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COMPARISON OF ADAPTIVE ANTENNA ARRAYS CONTROLLED BY GRADIENT ALGORITHMS

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Abstract

The paper presents the Simple Kalman filter (SKF) that has been designed for the control of digital adaptive antenna arrays. The SKF has been applied to the pilot signal system and the steering vector one. The above systems based on the SKF are compared with adaptive antenna arrays controlled by the classical LMS and the Variable Step Size (VSS) LMS algorithms and by the pure Kalman filter.

It is shown that the pure Kalman filter is the most convenient for the control of the adaptive arrays because it does not require any a priori information about noise statistics and excels in high rate of convergence and low misadjustment. Extremely high computational requirements are drawback of this filter. Hence, if low computational power of signal processors is at the disposal, the SKF is recommended to be used.

Computational requirements of the SKF are of the same order as the classical LMS algorithm exhibits. On the other hand, all the important features of the pure Kalman filter are inherited by the SKF.

The paper shows that presented Kalman filters can be regarded as special gradient algorithms. That is why they can be compared with the LMS family.

Keywords:

Adaptive antenna array. Array signal processing. Gradient algorithms. LMS algorithm. Kalman filter.

1. Introduction

An adaptive antenna array is an antenna system that automatically sets minims of its directivity pattern to directions from that the most powerful interferences come. Hence, the signal to interferences ratio (SIR) at the antenna system output can be significantly increased. This feature of adaptive arrays causes that such antennas are an important part of many radars, satellite radio communication receivers, military radio communication systems etc.

Properties of adaptive antennas are namely influenced by the quality of the antenna system, by the electronic control circuitry and by the control algorithm.

Whereas the design methods of antenna arrays and electronic control circuitry of adaptive antennas are in detail worked out at the present time, the area of the adaptive control is in the state of the very intensive development. This situation is caused by the disproportion between the computational requirements of the "nearly perfect" control algorithms and the computational power of the today signal processors.

Investigation of control algorithms has been so far oriented towards improving properties of the classical LMS algorithm [1], [2] in the most cases. In this work, a quite different approach has been elected. Instead of perfecting the LMS algorithm, the Kalman filter has been simplified. The Simplified Kalman filter's (SKF) computational requirements are of the same order as the LMS one. On the other hand, the SKF preserves most of the positive features of the pure Kalman filter. Hence, the SKF algorithm is suitable for the digital control of adaptive antennas especially.

Section II of the presented paper reminds the reader of the principles of adaptive antennas based on the pilot signal and the steering vector methods. Section III is devoted to gradient algorithms - the classical LMS and the Variable Step Size (VSS) LMS are presented here. Section IV deals with the pure Kalman filter and its simplification to the SKF one. It is shown that the pure Kalman filter can be considered as the gradient algorithm and the SKF as the step variable LMS. Hence, they can be compared with the LMS family. Section V describes results of performed computer simulations and parameters of the mentioned algorithms are compared here.

2. Principles of Adaptive Antenna Arrays

2.1 The Pilot Signal System

The adaptive antenna based on the pilot signal was developed in the late sixties by B. Widrow [3]. The principle of the system is very simple - transmitter transmits signal that is synchronously generated in the receiving system. If no interferences are received by the receiving antenna then the difference between the signal at the antenna output and the generated signal (difference is called error signal) is zero after the right amplification of the received signal. On the contrary, if strong interferences are received by the antenna system then the level of the error signal can be very high. Hence, the directivity pattern of the receiving antenna array is synthesized so as the mean squared error can be minimized. Minimal error signal means that minims of the directivity pattern are set to the directions from that the most powerful interferences come. By this way, the SIR at the antenna output is optimized.

Synthesis of the directivity pattern is performed by setting the proper amplitudes and phases at the outputs of the antenna elements. In broadband applications, "the complex weighting" of the antenna elements' outputs is realized by transversal filters with real adjustable weights. An antenna element completed by the complex weight is drawn in figure 1. All outputs of all antenna elements are then summed to form the output signal.

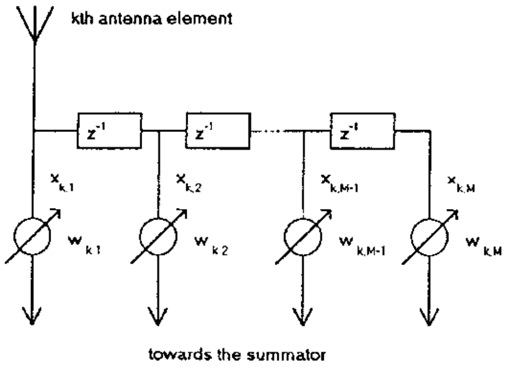

Fig.1
An antenna element completed by the adjustable broad-band complex weight. The adaptive array consists of N half-wavelength-spaced omnidirectional elements.

The output signal can be expressed by the equation

$$y(j) = \mathbf{W}^T(j)\mathbf{X}(j) \qquad (1)$$

where $y(j)$ is the sample of the antenna output signal, $\mathbf{W}(j)$ denotes the column vector of the transversal filters' weights $\mathbf{W}^T(j) = [w_{1,1}(j), w_{1,2}(j), \ldots, w_{1,M}(j), \ldots, w_{N,M}(j)]$, $\mathbf{X}(j)$ is the column vector of signals at the delay lines' taps $\mathbf{X}^T(j) = [x_{1,1}(j), x_{1,2}(j), \ldots, x_{1,M}(j), \ldots, x_{N,M}(j)]$, T denotes the transpose, N is number of antenna elements in the antenna array and M is number of taps of each transversal filter.

