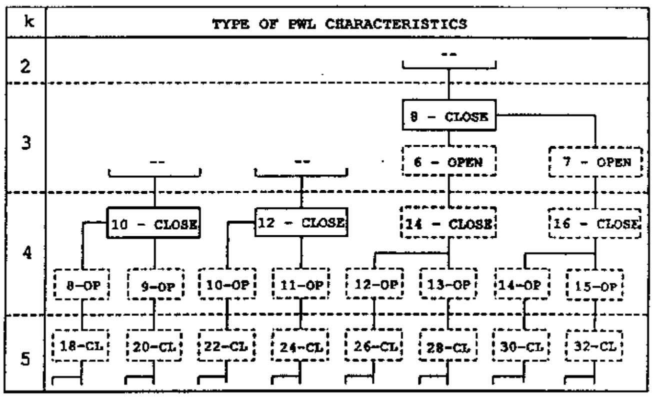
5. Repeat step 3 and step 4 until the required type is reached.

2.3 Example of the internal block design

To illustrate the design procedure of the internal block description a canonical case for k=3 is introduced.

An open PWL characteristic having 5 breakpoints is considered. It also represents the maximum attainable by

Table 3
Principle of the systematic hierarchical procedure for some attainable non-canonical forms.



k=3 (Table 1). Matrices M and q have the standard forms (Fig. 2) and the form of matrix N corresponds to the type: (2k-1) - open (Fig. 2). The complete description (3) of the internal block is then

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} = 0$$

and the corresponding symbolic diagram is introduced in Fig. 3. All the existing segments are drawn by the full lines, the individual partial states are indicated by related state vectors [1],[2], and for the "independent" breakpoints bold notation are used. The actual shape of the resultant PWL characteristic is given by the external block description.

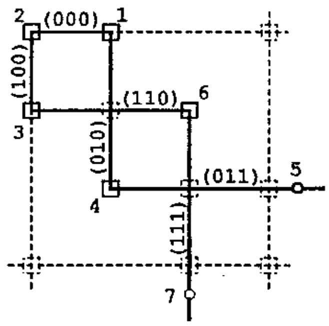


Fig. 3
Symbolic diagram of the type 5-open.

3. External block design

After finding matrices M,N and q the values of state variables u,j for the individual breakpoints can be substituted into the eqns (1),(2) describing the external block. The values of external variables x,y correspond to the co-ordinates of these breakpoints so that we can easily derive or choose the parameters of the individual matrices.

The external block description for the i-th breakpoint has the form

$$y^{(i)} = A'u^{(i)} + B'j^{(i)} + f',$$

$$x^{(i)} = C'u^{(i)} + B'j^{(i)} + g',$$

where $y^{(i)}$ and $x^{(i)}$ represent the required co-ordinates of the i-th breakpoint, $u^{(i)}$ and $j^{(i)}$ are the corresponding values of the state variables obtained from the internal block design. The number of independent vectors $[u^{(i)T}, j^{(i)T}]$ is only (k+2) so that only the co-ordinates of (k+2) breakpoints can be chosen independently. The corresponding algorithm contains the following steps:

1. Select (k+2) independent state vectors $[u^{(i)T}, j^{(i)T}]$.
2. Substitute these vectors into the above given formulas to obtain (m+1) sets of (k+2) eqns for (m+1) sets of (2k+1) model parameters.
3. Choose the values of suitable parameters in each set to ensure the unique solution of the set.

3.1 Example of the external block design

Using the described procedure the external part of the decomposed PWL model in the previous example can be designed (for a simplicity consider m = 1). Denoting the co-ordinates of the given breakpoints $1-(X_1, Y_1)$ to $5-(X_5, Y_5)$, choosing $f' = g' = 0$, and also all coefficients by voltage u_3 in eqns (1),(2) equal zero then the following form of the external part description is obtained.

$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 & 0 \\ X_1 & X_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} Y_3 & Y_4 - Y_1 & Y_5 - Y_4 \\ X_3 & X_4 - X_1 & X_5 - X_4 \end{bmatrix} \cdot \begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix}$$

The dependent co-ordinates of the remaining breakpoints

$$X_6 = X_3 + X_4 - X_1, \quad Y_6 = Y_3 + Y_4 - Y_1,$$

$$X_7 = X_5 + X_3 - X_1, \quad Y_7 = Y_5 + Y_3 - Y_1.$$

4. Conclusions

Present hierarchical procedure allows the direct and automatic design of the decomposed parametric state model for both the single and the multiple symmetric one-dimensional PWL characteristics. For the asymmetric types a similar algorithm is not known yet. The results obtained are valid for a certain existing combination of the particular states of variables u and j . This model can be modified for any other prescribed combination using the simple change of the related columns in matrices A', B', C', D', M, N (instead of the well known pivoting procedure).

The complete implicit description represents a decomposed parametric state model of an one-dimensional static PWL system. It can directly be utilized either for the dc analysis of networks containing ideal diodes or for the dynamic analysis of unconventional PWL systems. Decomposed state model is also suitable as a theoretical prototype for the derivation of any other type of PWL system state models as well as for their direct synthesis using multiple voltage-controlled voltage sources [9],[10]. In the synthesis procedure some additional restrictions of the resultant PWL characteristic shape appear due to the validity of the stability conditions of the whole network.

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About author, ...

Zdeněk Kolka was born in Brno, Czechoslovakia, in 1969. He received the M.E. degree in electrical engineering from the Faculty of Electrical Engineering, Technical University of Brno, in 1992. At present he is a PhD student supervised by Prof. Jiří Pospíšil at the Institute of Radioelectronics, Technical University of Brno. He is interested in PWL circuits modelling and design.