The error signal is given by the relation

$$e(j) = d(j) - y(j) \qquad (2)$$

(d is the pilot signal) and the mean squared error

$$E\{e^2(j)\} = E\{d^2\} + \mathbf{W}^T\mathbf{R}(x,x)\mathbf{W} - 2\mathbf{W}^T\mathbf{R}(x,d)\,. \qquad (3)$$

In the above equation, $E\{..\}$ denotes the expectation operator, $\mathbf{R}(x,x)$ is the autocorrelation matrix of the vector $\mathbf{X}(j)$ and $\mathbf{R}(x,d)$ is the cross correlation vector of the vector $\mathbf{X}(j)$ and the pilot signal d.

From the mathematical point of view, relation (3) describes a convex parabolic function in the MN -dimensional space. Hence, the mean squared error will be minimized if the gradient with respect to the weights' vector $\mathbf{W}(j)$ is zero

$$\nabla E\{e^2\} = 2\mathbf{R}(x,x)\mathbf{W} - 2\mathbf{R}(x,d) = 0\,. \qquad (4)$$

Solving (4) yields the optimal weights' vector

$$\mathbf{W}_{opt} = \mathbf{R}^{-1}(x,x)\mathbf{R}(x,d)\,. \qquad (5)$$

Relation (5) is called the Wiener-Hopf equation.

2.2 The Steering Vector System

Principle of adaptive antenna array based on the steering vector method was published in the beginning of seventies by O.L. Frost for the first time [4]. The advantage of the mentioned antenna is that it works without the pilot signal. Hence, the electronic circuitry of the system is significantly simpler.

The steering vector system is based on the minimization of the expected total output power

$$P = E\{y^2(j)\} = \mathbf{W}^T\mathbf{R}(x,x)\mathbf{W} \qquad (6a)$$

under the constraint of preserving properties of the receiving system in the direction of the main lobe of the antenna array

$$\mathbf{C}^T\mathbf{W} = \mathbf{F}\,. \qquad (6b)$$

In the relation (6a), $y(j)$ is sample of the output signal expressed by (1), $E\{..\}$ denotes the expectation operator, $\mathbf{W}(j)$ is the column vector of weights and $\mathbf{R}(x,x)$ is the autocorrelation matrix of the vector $\mathbf{X}(j)$.

Relation (6b) is a mathematical expression of the following idea: "Phase of the desired signal arriving from the main lobe direction is at all antenna outputs the same. Hence, the antenna system completed by a transversal filter at each antenna element output can be for the main lobe direction replaced by one antenna element with a transversal filter whose kth weight is the summa of kth weights of all the transversal filters of the antenna system.

If preserving antenna properties in the main lobe direction is required then weights of the "joined" transversal filter cannot change during the adaptation process." Now, understanding the meaning of symbols in (6b) is easy: **F** is the column vector of weights of the "joined" transversal filter, $\mathbf{W}(j)$ is the above described weight vector and **C** is the matrix of coefficients that performs the process of joining.

Solving the optimization problem (6) by the method of Lagrange multipliers results in the following equation for the optimal weights

$$\mathbf{W}_{opt} = \mathbf{R}^{-1}(x,x)\mathbf{C}[\mathbf{C}^T\mathbf{R}^{-1}(x,x)\mathbf{C}]^{-1}\mathbf{F}. \tag{7}$$

Symbols used in (7) has been explained above.

3. Gradient Algorithms

Relations (5) and (7) are totally unsuitable for the use in real-time systems because they require computation of correlation matrices, their inversion and multiplication. That is why the gradient algorithms have been developed.

Gradient algorithms use the instantaneous values of signal to the estimation of the gradient of the minimized function with respect to the weight vector. Then the actual setting of weights is changed in the opposite direction of the estimated gradient. By this way, weights are recursively changed until they are close enough to the optimal state. Recursive search for the optimal weights can be expressed as

$$\hat{\mathbf{W}}(j+1) = \hat{\mathbf{W}}(j) - \alpha\hat{\nabla}(j) \tag{8}$$

where $\hat{\mathbf{W}}(j)$ is the estimated optimal weight vector, α is the scalar constant controlling rate of convergence and stability, $\hat{\nabla}$ is the estimated gradient.

3.1 The LMS algorithm

Pilot signal system. Differentiating square of the instantaneous value of the error sample (2) by the weight vector $\mathbf{W}(j)$ yields

$$\hat{\nabla}(j) = -2e(j)\mathbf{X}(j) \tag{9}$$

where $e(j)$ is the error signal and $\mathbf{X}(j)$ denotes the column vector of signals at the transversal filters' taps.

The complete adaptation algorithm can be then expressed as

$$\hat{\mathbf{W}}(j+1) = \hat{\mathbf{W}}(j) + 2\alpha e(j)\mathbf{X}(j). \tag{10}$$

Equation (10) is the classical LMS algorithm for the pilot signal system.

Steering vector system. In the case of minimizing the quadratic function (6a) under the linear constraint (6b), the cost function

$$H(\mathbf{W}) = \tfrac{1}{2}\mathbf{W}^T\mathbf{R}(x,x)\mathbf{W} + \lambda^T(\mathbf{C}^T\mathbf{W} - \mathbf{F}) \tag{11}$$

has to be extremalized. In (11), λ is the column vector of Lagrange multipliers and the rest of symbols has the same meaning as before.

Setting of weights is changed in the contra direction of the cost function gradient related to the weight vector

$$\nabla = \mathbf{R}(x,x)\mathbf{W} + \mathbf{C}\lambda. \tag{12}$$

Computation of gradient according to (12) requires the knowledge of the autocorrelation matrix $\mathbf{R}(x,x)$. This problem can be solved as it has been done at the pilot signal system - expectation operator is removed and the gradient is estimated from the instantaneous values of the signal

$$\hat{\nabla}(j) = \mathbf{X}(j)\mathbf{X}^T(j)\mathbf{W}(j) + \mathbf{C}\lambda(j)$$
$$\hat{\nabla}(j) = y(j)\mathbf{X}(j) + \mathbf{C}\lambda(j) \tag{13}$$

In (13), $y(j)$ is the output signal given by (1).

Let us substitute (13) for $\hat{\nabla}(j)$ in (8). After performing certain mathematical arrangements, we obtain the resultant algorithm

$$\hat{\mathbf{W}}(j+1) = \mathbf{P}\{\hat{\mathbf{W}}(j) - \alpha y(j)\mathbf{X}(j)\} + \tilde{\mathbf{F}} \tag{15a}$$

where

$$\tilde{\mathbf{F}} = \mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{F} \tag{15b}$$

$$\mathbf{P} = \mathbf{I} - \mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T \tag{15c}$$

$$\hat{\mathbf{W}}(0) = \tilde{\mathbf{F}}. \tag{15d}$$

Algorithm (15) can be considered as the LMS for the steering vector system.

Summary. LMS algorithms are very simple and so they are easily implementable by today signal processors. On the other hand, they suffer from relatively slow convergence rate, high misadjustment and the adaptation stability problems - if the learning constant α is too high then the system oscillates, if α is too low then the system does not work properly; optimal value of α can be computed according to the relation

$$\alpha_{opt} = \frac{2}{3\,tr(\mathbf{R}(x,x))} \tag{16}$$

where *tr* denotes trace of the matrix.

Some new algorithms have been developed to remove described disadvantages of the LMS algorithms by additional control of α. Description of one of the most effective modified LMS algorithms is presented in the following subsection.

3.2 The Variable Step Size LMS algorithm

VSS LMS has been published in [1]. The adaptation constant is here changed in the dependence on the squared error

$$\alpha'(j+1) = \eta\alpha(j) + \gamma e^2(j) \tag{17a}$$

$$\alpha(j+1) = \begin{cases} \alpha_{max} & \alpha'(j+1) > \alpha_{max} \\ \alpha_{min} & \alpha'(j+1) < \alpha_{min} \\ \alpha'(j+1) & otherwise \end{cases} \tag{17b}$$

Here, η is a constant from the interval $0 < \eta < 1$. Its optimal value has been experimentally determined to 0.95. A constant γ should be a positive number close to zero; γ does not influence misadjustment and negligible influences rate of convergence. A constant α_{max} is given by (16), a constant α_{min} provides the minimum level of tracking ability.

Described algorithm shows quicker convergence rate and lower misadjustment than the classical LMS. In addition, it preserves the simplicity of the LMS. On the other hand, the problem of the adaptation stability is not solved here - if the algorithm shall be stable than the autocorrelation matrix $\mathbf{R}(x,x)$ has to be known for the proper determination of α_{max} and α_{min}.

4. The Kalman Filter

The Kalman filter is an identification method that can provide optimal estimate of state quantities of the state model to that it is applied. Since we are searching for the optimal weights the weights' vector $\mathbf{W}(j)$ is regarded as the state one in our case.

The fact that adaptive antennas are static systems (the working point is searched) significantly simplifies the state model. Nevertheless the adaptation algorithm requires still so high computational power of the control processors that its use is extremely complicated in today systems. Hence, simplification of the Kalman filter is required.

4.1 Pilot signal system

Application of the Kalman filter theory to the pilot signal system that can be described by the state model

$$\mathbf{W}(j+1) = \mathbf{W}(j) \tag{18a}$$

$$d(j) = \mathbf{X}^T(j)\,\mathbf{W}(j) + v(j) \tag{18b}$$

(v is called the residual error and has the same meaning as the above error signal)
leads to the following set of equations [5]

$$\hat{\mathbf{W}}(j+1) = \hat{\mathbf{W}}(j) + \mathbf{K}(j)[d(j) - \mathbf{X}^T(j)\,\hat{\mathbf{W}}(j)] \tag{19a}$$

$$\mathbf{K}(j) = \mathbf{P}(j)\mathbf{X}(j)[\mathbf{X}^T(j)\mathbf{P}(j)\mathbf{X}(j) + R]^{-1} \tag{19b}$$

$$\mathbf{P}(j+1) = \mathbf{P}(j) - \mathbf{K}(j)\mathbf{X}^T(j)\mathbf{P}(j) \tag{19c}$$

Here, $\hat{\mathbf{W}}$ denotes estimate of the optimal tap-weight vector, $\mathbf{K}$ is the Kalman gain vector, d denotes the pilot signal, $\mathbf{X}$ is the tap-input vector, $\mathbf{P}$ is the predicted state-error correlation matrix, R denotes the variance of the residual error.

In (19), if we consider

$$\alpha(j)\,\hat{\nabla}(j) = \mathbf{K}(j)\left[d(j) - \mathbf{X}^T(j)\hat{\mathbf{W}}(j)\right]$$

then the Kalman filter can be regarded as a gradient algorithm.

Computational requirements of the Kalman filter (19) can be significantly reduced by the use of only diagonal elements of the predicted state-error correlation matrix $\mathbf{P}$ in the iteration process [9], [10].

The i-th diagonal element of $\mathbf{P}$ can be understood as the variance of the i-th weight estimate error (mean of the error is supposed to be zero)

$$p_{i,i} = E\{(\hat{w}_i - w_i)^2\} \tag{20a}$$

where $\hat{w}_i$ is the actual estimate of the i-th optimal weight w_i and E denotes the statistical expectation operator.

Non-diagonal elements of $\mathbf{P}$ contain information about the "cross-variance" of the i-th a j-th optimal weights estimate errors

$$p_{i,j} = E\{(\hat{w}_i - w_i)(\hat{w}_j - w_j)\} \tag{20b}$$

Hence, if the cross-variances are not considered in the iteration process the loss of information negatively influences parameters of the Kalman filter - rate of convergence is lower and misadjustment higher. On the contrary, computational requirements drop from the order of $(M.N)^2$ to the order of $(M.N)$ where $(M.N)$ is the total number of weights. In comparison with the LMS algorithm, number of mathematical operations per iteration is approximately three times higher.

The Simplified Kalman filter (SKF) can be expressed by the following set of equations

$$\hat{w}_i(j+1) = \hat{w}_i(j) + k_i(j)e(j) \tag{21a}$$

$$k_i(j) = \frac{p_i(j)x_i(j)}{\sum_{m=1}^{M.N} p_m(j)x_m^2(j) + R} \tag{21b}$$

$$p_i(j+1) = p_i(j)[1 - k_i(j)x_i(j)] \tag{21c}$$

Here, $\hat{w}_i$ is the i-th component of the optimal weight estimate vector, k_i denotes the i-th component of the Kalman gain vector, e denotes the error signal (difference between the pilot signal and the antenna output one), x_i is the i-th component of the tap-input vector, p_i is the variance of the i-th weight estimate error, R denotes the variance of the residual error and $M.N$ is the number of weights.

In addition to the computer requirements' reduction, the use of only diagonal elements of the predicted state-error correlation matrix **P** solves the problem of the numerical non-stability of the Kalman filter too. All the quantities in the denominator (21b) are non-negative. If the variance of the residual error R is positive then the denominator can never equal zero. On the contrary, the non-zero denominator of the pure Kalman filter (19) cannot be guaranteed. Hence, the pure Kalman filter can oscillate from such a "non-adaptive reason" (see figure 2) whereas the SKF not.

Substituting (21b) to (21a) yields

$$\hat{w}_i(j+1) = \hat{w}_i(j) + \frac{p_i(j)}{\sum p_m x_m^2 + R} x_i(j) e(j) \qquad (22)$$

If the fraction in (22) is considered as the controlled adaptation step $\alpha(j)$ then the SKF can be classified as the step variable LMS algorithm.

Assume that the variance of the residual error

$$R \gg \max_{i=1..M} \left\{ \frac{1}{p_i} \right\} \sum_{i=1}^{M} p_i x_i^2 \qquad (23)$$

then the (22) can be rewritten

$$\hat{w}_i(j+1) = \hat{w}_i(j) + \frac{1}{R} x_i(j) e(j) \qquad (24)$$

that is the classical algorithm LMS. Condition (23) can be considered as a modification of the stability condition of the algorithm LMS (16).

It can be concluded that the simplified Kalman filter converges to the optimal LMS algorithm for very high R. In the opposite case (R is close to zero), the simplified Kalman filter converges to the pure one because the feedback estimation of the optimal Kalman gain vector is not suppressed by the variance of the residual error R. Validity of this idea has been verified by the computer simulations (see figures 3 and 5).

4.2 Steering vector system

The steering vector system can be described by the following state model [6]

$$\mathbf{W}(j+1) = \mathbf{W}(j) \qquad (25a)$$

$$\begin{bmatrix} 0 \\ \mathbf{F} \end{bmatrix} = \begin{bmatrix} \mathbf{X}^T(j) \\ \mathbf{C}^T \end{bmatrix} \mathbf{W}(j) + \begin{bmatrix} v_r(j) \\ \mathbf{V}_c(j) \end{bmatrix} \qquad (25b)$$

where **W** is the column vector of weights, **X** is the tap-input vector, **F** is the weight vector of the joined transversal filter, **C** is the matrix of coefficients that performs joining transversal filters (see equation. 6b) v_r is the residual error defined as

$$v_r(j) = -\mathbf{X}^T(j)\mathbf{W}(j)$$

and $\mathbf{V}_c(j)$ is the column vector of the constraint errors (kth element of the $\mathbf{V}_c(j)$ is defined as the difference between kth weight of the desired joined filter and the actual sum of weigths at kth taps of all transversal filters.

Application of the Kalman filter theory to the state model (25) yields

$$\hat{\mathbf{W}}(j+1) = \hat{\mathbf{W}}(j) + \mathbf{K}(j)\left\{ \begin{bmatrix} 0 \\ \mathbf{F} \end{bmatrix} - \begin{bmatrix} \mathbf{X}^T(j) \\ \mathbf{C}^T \end{bmatrix} \hat{\mathbf{W}}(j) \right\} \qquad (26a)$$

$$\mathbf{K}(j) = \mathbf{P}(j)[\mathbf{X}(j)\ \mathbf{C}] \left\{ \begin{bmatrix} \mathbf{X}^T(j) \\ \mathbf{C}^T \end{bmatrix} \mathbf{P}(j)[\mathbf{X}(j)\ \mathbf{C}] + \mathbf{Q} \right\}^{-1} \qquad (26b)$$

$$\mathbf{P}(j+1) = \mathbf{P}(j) - \mathbf{K}(j) \begin{bmatrix} \mathbf{X}^T(j) \\ \mathbf{C} \end{bmatrix} \mathbf{P}(j) \qquad (26c)$$

Here, $\hat{\mathbf{W}}$ denotes estimate of the optimal tap-weight vector, **K** is the Kalman gain matrix, **X** is the tap-input vector, **F** is the weight vector of the joined transversal filter, **C** is the matrix of coefficients, **P** is the predicted state-error correlation matrix and **Q** is the correlation matrix of $[v_r(j)\ \mathbf{V}_c^T(j)]^T$.

Evidently, computational requirements per iteration of the above algorithm are enormous. Hence, we will use in the iteration process diagonal elements of the predicted state-error correlation matrix only. In this way, the computational requirements of the algorithm will be significantly reduced without any dramatic degradation of the positive properties of the constrained Kalman filter as it has been shown at the pilot signal system.

Taking in mind that iterative search for i-th optimal weight is controlled first by the power of the total output signal and second by the constraint that the sum of kth weights of all filters equals kth weight of the desired joined transversal filter, we can conclude with respect to (26a) that only two elements in each row of the Kalman gain matrix are necessary to be computed. Implementation of this idea involves another decrease of computational requirements.

Finally, computational requirements of the algorithm can be significantly influenced by the method of the matrix inversion in (26b). It can be shown that desired high effectiveness of the inversion computations is guaranteed by the method of algebraic complements [8].

After realizing all the above ideas, we can express the constrained simplified Kalman filter by the following set of equations (for details see [10])

$$\hat{w}_l(j+1) = \hat{w}_l(j) - k_{l,1}(j) \sum_{i=1}^{M.N} \hat{w}_i(j) x_i(j) + k_{l,m+1}(j) \left[f_m - \sum_{n=1}^{N} \hat{w}_l(j) \right]$$

$$\alpha_{1,1}(j) = \frac{1}{\Delta(j)} \qquad (27b)$$

$$m \neq 1 \qquad a_{1,m}(j) = a_{m,1}(j) = -\frac{1}{\Delta(j)} \frac{\sum_{n=1}^{N} p_l(j) x_l(j)}{\sigma_{cm}^2 + \sum_{n=1}^{N} p_l(j)} \qquad (27c)$$

$$m \neq 1 \qquad a_{m,m}(j) = \frac{1}{\left[\sigma_{cm}^2 + \sum\limits_{n=1}^{N} p_l(j)\right]^2} \cdot \left\{ \sigma_{cm}^2 + \sum_{n=1}^{N} p_l(j) + \frac{\left[\sum\limits_{n=1}^{N} p_l(j) x_l(j)\right]^2}{\Delta(j)} \right\}$$

$$\Delta(j) = \left\{ \sum_{i=1}^{MN} p_i(j) x_i^2(j) + \sigma_r^2 - \sum_{m=1}^{M} \frac{\left[\sum\limits_{n-1}^{N} p_l(j) x_l(j)\right]^2}{\sigma_{cm}^2 + \sum\limits_{n=1}^{N} p_l(j)} \right\} \prod_{m=1}^{M} \left[\sigma_{cm}^2 + \sum_{n=1}^{N} p_l(j) \right]$$

$$k_{l,1}(j) = p_l(j)[x_l(j) a_{1,1}(j) + a_{m+1,1}(j)] \qquad (27f)$$

$$k_{l,m+1}(j) = p_l(j)[x_l(j) a_{1,m+1}(j) + a_{m+1,m+1}(j)] \qquad (27g)$$

$$p_l(j+1) = p_l(j)[1 - k_{l,1}(j) x_l(j) - k_{l,m+1}(j)] \qquad (27h)$$

Coefficients of the equations (27) can move in the following intervals:

$m = 1..M$ (M is the number of the transversal filter taps)

$n = 1..N$ (N is the number of antenna elements) and

$l = n + (m-1)N$.

Symbols in (27) denote: $\hat{w}_l$ is the l-th optimal weight estimate, $k_{l,1}$ is the element of the Kalman gain matrix, x_l is the sample at the i-th tap, f_m is m-th weight of the joined transversal filter, $a_{1,m}$ is the element of the inversion matrix, Δ denotes the determinant of the inverted matrix in (26b), p_l is the variance of the l-th weight estimate error, σ_r^2 is the variance of the residual error and σ_{cm}^2 is the variance of the constraint error of the m-th weight of the joined transversal filter.

Since the theoretical analysis of the adaptation stability of the proposed algorithm is rather complicated let us state here only that the simplified filter is not sensitive to the initial values of the weights and variances of weights' errors and that no stability problems occur if the variance of the residual error is positive and variances of the constraint errors are not lower than one [10]. Other adaptation properties of the algorithm will be demonstrated by the computer simulations.

With regard to the numerical stability of the algorithm, the relation (27e) shows that the inverted matrix is singular if and only if

$$\sum_{m=1}^{M} \frac{\left[\sum\limits_{n=1}^{N} p_l(j) x_l(j)\right]^2}{\sigma_{cm}^2 + \sum\limits_{n=1}^{N} p_l(j)} - \sigma_r^2 = \sum_{i=1}^{MN} p_i(j) x_i^2(j) \qquad (28)$$

Hence, the numerical stability of the simplified constrained Kalman filter is not guaranteed but it can be shown that the probability that the inverted matrix is singular is significantly lower in comparison with the pure constrained Kalman filter.

5. Computer Simulations

5.1 Pilot signal system

At the first time, computer simulations of a two-element antenna array (omnidirectional elements spaced the half wavelength) with a four-tap transversal filter at each antenna element output have been performed. Desired signal (white, variance 1) has come from the direction that has been perpendicular to the array aperture. Interference signal (white, variance 50) has arrived from the direction departing 45 degrees from the desired signal direction. Thermal noise of variance 0.5 has been considered.

Figure 2 illustrates the numerical stability problems of the pure Kalman filter (dashed line) when the variance of the residual error is too low ($R = 10^{-4}$) and the short mantissa is used (7 digits). On the contrary, simplified Kalman filter (solid line, $R = 10^{-4}$, 7 digits' mantissa) exhibits no oscillations.

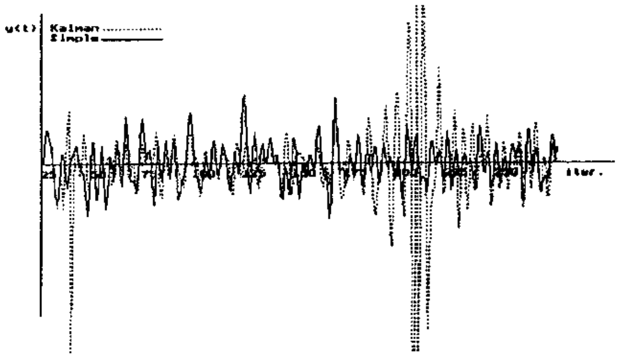

Fig.2

Pilot signal system. Time course of the output signal of the adaptive antenna controlled by the pure Kalman filter (dotted) and the simplified one (solid).

Figure 3 shows the time course of the ensemble-averaged squared error (20 ensembles) of the classical LMS algorithm (learning constant $\alpha = 10^{-6}$) and of the simplified and pure Kalman filters (variance of the residual error $R = 10^{-1}$ for the pure Kalman filter and $R = 10^{-6}$ for the simplified one). Evidently, both the rate of convergence and the misadjustment of the simplified Kalman algorithm are lying between the LMS and Kalman's ones.

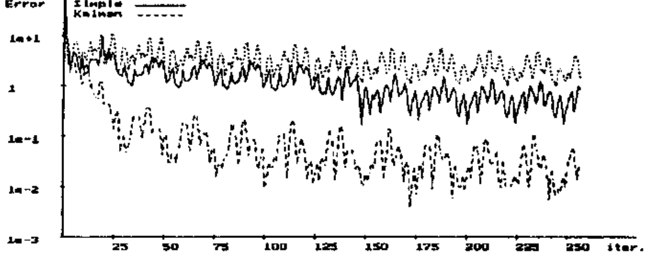

Fig.3

Pilot signal system. Comparison of the time courses of the ensemble-averaged squared error of the adaptive antenna controlled by the LMS algorithm (dotted), pure Kalman filter (dashed) and simplified one (solid).

Figure 4 provides comparison of the variable step size LMS algorithm ($\eta = 0.95$; $\gamma = 10^{-7}$; $\alpha_{max} = 10^{-5}$) and the simplified Kalman filter ($R = 10^{-6}$). It can be concluded that the SKF and VSS LMS have approximately the same rate of convergence. SKF excels in lower misadjustment and independence on a priori information about noise statistics. VSS LMS exhibits lower computational requirements (see table 1). Squared error has been averaged through 20 ensembles.

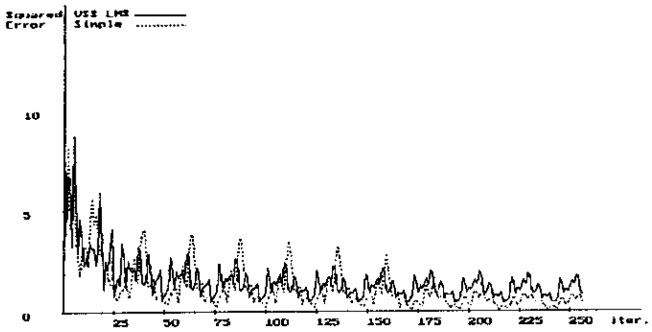

Fig.4
Pilot signal system. Comparison of the time courses of the ensemble-averaged squared error of the adaptive antenna controlled by the variable step size (VSS) LMS algorithm (solid) and the simplified Kalman filter (dotted).

Figure 5 shows the time course of the ensemble-averaged squared error (20 ensembles) of the simplified Kalman's algorithm for $R = 10^{-5}$ and $R = 10^{+5}$. Evidently, no stability problems occur. If R is increased then the level of misadjustment grows and the rate of convergence drops.

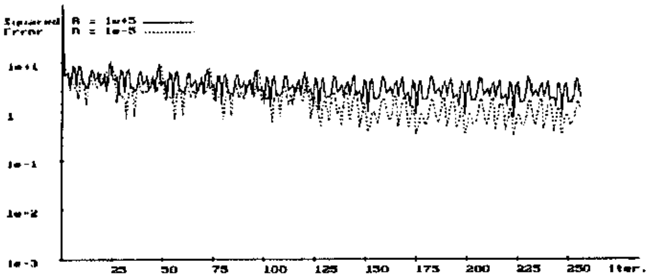

Fig.5
Pilot signal system. Comparison of the time courses of the ensemble-averaged squared error of the adaptive antenna controlled by the simplified Kalman filter with different variances of the residual error: $R = 10^{-5}$ (dotted) and $R = 10^{+5}$ (solid).

Second kind of simulations has been performed with the 8-element antenna array that has been completed by the narrow-band weights (2-tap transversal filters). Desired signal (harmonic, amplitude 1) has come from the main lobe direction, first and second interferences (harmonic, amplitude 50) arrived from directions 22 degrees and 68 degrees. In these directions, local maxims of the directivity pattern of the non-adaptive array appear (figure 6a). After 5 iteration cycles, systems based on the Kalman filter and the SKF show expressive minims in the interferences' directions. On the contrary, the LMS based systems set directivity pattern minims to the desired directions evidently more slowly (figure 6b). Figure 6c shows the directivity pattern in the steady state.

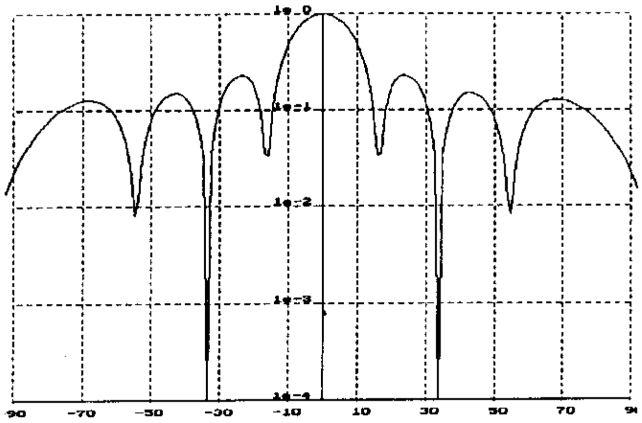

Fig.6a
Directivity pattern of a non-adaptive antenna array consisting of 8 omnidirectional elements spaced half wavelength.

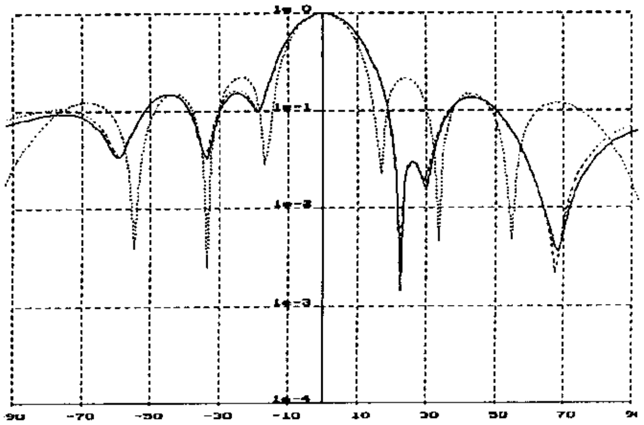

Fig.6b
Pilot signal system. Directivity pattern of the 8-element half-wavelength-spaced antenna array after 5 iteration cycles. Control algorithms: LMS (dotted), pure Kalman filter (dashed) and the simplified Kalman filter (solid).

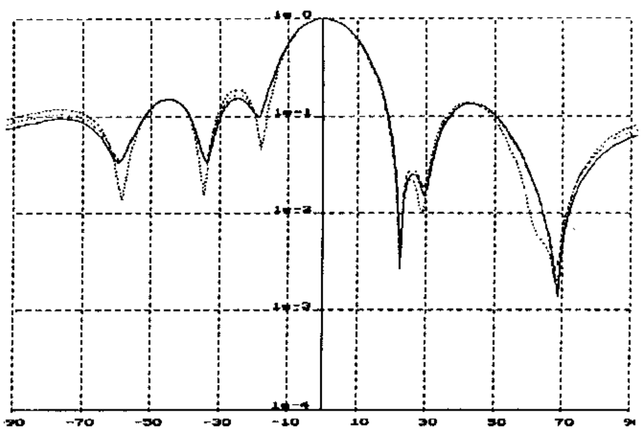

Fig.6c
Pilot signal system. Directivity pattern of the 8-element half-wavelength-spaced antenna array after 50 iteration cycles. Control algorithms: LMS (dotted), pure Kalman filter (dashed) and the simplified Kalman filter (solid).

5.2 Steering vector system

Steering vector system has been tested by the same kind of simulations as the pilot signal one.

Results of the broadband simulations (two-element antenna array with a four-tap transversal filter at each antenna element output; white desired signal with variance 1 arriving from the main lobe direction, white interference signal with variance 50 arriving from the direction 45 degrees) are presented in figures 7 - 9.

Figure 7 shows the time course of the ensemble-averaged squared error (20 ensembles) of the LMS algorithm (learning constant $\alpha = 10^{-5}$) and of the simplified and pure Kalman filters (variances of both the residual and the constraint errors 1 for both filters). Evidently, both the rate of convergence and the misadjustment of the SKF are lying between the LMS algorithm and the pure Kalman filter.

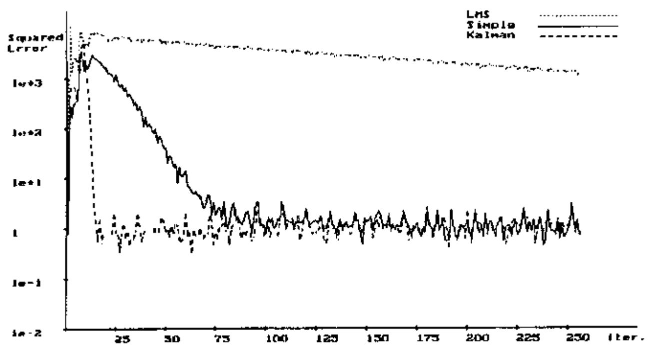

Fig.7
Steering vector system. Comparison of the time courses of the ensemble-averaged squared error of the adaptive antenna controlled by the LMS algorithm (dotted), pure Kalman filter (dashed) and the simplified one (solid).

Figure 8 shows the time courses of the ensemble-averaged squared error (20 ensembles) of the simplified constrained Kalman filter for $\sigma_r^2 = 10^{-5}$ and $\sigma_r^2 = 10^{+5}$ (variances of the constraint error have been all the time set to 1). Evidently, no stability problems occur. If the variance of residual error grows then parameters of the adaptation process become worse.

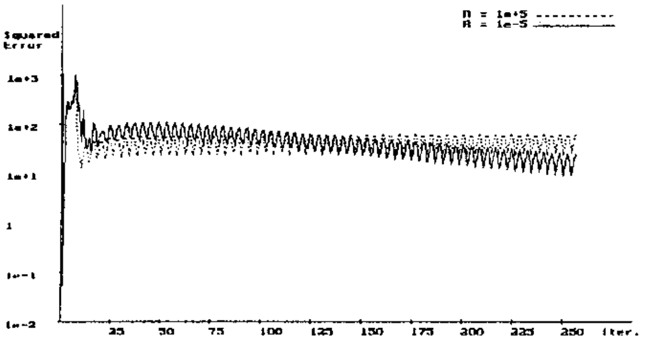

Fig.8
Steering vector system. Comparison of the time courses of the ensemble-averaged squared error of the adaptive antenna controlled by the simplified Kalman filter with different variances of the residual error: $R = 10^{+5}$ (dotted) and $R = 10^{-5}$ (solid).

Figure 9 illustrates the influence of the change of the variances of the constraint errors. It can be seen that the growth of the variances positively influences behavior of the system.

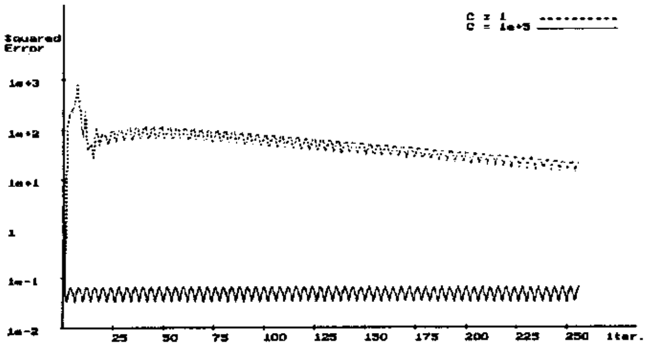

Fig.9
Steering vector system. Comparison of the time courses of the ensemble-averaged squared error of the adaptive antenna controlled by the simplified Kalman filter with different variances of the constraint errors: (dotted) and (solid).

Figure 10 presents result of the narrowband simulation (8-element antenna array completed by the 2-tap transversal filters; desired signal harmonic with amplitude 1 arriving from the main lobe direction, first and second interferences harmonic with amplitude 50 arriving from directions 18 degrees and 68 degrees). It can be seen that LMS has some problems in the direction 68 degree in the steady state. This effect is probably caused by the.relatively high level of misadjustment of LMS algorithm.

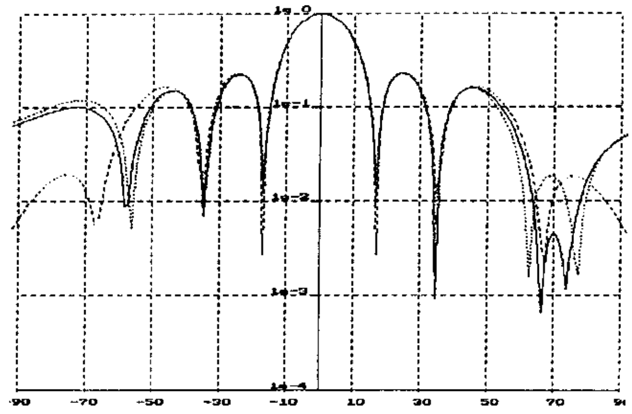

Fig.10
Steering vector system. Directivity pattern of the 8-element half-wavelength-spaced antenna array after 50 iteration cycles. Control algorithms: LMS (dotted), pure Kalman filter (dashed) and simple Kalman filter (solid).

Computational requirements of the presented algorithms have been tested by the Turbo Profiler, version 2.2, Borland International. Measurement results are summarized in the table 1. It can be seen that computational requirements of the simplified Kalman filter are approximately three times higher in comparison with the LMS algorithm in the pilot signal system and nearly the same as the LMS shows at the steering vector system. The pure Kalman filter consumes incomparable longer computation time of the processor and its requirements grow approximately quadratically with the number of weights. On the contrary, computational requirements of the VSS LMS algorithm are very close to the LMS one.

	PILOT SIGNAL		STEERING VE	
algorithm	8 taps	16 taps	8 taps	16 taps
LMS	100	190	899	$3{,}31.10^3$
VSS LMS	110	195	---	---
Kalman	$1{,}03.10^4$	$7{,}02.10^4$	$2{,}54.10^4$	$1{,}74.10^5$
SKF	331	644	853	$1{,}70.10^3$

Tab.1
Computational requirements of the presented algorithms. 8-tap pilot signal system controlled by the classical LMS algorithm is taken as the reference (100%).

6. Conclusion

Presented paper compares algorithms that can be used for the control of adaptive antenna arrays based on the pilot signal method and the steering vector one. The classical LMS, the variable step size LMS, Kalman filter and simplified Kalman filter have been investigated.

It has been shown that the control algorithm based on the pure Kalman filter can be considered as the gradient one and that the simplified Kalman filter behaves as the step variable LMS algorithm.

Computer simulations have proved that the simplified Kalman filter is the most convenient algorithm for digital control of adaptive antennas: SKF excels in relatively high rate of convergence, low misadjustment and high total stability, is not sensitive to the a priori information about noise statistics, and does not require high computational power of signal processors.

The VSS LMS algorithm exhibits similar properties. In addition, computational requirements are approximately on the same level as LMS. Unfortunately, insurance of the adaptation stability of the VSS LMS requires knowledge of a priori information about noise statistics.

7. Acknowledgment

Presented work is based on my Ph.D. dissertation "Stability of digital adaptive antenna arrays". At this place, I would like to thank my supervisor Prof.Ing. Dušan Černohorský, CSc. for his encouragement, help and care he has devoted me during my doctoral study. I am very obliged to him to all his useful suggestions that has helped me to work up my Ph.D. dissertation, this paper and other my works.

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